Logs & Exponents

FAMAT State Convention - 2002

For all questions, answer E. "NOTA" means none of the above answers is correct.

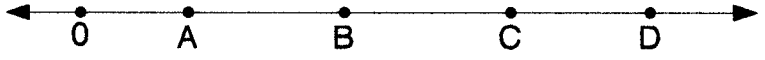
1. Find a decimal approximation correct to four decimal places for $\log_{21} 42$.

   A. 0.8146 B. 1.2277 C. 1.6232 D. 3.7377 E. NOTA

2. Given: $15 \times 2.1^{\frac{x}{3}} = 74.6$, solve for x and give your answer correct to four decimal places.

   A. 3.6428 B. 3.7497 C. 6.4861 D. 16.5283 E. NOTA

3. On the given number line, $A = \log_b 8$, $B = \log_b 9$, $C = \log_b 800$, & $D = \log_b 900$ where b is an acceptable constant for the base of a logarithm. In relation to the lengths of segments AB and CD, which of the following must be true?



Note: Figure not to scale

   A. $|AB| < |CD|$ B. $|AB| = |CD|$ C. $|AB| > |CD|$
   D. It is not possible to determine relative size from the information given. E. NOTA

4. Banks that compound interest use an exponential function to calculate the amount of money in an account at any time. Suppose that you deposited \$542 in a bank account and one year later with no additional deposits the balance had grown to \$580. To the nearest penny, how much would be in the account after ten years assuming the bank pays compound interest at the same rate over the ten years?

   A. \$922.00 B. \$1066.20 C. \$1067.30 D. \$1142.13 E. NOTA

5. Simplify : 2001^{2001}. Give your answer exactly or in scientific notation to four significant digits.

   A. ∞ B. 0 C. 2.002×10^{2004} D. 6.243×10^{6605} E. NOTA

6. If $\log_a 2 = x$, $\log_a 3 = y$, & $\log_a 5 = z$, write $\log_a \left[\left(\frac{9000}{a^2} \right)^3 \right]$ as an expression in x, y, & z.

   A. $3(x+y+z-2)$ B. $3(3x+2y+2z-2)$ C. $(3x+2y+3z-2)^3$
   D. $3(3x+2y+3z)-1^2$ E. NOTA

7. If $\log_b t = 1.44$ give an exact numerical value for $\log_b \sqrt[4]{t^3}$.

A. 1.08 B. 1.2 C. 1.314534138 D. 1.92 E. NOTA

8. Simplify : $2^{10000} + 2^{10000}$

A. 4^{10000} B. 2^{20000} C. 2^{10001} D. 4^{10001} E. NOTA

9. Simplify : 1999^{-1999}. Give your answer exactly or in scientific notation to four significant digits.

A. ∞ B. 0 C. 4.734×10^{-6599} D. 1.999×10^{-2002} E. NOTA

10. Radioactive elements decay exponentially. If 12.4 grams of a radioactive element decays to 11.1 grams in 3 days and 2 hours, what is the half-life (amount of time for half of the element to decay) of the particular radioactive element? Give your answer correct to four significant figures.

A. 6.259 hours B. 463.1 hours C. 463.2 hours D. 463.2 days E. NOTA

11. An exponential function's graph is given. If the function's equation is $f(x) = a \times b^x$, then which of the following must be true?

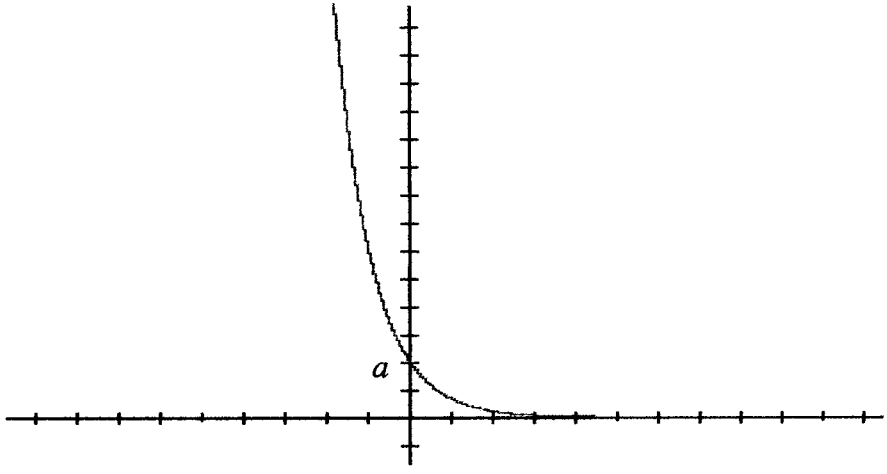
A. $b < -1$

B. $-1 < b < 0$

C. $0 < b < 1$

D. $1 < b$

E. NOTA



12. When a hot liquid is placed in a cool place, the difference between the temperature of the hot liquid and the ambient temperature is an exponential function of time. A cup of coffee is poured and placed in a room with an ambient temperature of 29.0°C. Three minutes later the coffee is measured to be 83.0°C, after an additional 9 minutes, the temperature of the coffee is measured to be 54.5°C. To the nearest tenth of a degree, how hot was the coffee when it was poured?

A. 99.9°C B. 98.3°C C. 89.7°C D. 70.4°C E. NOTA

13. If $\log_b a = 5$, then $\log_a 10 = \ldots$

A. $\sqrt{2}$ B. $5\log_a 10$ C. $.2\log_a 10$ D. $.4\log_b \sqrt{10}$ E. NOTA

14. Given: $\sum_{i=1}^{n}\left(3\times.5^{i}\right)=1.5\times\frac{\left(1-.5^{n}\right)}{.5}$. If $\sum_{i=1}^{n}\left(3\times.5^{i}\right)=2.994140625$ what is n?

A. 9 B. 10 C. 11 D. 12 E. NOTA

15. What exponent (correct to four decimal places) on 21 gives 42?

A. 0.8146 B. 1.2277 C. 1.6232 D. 3.7377 E. NOTA

16. Solve for x. $\log_{12} x = 4 \Rightarrow x = \ldots$

A. $\frac{1}{3}$ B. 3 C. 48 D. 20736 E. NOTA

17. Simplify: $\log_{a^3}\sqrt{a}$

A. $\frac{2a}{3}$ B. $\frac{a}{3}$ C. $\frac{a}{6}$ D. $\frac{1}{6}$ E. NOTA

18. Solve for x. $\log_2(x-2)+\log_2(x-6)=5$

A. $x\in\{10\}$ B. $x\in\{-2,10\}$ C. $x\in\{6\}$ D. $x\in\{2,6\}$ E. NOTA

19. How many digits does 65432^{9876} have?

A. 9876 B. 9877 C. 9880 D. 47561 E. NOTA

20. Given a set X={□, ◰, ◂, ▸, ▲, ▼} with operation ⊗ defined by the following chart. For example, ◰ ⊗ ▸ = □. Further, let the operation q2p(a) for any element in set X be defined as $a \otimes a$. By extension, q3p(a) would be $(a \otimes a) \otimes a$. Find q987p(◂).

⊗	□	◰	◂	▸	▲	▼
□	□	◰	◂	▸	▲	▼
◰	◰	▸	▼	□	◂	▲
◂	◂	▼	◰	▲	□	▸
▸	▸	□	▲	◰	▼	◂
▲	▲	◂	□	▼	▸	◰
▼	▼	▲	▸	◂	◰	□

A. ◰ B. ▼ C. ▸
D. □ E. NOTA

21. To the nearest penny, how much money would be on deposit in ten years if $50.00 were left in an account that pays 5.25% annual interest compounded quarterly?

A. $126.25 B. $84.23 C. $83.40 D. $76.25 E. NOTA

22. Addition : Coefficient :: Multiplication : [redacted]

A. Subtraction B. Division C. Exponent D. Index E. NOTA

23. At the time of the writing of this test, $2^{6972593}-1$ was the largest known prime number. It is a prime number known as a Mersenne prime. Every time a Mersenne prime is found we also find a perfect number. All known perfect numbers can be written in the form $\left(2^{n}-1\right)\left(2^{n-1}\right)$ where $\left(2^{n}-1\right)$ is a prime number. How many digits would this largest perfect number have? Give your answer exactly or in scientific notation to four significant digits.

A. 13945186 B. 4197918 C. 4197919 D. 9.552×10^{4197918} E. NOTA

24. Solve for x. $21^x = 42 \Rightarrow x = \ldots$

A. 0.8146 B. 1.2277 C. 1.6232 D. 3.7377 E. NOTA

25. Solve for x. $\log_7(9-x) + \log_7(8-x) = \log_7 2$

A. $x \in \{7\}$ B. $x \in \{10\}$ C. $x \in \{7,10\}$ D. $x \in \{8,9\}$ E. NOTA

26. What would you multiply $\log_{10} \pi$ by to convert it to $\ln \pi$?

A. $\log_\pi 10$ B. $\ln \pi$ C. $\ln 10$ D. $\log_{10} e$ E. NOTA

27. Log(100!) must be...

A. < 100 B. between 100 & 150
C. between 150 & 200 D. between 200 & 250 E. NOTA

28. The formula for the fixed payment required to retire a loan in a specific period of time is:

$$P = B\frac{\frac{i}{n}}{\left(1-\left(1+\frac{i}{n}\right)^{(-n\times t)}\right)}$$

Where B represents the amount borrowed, i represents the annual interest rate expressed as a decimal, n represents the number of payments made per year, and t represents the number of years payments will be made. To the nearest dollar, what fixed payment must be made each month to retire a debt of \$30,000 in five years if the interest rate is 8%?

A. \$1586 B. \$608 C. \$540 D. \$500 E. NOTA

29. In the change of base formula, $\log_b x = \dfrac{\log_c x}{\log_? b}$, what variable belongs where ? is?

A. a B. b C. c D. x E. NOTA

30. Simplify: $7^{\log_{49} 343}$.

A. $7^{\frac{2}{3}}$ B. $7^{\frac{3}{2}}$ C. $\sqrt[3]{49}$ D. 18.52 E. NOTA