FAMAT State Convention 2002

Note: "NOTA" means "none of these answers."

1. What is the y-intercept of the line which contains the points $\left(\frac{3}{4}, 2\right)$ and $\left(\frac{1}{8}, -\frac{1}{2}\right)$?

(A) $-\frac{1}{4}$ (B) -1 (C) $-\frac{5}{4}$ (D) $-\frac{3}{4}$ (E) NOTA

2. If a relation is a "constant function," then it must be linear and ___?___.

(A) have a slope of zero (B) a direct variation (C) contain the origin (D) have an undefined slope (E) NOTA

3. Suppose A varies directly as r^2 and A is π when $r = \frac{1}{2}$. What is the value of A when r is 5?

(A) 40π (B) $\frac{\pi}{100}$ (C) 100π (D) $\frac{\pi}{40}$ (E) NOTA

4. Suppose $f: x \to \frac{2x-1}{3}$. Find the value of $f(-3)$.

(A) $-6\frac{1}{3}$ (B) $1\frac{2}{3}$ (C) $5\frac{2}{3}$ (D) $-2\frac{1}{3}$ (E) NOTA

5. Given: $\left(a, \frac{a}{2}\right) \in \{(x,y) : y = ax^2\}$ Find the value(s) of **a**.

I. $\frac{\sqrt{2}}{2}$ **II.** $\frac{-\sqrt{2}}{2}$ **III.** 0

(A) I only (B) I & II only (C) III only (D) I, II, & III (E) NOTA

6. If $f(x) = 3x$ and $g(x) = x^2 - 1$, then find $g\left(f\left(\frac{-1}{a}\right)\right)$. (note: $a \neq 0$)

(A) $\frac{9-a^2}{a^2}$ (B) 9 (C) $\frac{3-3a^2}{a^2}$ (D) 0 (E) NOTA

7. If f is a function such that $f(x) = -x^2 + 1$ for all $x \in \Re$, then $f(2-h) - f(h)$ is equal to ___?___.

(A) $-4h^2 + 8h - 3$ (B) $4h - 2$ (C) $2h^2 + 4h - 4$ (D) $4h - 4$ (E) NOTA

8. Suppose f is a function such that $f(x) = \mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$, where $\mathbf{a} \neq 0$. The graph of the function contains the points (1, 8) and (0, 17). The axis of symmetry is $x = 2$. Find the value of **c**.

(A) -12 (B) 17 (C) 5 (D) 3 (E) NOTA

9. The publisher of a magazine that has a circulation of 80,000 and sells for \$1.60 a copy decides to raise the price of the magazine because of increased production costs. By surveying the readers of the magazine, the publisher finds that the magazine will lose 10,000 readers for each \$0.40 increase in price. What price per copy maximizes the revenue?

(A) \$2.40 (B) \$3.00 (C) \$3.20 (D) \$4.80 (E) NOTA

10. For which of the following functions does $f(a+b) = f(a) + f(b)$?

(A) $f(x) = x^2$ (B) $f(x) = \frac{1}{x}$ (C) $f(x) = 4x + 1$ (D) $f(x) = 3x$ (E) NOTA

11. Suppose **w** varies inversely as the square of **z** and **z** varies directly to the cube of **x**. What is the relationship between **w** and **x**?
(A) **w** is directly proportional to the sixth root of **x**
(B) **w** is inversely proportional to the sixth root of **x**
(C) **w** is directly proportional to the sixth power of **x**
(D) **w** is inversely proportional to the sixth power of **x**
(E) NOTA

12. The cost of parking a car in a municipal parking lot is \$3 for the first hour or any part thereof, plus \$2 for each additional hour or part thereof. Find a rule for the cost **C** as a function of time *t*.
(note: [t] represents the "**greatest integer function**" and $\lceil t \rceil$ represents the "**rounding-up function**")
(A) $\mathbf{C}(t) = 3 + 2\lceil t - 1 \rceil;\ t > 0$ (B) $\mathbf{C}(t) = 3 + 2[t - 1];\ t > 0$
(C) $\mathbf{C}(t) = 3 + 2\lceil t \rceil;\ t > 0$ (D) $\mathbf{C}(t) = 3 + 2[t];\ t > 0$ (E) NOTA

13. Find an integer **c** such that the function $f(x) = 4x^3 + \mathbf{c}x - 27$ has a double root.
(A) 27 (B) −27 (C) −36 (D) 9 (E) NOTA

14. The function $h(\mathbf{x}) = -16\mathbf{x}^2 + v_0\mathbf{x} + s_0$ models the height of an object tossed upward where v_0 represents the initial velocity, s_0 represents the initial height, and **x** represents time. A golf ball is hit upward from ground level with an initial velocity of 72ft/sec. Find the maximum height that the ball reaches.
(A) 79 ft. (B) 80 ft. (C) 81 ft. (D) 82 ft. (E) NOTA

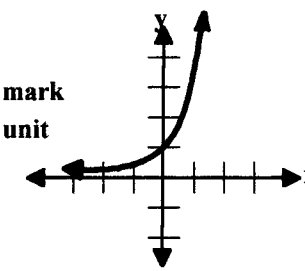
15. If **f** is a function such that $\mathbf{f}(x) = \log_2 x$, then $\mathbf{f}(2x) =$ ___?___ .
(A) $2\mathbf{f}(x)$ (B) $\mathbf{f}^2(x)$ (C) $\mathbf{f}(x+1)$ (D) $\mathbf{f}(x) + 1$ (E) NOTA

16. If $\mathbf{f}(x) = \frac{x}{x+1}$ where $x \neq -1$, then find $\mathbf{f}^{-1}(x)$ where $\mathbf{f}^{-1}$ denotes the inverse function of **f**.

(A) $\frac{x+1}{x}$; $x \neq 0$ (B) $\frac{x}{x+1}$; $x \neq -1$ (C) $\frac{x}{1-x}$; $x \neq 1$ (D) $\frac{x}{x-1}$; $x \neq 1$ (E) NOTA

17. If $g(x) = g(x-2) + 3x$ and $g(3) = -5$, then find g(9).
(A) 91 (B) 58 (C) 31 (D) 10 (E) NOTA

18. Which equation best describes the graph shown to the right? **note: each tick mark represents one unit**
(A) $y = \log(x)$ (B) $x = \ln(y)$
(C) $y = \left(\frac{1}{2}\right)^{-x}$ (D) $x = \log(y)$ (E) NOTA



19. If $f(\mathbf{x}) = 2^{\mathbf{x}}$, then which expression is equal to $f(\mathbf{x} + 1)$?
(A) $2^{\mathbf{x}} + 2$ (B) $3^{\mathbf{x}}$ (C) $2 + f(\mathbf{x})$ (D) $2 \bullet f(\mathbf{x})$ (E) NOTA

20. Suppose $f(\mathbf{x})$ is positive for all real **x** and $f(\mathbf{x} + \mathbf{y}) = f(\mathbf{x}) \bullet f(\mathbf{y})$ for all **x** and **y**. Find an equation for a function that has the property that $f(\mathbf{x} + \mathbf{y}) = f(\mathbf{x}) \bullet f(\mathbf{y})$ for all real **x** and **y**.
(A) $f(\mathbf{x}) = 2^{\mathbf{x}}$ (B) $f(\mathbf{x}) = 2\mathbf{x}$ (C) $f(\mathbf{x}) = \mathbf{x}^2$ (D) $f(\mathbf{x}) = \frac{2}{\mathbf{x}}$; $\mathbf{x} \neq 0$ (E) NOTA

21. A polynomial function f(x) of degree four has roots at -4, 0, 2, and 4. The function $g(x) = f(x) + K$ has only one root, at $x = a$. What is the minimum value of f(x)? (note: $K > 0$)
(A) K (B) $-K$ (C) a (D) $-a$ (E) NOTA

22. The table contains some ordered pairs of a function. The relationship is best described as _?_.

x	-2	-1	0	1	2
y	-5.5	1.5	2.5	3.5	10.5

(A) linear (B) quadratic (C) cubic (D) exponential (E) NOTA

23. Suppose the amount of money that you have accumulated in a bank account is a function of time and the account pays an 8% annual interest, compounded monthly. Also assume that you make only one deposit into this account. How much money must you initially deposit into this account in order to have a balance of \$1000 after one year? **(Assume the bank will round to the nearest penny at the end of one year.)**
(A) \$923.07 (B) \$923.12 (C) \$923.36 (D) \$925.93 (E) NOTA

24. A piece of cardboard six inches by ten inches is made into an open box (with no top) by cutting out squares (with sides of length x) from each corner. Which expression represents V(x), where V(x) is equal to the volume of the box?
(A) $x^3 - 16x^2 + 60x$ (B) $2x^3 - 22x^2 + 60x$ (C) $2x^3 - 26x^2 + 60x$ (D) $4x^3 - 32x^2 + 60x$ (E) NOTA

25. Suppose $f(x) = \frac{2x^2 - x - 6}{x^2 - 4}$. Find the equation(s) of the asymptotes for the graph of f.
I. $x = -2$ **II.** $x = 2$ **III.** $y = 2$
(A) I only (B) I & II only (C) I & III only (D) I, II & III (E) NOTA

26. The graphs of $f(x) = \mathbf{a}x + \mathbf{b}$ and $g(x) = \mathbf{b}x - \mathbf{a}$ are perpendicular lines. If $\frac{g(1)}{f(0)} = 10$, then what is the value of **a** ?
(A) 3 (B) ± 3 (C) $\frac{1}{3}$ (D) $\pm\frac{1}{3}$ (E) NOTA

27. Suppose that $\boldsymbol{h}$ is an exponential function of the form $\boldsymbol{h}(x) = a^x$ and $\boldsymbol{h}(3) = 5$. What is the value of $\boldsymbol{h}(-6)$?
(A) -25 (B) -10 (C) $\frac{1}{25}$ (D) $\frac{1}{10}$ (E) NOTA

28. If $g(x) = x^2 - 5$ and $f(x) = 4x$, then find the sum of the ordinates for the intersection points of f and g.
(A) 16 (B) 4 (C) 20 (D) -1 (E) NOTA

29. A function is said to be "*even*" if $f(x) = f(-x)$. Which of the following are **not** even functions?
I. $f(x) = -|x^3|$ **II.** $f(x) = -x^4 + 2x^2 + 13$ **III.** $f(x) = 3x$ **IV.** $f(x) = 2x^3$
(A) I & II (B) III & IV (C) II, III, & IV (D) I & IV (E) NOTA

30. Consider a function $\boldsymbol{P}$ where $\boldsymbol{P}(x) = \frac{1}{6}x^2 - \frac{4}{3}x + \frac{2}{3}$. Now, find the equation of the line which contains the focus of the graph of $\boldsymbol{P}$ and the y-intercept of the graph of $\boldsymbol{P}$.
(A) $7x + 24y = 16$ (B) $2x + 3y = 2$ (C) $x + 24y = 16$ (D) $x - 6y = -4$ (E) NOTA