ALPHA GEMINI TEST

Note: For each problem, where there is no choice (e), assume (e) none of the above. Please bubble in one student code in the appropriate place on the answer sheet. If you have a partner, write his/her code on the answer sheet just inside the square for name.

1. State the domain of the function: $f(x) = \dfrac{x-5}{x+2}$

$(a)\ \{all\ \Re\ except\ 5\}$ $(b)\ \{all\ \Re\ except\ -5\}$ $(c)\ \{all\ \Re\ except\ 2\}$ $(d)\ \{all\ \Re\ except\ -2\}$

2. Simplify to a ***single*** trig function. $\left(\cos x \tan x\right)^2 - \dfrac{\sin^2 x}{\sec^2 x}$

(a) $\cos^6 x$ (b) $\cot^2 x$ (c) $\cos^2 x$ (d) $\sin^4 x$

3. Given $f(x) = \dfrac{x^2-7x+12}{x^3-4x^2-x+4}$. $f(x)$ has vertical asymptotes at x =

(a) -1 and 1 (b) -1, 1 and 4 (c) 1 (d) - 1 (e) 4

4. Solve for x: $\ln(2-x) + \ln x = 0$ (a) -1 (b) 1 (c) e (d) $\dfrac{1}{e}$ (e) no solution

5. According to the Rational Root Theorem, which of the following is NOT a possible rational root for $6x^3 + 5x^2 - 73x + 12 = 0$?

(a) 12 (b) $\dfrac{1}{2}$ (c) $\dfrac{2}{3}$ (d) $\dfrac{3}{4}$ (e) $-\dfrac{1}{6}$

6. Solve over $\Re$. Express solution using interval notation. $x^2 - x - 6 \le 0$

(a) [- 5, - 1] (b) [- 2, 3] (c) $(-\infty,-2] \cup [3,\infty)$ (d) $(-\infty,-5] \cup [-1,\infty)$

7. A snap together cube has a protruding snap on one side and receptacle holes on the other 5 sides. What is the smallest number of these cubes that can be snapped together so that only receptacle holes are showing? (a) 3 (b) 4 (c) 5 (d) 6 (e) 8

8. If $f(x) = \dfrac{3x+6}{x+3}$, then find $f(x+h) - f(x)$

(a) 3 (b) 2h (c) $\dfrac{3h}{(x+3)(x+h+3)}$ (d) $\dfrac{6xh(x+1)}{(x+3)^2}$

9. Evaluate: $\log_2 16 + 2\log_3 9 - \log_{\frac{1}{5}} 25$

(a) 8 (b) 10 (c) 12 (d) 16 (e) not possible without a calculator

10. Which one of the following does not belong, since it corresponds to a different unit circle angle?

(a) $-\frac{\pi}{4}$ (b) $\frac{15\pi}{4}$ (c) 315° (d) -225° (e) -405°

11. If $\cos\theta = -\frac{3}{7}$ and $\sin\theta < 0$, then $\cot\theta =$

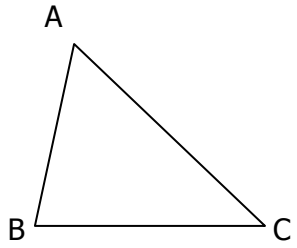
(a) $-\frac{2\sqrt{10}}{7}$ (b) $\frac{3\sqrt{10}}{20}$ (c) $-\frac{7}{3}$ (d) $\frac{2\sqrt{10}}{3}$ (e) $\frac{3}{4}$

12. Given that E is an acute angle with $\tan(E+42^\circ) = \cot E$, find E.

(a) 42° (b) 24° (c) 48° (d) 138° (e) 69°

13. In $\triangle ABC$, $m\angle A = 60^\circ, m\angle B = 45^\circ$ and BC = 24.
Find length of side AC.

(a) 24 (b) $8\sqrt{6}$ (c) $24\sqrt{2}$ (d) $96\sqrt{3}$



14. Find the area of a triangle whose sides are 2, 3 and 4.

(a) $8\sqrt{3}$ (b) $12\sqrt{30}$ (c) $3\sqrt{15}$ (d) $\frac{3\sqrt{15}}{4}$

15. Solve over $[0, 2\pi)$ for x: $2\sin x\cos x + \cos x = 0$

(a) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$ (b) $\frac{\pi}{2}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$ (c) $\frac{5\pi}{6}, \frac{11\pi}{6}$ (d) $0, \pi$

16. The polar coordinates of the ordered pair $(\sqrt{3}, 1)$ would be: (a) $(2, 30^\circ)$ (b) $\sqrt{3}x + y$

(c) $2(60^\circ, 120^\circ)$ (d) $2(\cos 30^\circ + i\sin 30^\circ)$

17. An angle of 30° of elevation is made with the string of a flying kite and Pablo's hand which is approximately 1m above the ground and parallel to the ground. If he uses 30 m of string to fly his kite, how high above the ground is the kite?

(a) $31\sqrt{3}$ m (b) 16 m (c) 15.5 m (d) 62 m (e) not possible without a calculator

18. A rope is tied tight around the equator. A second rope is placed 1 foot directly above the first at each point. About how much longer is the 2nd rope than the 1st? (ans. in ft)

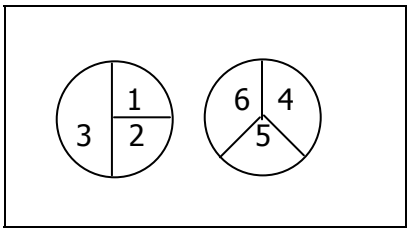
a) 6 b) 60 c) 6000 d) 60000 e) 6 million

19. When Armish started out on his trip his odometer read 45973 and his fuel gauge read $\frac{7}{8}$ full. Exactly one hour and 48 minutes later, the odometer read 46081 and the fuel gauge read $\frac{1}{2}$ full. If Armish needs to continue on his trip and doesn't have time to fill his car with gas, how far can he travel, assuming the same constant speed?
a) 36 b) 72 c) 144 d) 288 (answers in miles)

20. A function from the integers to the integers is defined as follows:
$f(n)=\begin{cases} n+3 \text{ if } n \text{ is odd} \\ \frac{n}{2} \text{ if } n \text{ is even} \end{cases}$ Suppose that k is odd and $f(f(f(k)))=27$. What is the sum of the digits of k? a) 3 b) 6 c) 9 d) 12 e) 15

21. Positive integers A, B, and C, with no common factor greater than 1 exist such that $A\log_{200}5+B\log_{200}2=C$. What is A + B + C? a) 6 b) 7 c) 8 d) 9 e) 10

22. The two wheels shown are spun, and the two resulting numbers are added. What is the probability that the sum of the two numbers is even?



a) $\frac{1}{6}$ b) $\frac{1}{4}$ c) $\frac{1}{3}$ d) $\frac{5}{12}$ e) $\frac{4}{9}$

23. A list of 5 positive integers has mean 12 and range 18. The mode and median are both 8. How many different values are possible for the second largest element of the list? a) 4 b) 6 c) 8 d) 10 e) 12

24. Sarah works from Wednesday to Saturday as a receptionist for a health spa. Each day she works 2 hours more than the previous day. If she works a total of 24 hours during the 4th period, how many hours does she work on Fridays?
a) 6 b) 7 c) 8.5 d) 20

25. A fair standard 6-sided die is tossed 3 times. Given that the sum of the first 2 tosses equals the 3rd, what is the probability that at least one "2" is tossed?
a) $\frac{1}{6}$ b) $\frac{91}{216}$ c) $\frac{1}{2}$ d) $\frac{8}{15}$ e) $\frac{7}{12}$

26. For how many 3-digit whole numbers does the sum of the digits equal 25?
a) 2 b) 4 c) 6 d) 8 e) 10

27. If M is 30% of Q, Q is 20% of P, and N is 50% of P, then $\frac{M}{N}$ equals which of the following? a) $\frac{3}{250}$ b) $\frac{3}{25}$ c) 1 d) $\frac{6}{5}$ e) $\frac{4}{3}$

28. For $1 < x < y < x+y$, let $S = \{1,\ x,\ y,\ x+y\}$. What is the difference between the mean and the median of S? a) x b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) $\frac{1}{6}$ e) $\frac{y}{2}$

29. At Luvamathville Junior High 30 % of the students in the math club are in the science club, and 80% of the students in the science club are in the math club. Fifteen students are in the science club. How many students are in the math club?
(a) 12 (b) 16 (c) 30 (d) 36 (e) 40

30. A sequence is defined as $a_1 = 10$ and $a_n = \begin{cases} \frac{a_{n-1}}{2} & if\ \ a_{n-1}\ is\ even \\ 3(a_{n-1})+1 & if\ a_{n-1}\ \ is\ \ odd \end{cases}$, find the 10th term. (a) 1 (b) 2 (c) 4 (d) 5 (e) 8

31. During a recent span of time, eleven days had some rain. A morning rain was always followed by a clear afternoon, and an afternoon rain was always preceded by a clear morning. In all, nine mornings and twelve afternoons were clear. How many days had no rain at all?
(a) 4 (b) 5 (c) 6 (d) 9 (e) 11

32. P(x) is an odd function such that P(1) = 2, P(3) = 5 and P(-5) = -1. What is the value of P(P(P(- 3))) ? (a) - 2 (b) - 1 (c) 1 (d) 2 (e) 5

33. These numbers are pumpwarts: 16325 34721 52163 90341 50381
These numbers are not pumpwarts: 2564 12345 854 12635 34325 45026
Which of the following is a pumpwart? a) 72521 b) 72341 c) 4562 d) 13562

34. What is the value of the expression $\frac{1}{\log_2 100!}+\frac{1}{\log_3 100!}+\frac{1}{\log_4 100!}+\ldots+\frac{1}{\log_{100} 100!}$?
(a) 0.01 (b) 0.1 (c) 1 (d) 2 (e) 10

35. Find x, where $0 < x < 90^\circ$: $81^{\cos^2 x}\, 9^{\cos x} = 729^{\frac{1}{3}}\, 3^{2\cos x}$
(a) 30 (b) 45 (c) 60 (d) 90

36. Three friends each have a red, a white, a yellow, a blue and a green T-shirt. If each of them randomly chooses a T-shirt to wear, what is the probability that they all choose different colors? (a) $\frac{2}{5}$ (b) $\frac{5}{9}$ (c) $\frac{1}{125}$ (d) $\frac{2}{3}$ (e) $\frac{5}{12}$

37. How many integers x in {1, 2, 3, ..., 99, 100} are there such that $x^2 + x^3$ is the square of an integer? (a) 6 (b) 7 (c) 8 (d) 9 (e) 10

38. What number(s) have the property that its reciprocal is one less than the number?

(a) $\frac{1}{3}, 4$ (b) $\frac{1 \pm \sqrt{5}}{2}$ (c) $-\frac{1 \pm \sqrt{5}}{2}$ (d) $\frac{1}{2}, 1$

39. In a series of 3 races, a student earns 5 points a win, 3 points 2nd place, and 1 point for 3rd place; no ties allowed. How many points must a student earn in the 3 races to be guaranteed of earning more points than any other student?
(a) 9 (b) 10 (c) 11 (d) 13 (e) 15

40. An "unfair" coin has a $\frac{2}{3}$ probability of turning up heads. If this coin is tossed 50 times, what is the probability that the total number of heads is even?

(a) $25\left(\frac{2}{3}\right)^{50}$ (b) $\frac{1}{2}\left(1-\frac{1}{3^{50}}\right)$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}\left(1+\frac{1}{3^{50}}\right)$ (e) $\frac{2}{3}$