Gemini Topic Test
FAMAT State Convention 2002

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. Simplify the following: $(1-i)^{2002}$

   A. 2^{1001} B. -2^{1001} C. $2^{1001}i$ D. $-2^{1001}i$ E. NOTA

2. What is the probability that in a room of 9 people, at least 2 people have the same birthday? (Round your answer to the nearest hundredth)

   A. 0.00 B. 0.22 C. 0.09 D. 0.10 E. NOTA

3. Which number is nearest to $100-\sqrt{9999}$?

   A. 0.0025 B. 0.005 C. 0.009 D. 0.01 E. NOTA

4. Aswath and Jerry are trying to paint the living room of the apartment they live in. Unfortunately, the only help they can find is their friend Dave, who is very uncoordinated and very prone to distractions, hence counter productive. Dave could ruin the entire paint job in exactly 12 hours, and so, painting the room takes 2 hours longer, since he is "helping." How many hours would it have taken Aswath and Jerry to paint the room without the aid of their good friend Dave?

   A. 1 B. 2 C. 4 D. 6 E. NOTA

5. The coefficient of $x^6y^3z^4$ in $(3x+2y+5z)^{13}$ is:

   A. $\binom{13}{6\ 3\ 3}$ C. $\binom{13}{6\ 3\ 3}\cdot 3^6\cdot 2^3\cdot 5^4$ E. NOTA

   B. $\binom{13}{6}\cdot\binom{13}{3}\cdot\binom{13}{4}$ D. $\binom{13}{6}\cdot\binom{13}{3}\cdot\binom{13}{4}\cdot 3^6\cdot 2^3\cdot 5^4$

6. $\sqrt{19+\sqrt{19+\sqrt{19+\sqrt{19+\ldots}}}}=\dfrac{A+\sqrt{B}}{C}$. A and C are integers with no common factors greater than one. What is the value of $\dfrac{B}{2AC}$?

   A. $\frac{77}{4}$ B. $\frac{99}{4}$ C. $\frac{101}{4}$ D. $\frac{111}{4}$ E. NOTA

7. Find $\dfrac{2AC}{B}$, if

   A = the number of different ways that 6 ogre's and 2 donkeys can be arranged in a straight line so that the 2 donkeys are not next to each other.
   B = the number of possible arrangements of 9 keys on a key ring.
   C = the number of sides in a regular polygon with 90 diagonals.

   A. $\frac{45}{2}$ B. 45 C. $\frac{135}{2}$ D. 135 E. NOTA

8. Let A be a square n by n matrix. In the statements below, one statement (if there is any) needs to be changed such that after it is changed, all the statements become true of the Invertible Matrix Theorem. Which statement needs to be changed?

1. A is an invertible matrix.
2. A is row equivalent to the n by n identity matrix.
3. The columns of A span $\Re^n$
4. The columns of A form a linearly dependent set.

A. 1 B. 2 C. 3 D. 4 E. NOTA

9. The arithmetic mean of x and y is 4, while the geometric mean of the two numbers is 5. Find the sum of the squares of x and y.

A. 5 B. 7 C. 14 D. 28 E. NOTA

10. How many digits does the base ten expansion of 2^{2001} have?

A. 32 B. 64 C. 128 D. 603 E. NOTA

11. Find the eccentricity of $9x^2 + 4y^2 + 36x - 24y + 36 = 0$.

A. $\frac{\sqrt{5}}{3}$ B. $\frac{\sqrt{5}}{2}$ C. $\frac{5}{9}$ D. $\frac{5}{4}$ E. NOTA

12. Which of the statements regarding points of concurrency must lie on the interior of a triangle?

1. circumcenter
2. centroid
3. incenter
4. orthocenter

A. 2 only
B. 1 and 2 only
C. 2 and 3 only
D. 2, 3 and 4 only
E. NOTA

13. Which of the following best describes the graph of $r = \frac{3}{4 + 5\cos\theta}$?

A. circle B. ellipse C. parabola D. hyperbola E. NOTA

14. Rationalize $\frac{11}{\sqrt[3]{4} + \sqrt[3]{6} + \sqrt[3]{9}}$ into the form $A\sqrt[3]{B} + C\sqrt[3]{D}$ where B and D are the smallest integers possible. Compute the value of $AB - CD$.

A. 0 B. 10 C. 11 D. 12 E. NOTA

15. Which density lets X = the number of failures before the α^{th} success?
A. Binomial C. Hypergeometric E. NOTA
B. Geometric D. Poisson

16. How many petals does the graph $r = 6\cos 4\theta$ have if $0 < \theta < 2\pi$?
A. 6 B. 8 C. 10 D. 12 E. NOTA

17. How many different squares can be formed by connecting 4 vertices chosen from among lattice points (x, y) with $0 \le x, y \le 4$?
A. 16 B. 30 C. 50 D. 74 E. NOTA

18. How many eigenvalues exist, if there are any, when $A = \begin{bmatrix} 2 & 3 \\ 3 & -6 \end{bmatrix}$?
A. 1 B. 2 C. 3 D. 4 E. NOTA

19. For all positive integers $n \ge 3$, which of the following cannot be a value of $\frac{n!}{(n-3)!}$?
A. 124,251,000 C. 214,921,200 E. NOTA
B. 90,518,400 D. 165,468,600

20. Find the thousandths digit of the distance between the functions $5x + 6y = 8$ and $5x + 6y = 11$.
A. 1 B. 2 C. 3 D. 4 E. NOTA

21. For x and y real numbers, how many times do the graphs $y = \log_2 x$ and $x = \log_5 y$ intersect?
A. 0 B. 1 C. 2 D. 3 E. NOTA

22. A plane travels from Grenelefe to Oklahoma City at a speed of 350 miles per hour. That same plane takes the same route and returns to Grenelefe, this time at a speed of 400 miles per hour. Finally, it flies back to Oklahoma City one last time, on the same route, this time at a speed of 375 miles per hour. What is the average speed of the plane over the three segments to the nearest tenth mph?
A. 372.9 B. 373.9 C. 375.0 D. 378.4 E. NOTA

23. The sum $\sum_{k=0}^{2n} \binom{2n}{k}^2$ equals, for $n \ge 1$:
A. $\binom{2n}{n}$ B. $\binom{2n}{2n}$ C. $\binom{4n}{2}$ D. $\binom{4n}{2n}$ E. NOTA

24. How many zeros are there at the end of 100! ?

A. 10 B. 20 C. 24 D. 35 E. NOTA

25. A ball is dropped from a height of x ($x \geq 0$) and bounces half its previous height on each bounce. The total distance traveled by the ball is equal to the product of the first 7 numbers in the Fibonacci sequence. From what height (x)did the ball fall?

A. 1040 B. 2080 C. 3120 D. 4160 E. NOTA

26. Find the product of all the real solutions of the equation $\frac{60x-10x^2-50}{x^2-5x} = x^2-8x+7$

A. 0 B. 2 C. 5 D. 10 E. NOTA

27. Which statement is logically equivalent to "if $x \neq 3$ then $y > 7$ "?

A. if $x > 7$ then $y \neq 3$ C. if $x = 3$ then $y = 7$ E. NOTA
B. if $y > 7$ then $x \neq 3$ D. if $y \leq 7$ then $x = 3$

28. Find the logical expression that is equivalent to the sentence "Some trucks crash into other trucks," if
T(x) denotes "x is a truck"
V(x) denotes "x is a vehicle"
C(x,y) denotes "x crashes into y"

Note: x and y can denote any object from the replacement set, where the replacement set of n elements has $\{C(n,2)+n\}$ subsets of two elements.

A. $\exists x \exists y \left((T(x) \wedge T(y)) \rightarrow C(x,y) \right)$ D. $\exists x \exists y \left(T(x) \wedge T(y) \wedge C(x,y) \right)$
B. $\exists y \exists x \left(C(x,y) \rightarrow (T(x) \wedge T(y)) \right)$ E. NOTA
C. $\exists y \exists x \, C(x,y)$

29. On a train, Huey, Dewey and Louie are the fireman, brakeman, and engineer, but not respectively. Also In addition to the three previous mentioned men, aboard the train are three businessmen who have the names: Mr. Huey, Mr. Dewey, and Mr. Louie. You are also given these facts:
-Mr. Dewey lives in Detroit.
-The brakeman lives exactly half way between Chicago and Detroit.
-Mr. Louie earns exactly $20,000 per year.
-The brakeman's nearest neighbor, one of the passengers, earns exactly 3 times as much as the brakeman.
-Huey beats the fireman at badminton.
-The passenger whose name is the same as the brakeman's lives in Chicago.

Based on the facts above, who is the engineer? (Every fact is important, and must be considered)
A. Huey B. Dewey C. Louie D. Mr. Dewey E. NOTA

30. Use the answers you obtained in questions 1-29 to answer this question.

Let R = the number of A's
Let S = the number of B's
Let T = the number of C's
Let U = the number of D's
Let V = the number of E's

Compute $\frac{{}_{S}C_{V}}{{}_{R}C_{U}} \cdot T$

A. 10 B. 20 C. 30 D. 40. E. NOTA