National MAθ Convention 2002
Hustle.....Precalculus/Calculus

1 When $p(x) = x^4 - 2x^3 - 3x^2 + 8x - 4$ is divided by which factor is the remainder the greatest?

a. $(x-1)$ b. $(x+1)$ c. $(x-2)$ d. $(x-3)$

Ans. d

2 Solve for *x*: $9x + 4 = 8^{1+\log_8 5}$

Ans. x = 4

3 As a cannonball travels through the air, its height in meters above the ground at t seconds is $h(t) = -9.8(t-10)^2 + 980$. For what time does this equation have physical meaning? Express answer in interval notation.

Ans. [0, 20]

4 Find $f''(-4)$ if

$f(x) = x^4 - 3x^3 + 2x - 7$

Ans. 264

5 Find the limit, if it exists.

$$\lim_{x \to -6} \frac{x^2 - 36}{x^2 + x - 30}$$

Ans. $\frac{12}{11}$

6 A point is moving along the graph of $y = \frac{1}{1+x^2}$ so that $\frac{dx}{dt} = 2$ cm/min.

Find $\frac{dy}{dt}$ when x = -2.

Ans. $\frac{8}{25}$

7 How many terms are in the sequence 3, 7, 11, … , 39 ?

Ans. 10

8 Solve for *x*:

$\left(2^{8x^2}\right)\left(2^{4x}\right)\left(2^{-2}\right) = \left(2^{2x^2}\right)\left(2^{5x}\right)$

Ans. $x = \frac{2}{3}$ *or* $-\frac{1}{2}$

9 Solve for x:

$$\begin{vmatrix} 4 & x^2 & x \\ 3 & 1 & 0 \\ -1 & -2 & 3 \end{vmatrix} = 8$$

Ans. $\frac{4}{9}$ *or* -1

10 Find $\lim_{x \to \infty} \left(\frac{1}{x} + \frac{1}{x^2} + 1 \right)$

Ans. 1

11 Find $f'(x)$ if

$f(x) = 3\cos^2 5x$

Ans. $-30\cos(5x)\sin(5x)$

12 If the coefficient of the 5th and 6th terms in the expansion of $(x-y)^n$ are equal, find the 3rd term.

Ans. $36x^7y^2$

13 Find the dot product given $\vec{v} = -3i + 4j - 7k$ and $\vec{w} = 3i - 6j - 3k$.

Ans. - 12

14 Simplify:

$$\frac{q!(q-3)!}{(q-5)!(q+2)!}$$

Ans. $\dfrac{(q-3)(q-4)}{(q+1)(q+2)}$

15 Find the mean of the first *500,000* odd numbers.

Ans. 500,000

16 The height, *h* in meters, of a projectile launched from under water is given by the portion of the curve of $h(t) = -t^5 - 2t^4 + 10t^3 + 20t^2 - 9t - 18$ where $t \geq 0$, *t* is in seconds. At what time does the projectile leave the water?

Ans. 1 second

17 Solve and express the answer in interval notation:

$(x+4)^2(x-1)^2(x+7)^2 > 0$

Ans.

$(-\infty,-7) \cup (-7,-4) \cup (-4,1) \cup (1,\infty)$

18 If

$f(x) = -8(x-1)^3(x+2)(x-3)^7(x+1)^4$

at which *x* intercept does the function just touch the *x* – axis?

Ans. at x = -1

19 Which of the functions are even?

I. $y = 3x^2$

II. $y = 5x^4 - 4$

III. $y = 4x^2 + 8x + 4$

IV. $y = x^4 + 3x^2 + 7$

V. $y = \cos 5x$

Ans. I, II, IV & V

20 State the domain for $f(x)$ in interval form if $f(x) = \sqrt{4 - x^2}$

Ans. [-2, 2]

21 What test would be used to prove convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n^5}$

Ans. P-Series

22 A particle is moving along a horizontal line according to the equation $s(t) = 2t^3 - 4t^2 + 2t - 1$. Find the velocity at the instant t = 3 seconds.

Ans. 32

23 Find $\int \sin\frac{1}{3}x \; dx$

Ans. $-3\cos\frac{1}{3}x + C$

24 Solve for x:

$\ln x + \ln(x-2) = 3\ln 2$

Ans. 4

25 Exactly evaluate: $\int_0^2 x\, e^{3x^2}\, dx$.

Ans. $\frac{1}{6}\left(e^{12} - 1\right)$