Solutions - Alpha - Individual Test

1. The difference will be a number of the form (2n+1)+(2n+3)...+(2n+2k-1) with k>1. One can establish by induction that this is k(2n+k). Hence 43 is a possible answer. On the other hand

44=12^{2}-10^{2}
45=7^{2}-2^{2}
48=8^{2}-4^{2}

Thus the correct answer is A.

2. Sol: Each number n is precisely of the form p^k*q^m with –1<k<14 and –1<m<6.
Hence d=14*6=84. Hence the answer is A.

3. Sum $= \frac{-b}{a} = \frac{500}{1000} = \frac{1}{2}$ $\boxed{A}$

4. $\binom{20}{4} = 4845$ $\boxed{D}$

5. Sum $= \frac{-b}{a} = \frac{1}{1} = 1$ $\boxed{E}$

6. Sol: It is useful to factor as (x-y)(x+y). The answer is A

$(105311)^2 - (105305)^2 = (105311 - 105305)(105311 + 105305)$
$= 6\,(210616) = 1263696$ $\boxed{A}$

7. $P(2W \text{ or } 2B \text{ or } 2 \text{ green}) = \frac{\binom{4}{2}}{\binom{18}{2}} + \frac{\binom{6}{2}}{\binom{18}{2}} + \frac{\binom{8}{2}}{\binom{18}{2}} = \frac{6+15+28}{153}$

$= \frac{49}{153}$ A

8. Sol: The vertex abscissa solves –2x+200=0. Thus x=100, also the average root. Hence y=40,000 and D is the correct answer.

9. Use long division. Remainder is $x+2$ (D)

10. ${}_{100}C_{50}\, x^{50}(1)^{50} = \frac{100!}{50!\,50!}\, x^{50}$ (B)

Sum = 16 (B)

11. $x = 42,\ y = -31,\ z = 5$

12. Sol: A given rectangle corresponds to a choice of two nontrivial interval subsets of {0,…8}. Thus there ("9 choose 2" squared) rectangles. This is 1296. Hence the answer is C.

13. Sol: Both my calculations and those at http://ecademy.agnesscott.edu/~lriddle/ifs/ksnow/area.htm indicate that the answer is 8*sqrt(3)/5. Thus the correct answer is E.

14. $x = \frac{1}{2+\frac{1}{3+x}} = \frac{3+x}{2(3+x)+1} = \frac{3+x}{6+2x+1} = \frac{3+x}{7+2x}$

$x = \frac{3+x}{7+2x}$ $\quad 7x+2x^2 = 3+x$ $\quad 2x^2+6x-3=0$ $\quad \frac{-6\pm\sqrt{36+24}}{4} = \frac{-6\pm\sqrt{60}}{4} = \frac{-6\pm 2\sqrt{15}}{4} = \frac{-3\pm\sqrt{15}}{2}$

answer is positive $\frac{-3+\sqrt{15}}{2}$ (C)

15. Sol: The solutions form a circle of diameter 2*sqrt(1000). This is approximately 62 and thus the answer is C.

16. Eq. of line: $m = \frac{70}{30} = 7/3$ $\quad y-100 = 7/3\,(x-50)$

When $x=0$, $y = ??$

$y-100 = 7/3(-50)$

$y = 100 - \frac{350}{3} = \frac{-50}{3}$ (A)

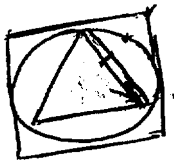
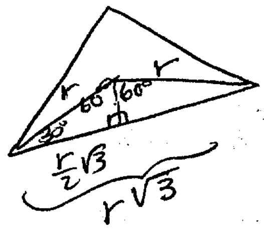
17. $\frac{n(n+1)}{2} = \frac{1000(1001)}{2} = 500(1001) = 500{,}500$ $\boxed{C}$

18. There are 21 different ways the treasure can be divided. $\boxed{B}$

19. Draw 2 evens or Draw an odd and a multiple of 4

$\frac{1}{2}\left(\frac{1}{2}\right) + 2\left[\frac{1}{2}\left(\frac{1}{5}\right)\right] \quad \frac{9}{20}$ $\boxed{C}$

$\frac{1}{4} + \frac{1}{5} = \frac{5}{20} + \frac{4}{20} =$

20.   

$A = \frac{s^2\sqrt{3}}{4} = \frac{(r\sqrt{3})^2\sqrt{3}}{4}$

$\frac{3r^2\sqrt{3}}{4}$

area of square =  $(2r)^2 = 4r^2$

ratio $= \frac{\frac{3r^2\sqrt{3}}{4}}{4r^2} = \frac{3\cancel{r^2}\sqrt{3}}{4} \cdot \frac{1}{4\cancel{r^2}} = \frac{3\sqrt{3}}{16}$ $\boxed{B}$

21. $9 \cdot \underline{10} \cdot \underline{10} \cdot \underline{1} \cdot \underline{1} = 900$ $\boxed{D}$

22. Since $\{a_n\}$ is a bounded and monotonic function, it converges to L. Then $\lim_{a \to \infty} a_n = \lim_{a \to \infty} a_{n+1}$

Thus, $L = \frac{1}{3-L} \Rightarrow 3L - L^2 = 1 \Rightarrow L^2 - 3L + 1 = 0$

$L = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$ Since $a_n \leq 2$, $\frac{3-\sqrt{5}}{2}$ $\boxed{C}$

23. The numbers can't be more than 10 places long.
Thus we have $9+9^2+9^2\cdot 8+9^2\cdot 8\cdot 7+9^2\cdot 8\cdot 7\cdot 6+$
$\dots+9^2\cdot 8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1=$ $8{,}877{,}690$ $\boxed{D}$

24.

$100+2\left[\frac{1}{2}\cdot 100+\frac{1}{2}^2(100)+\left(\frac{1}{2}\right)^3(100)+\dots\right]$

$100+2\left[\frac{50}{1-\frac{1}{2}}\right]$

$100+2\left(\frac{50}{1/2}\right)=$ $100+2(100)=$ 300 $\boxed{B}$

25. Net Revenue = Revenue − Cost

$12(x)+2(10-x)-60=$ Net $\boxed{A}$

$12x+20-2x-60$

$10x-40=$ Net

26. When determinant $=0$.

$$\left|\begin{matrix} 1 & 4 & c \\ 2 & -1 & 7 \\ 3 & -2 & 11 \end{matrix}\right| \begin{matrix} 1 & 4 \\ 2 & -1 \\ 3 & -2 \end{matrix} =$$

$-11+84-4c-(-3c-14+88)$

$-11+84-4c+3c+14-88$

$-11+84-4c+3c$

$-1-c=0$

$-1=c$ $\boxed{B}$

27. $x=\sqrt{2+\sqrt{2}}-\sqrt{2-\sqrt{2}}$

$x^2=2+\cancel{\sqrt{2}}-2\left(\sqrt{2+\sqrt{2}}\right)\left(\sqrt{2-\sqrt{2}}\right)+2-\cancel{\sqrt{2}}$

$4-2\left(\sqrt{4-2}\right)$

$4-2\sqrt{2}$

$x^4=$ $16-16\sqrt{2}+8=$ $24-16\sqrt{2}$

$x^8=$ $576-768\sqrt{2}+512=$ $1088-768\sqrt{2}$

$384x^2-x^8=$ $384(4-2\sqrt{2})-1088+768\sqrt{2}$

$384(4-2\sqrt{2})-1088+\cancel{768\sqrt{2}}$

$1536-\cancel{768\sqrt{2}}-1088$

448 $\boxed{C}$

28. minimum at vertex

$x = -\frac{c}{2}$

$f\left(\frac{-c}{2}\right) = \frac{c^2}{4} - \frac{c^2}{2} + 2 = -1$

$-\frac{c^2}{4} + 3 = 0$

$-\frac{c^2}{4} = -3$

$\frac{c^2}{4} = 3$

$c^2 = 12$

$c = \pm\sqrt{12} = \pm 2\sqrt{3}$

c is positive $\quad 2\sqrt{3} \quad \boxed{E}$

29. $\log_3 \frac{45}{7} = \log_3 45 - \log_3 7$

$\log_3 9 + \log_3 5 - \log_3 7$

$2 + x - y \quad \boxed{D}$

30. $xy + y^2 = 12 \qquad x = \frac{12 - y^2}{y}$

$-xy + x^2 = 8$

$y^2 + x^2 = 20$

$y^2 + \left(\frac{12-y^2}{y}\right)^2 = 20$

$\cancel{y^2 + 12 - y^2 = 20y}$

$y^2 + \frac{144 - 24y^2 + y^4}{y^2} = 20$

$y^4 + 144 - 24y^2 + y^4 = 20y^2$

$2y^4 - 44y^2 + 144 = 0$

$y^4 - 22y^2 + 72 = 0$

product $= \frac{k}{a} = \frac{72}{1} = 72$

$\boxed{D}$

31.) $\cot \frac{1}{2}\theta = \sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$

$\cot \frac{1}{2}\left[\arcsin -\frac{3}{5}\right] =$

$\cot \frac{1}{2}\theta$ where $\theta = \arcsin -\frac{3}{5}$

$= \sqrt{\frac{1+4/5}{1-4/5}} = \sqrt{\frac{9/5}{1/5}} = \sqrt{9} = 3$

but θ is in Quadrant IV

so $\cot \frac{1}{2}\theta = -3$

$\boxed{A}$

32 $\boxed{A}$

P.5

33. Heron's formula: area $= \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(10+12+14) = 18$

area $= \sqrt{18(8)(6)(4)} = \sqrt{3456} =$

$= 3\sqrt{2} \cdot 2\sqrt{2} \cdot \sqrt{6} \cdot 2$

$= 12 \cdot 2 \cdot \sqrt{6} = 24\sqrt{6}$

perimeter $= 10+12+14 = 36$

Ratio $= \frac{24\sqrt{6}}{36} = \frac{2\sqrt{6}}{3}$ $\boxed{A}$

34. $$\frac{(x+y)^3 \cancel{(z-1)}\cancel{(z+1)}}{(x+y)(x^2-xy+y^2)\cancel{(z-1)}\cancel{(z+1)}(z^2+1)(x+y+z)} = \frac{(x+y)^2}{(x^2-xy+y^2)(x+y+z)(z^2+1)}$$ $\boxed{E}$

35. $\underline{9} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} \cdot \underline{3} = 9(3^5) = 2187$ $\boxed{A}$

choose 1 less, 1 more, or keep the same

36. 

radius of small circle $= r$

radius of large circle $= r\sqrt{2}$

$$\frac{\cancel{\pi} r^2}{\cancel{\pi}(r\sqrt{2})^2} = \frac{r^2}{2r^2} = \frac{1}{2}$$ $\boxed{A}$

37. $|x^2-3| = |3x+1|$

$x^2-3 = \pm(3x+1)$

$x^2-3 = 3x+1$

$x^2-3x-4=0$

$(x-4)(x+1)=0$

$x = 4, -1$

$x^2-3 = -3x-1$

$x^2+3x-2=0$

$x = \frac{-3 \pm \sqrt{9+8}}{2} = \frac{-3 \pm \sqrt{17}}{2}$

$$4 + -1 + \frac{-3+\sqrt{17}}{2} + \frac{-3-\sqrt{17}}{2} = 3 - 3 = \boxed{0}$$

$$3 + -\frac{3}{2} + \cancel{\frac{\sqrt{17}}{2}} - \frac{3}{2} - \cancel{\frac{\sqrt{17}}{2}}$$

$\boxed{C}$

38. $f(g(2)) = 2^5 - 3(2)^4 + 2^2 - 1 = 32 - 48 + 4 - 1 = -13$

$f(y) = y^3 + 3y - 5$

$f(y) = -13$

$y^3 + 3y - 5 = -13$

$y^3 + 3y + 8 = 0$ NOTA $\boxed{E}$

$\frac{-3 \pm \sqrt{9-32}}{2}$

39.) $\binom{1024}{2} = 523{,}776$ $\boxed{A}$

40.) $P(E \,\&\, M \text{ in seats } 1 \text{ and } 2) = \frac{2\cdot 1\cdot \cancel{4\cdot 3\cdot 2\cdot 1}}{6\cdot 5\cdot \cancel{4\cdot 3\cdot 2\cdot 1}} = \frac{1}{15}$

$P(E \,\&\, M \text{ next to each other}) = 5\left(\frac{1}{15}\right) = \frac{1}{3}$ $\boxed{C}$

41.) 3 men work for $\frac{108}{4}$ or \$27/week

Each man works for \$9/week

Thus, 5 men work for \$45/week $\boxed{B}$

$\frac{135}{45} = 3$ weeks

	r	t	d
down river	r+c	$\frac{6}{r+c}$	6
up river	r−c	$\frac{6}{r-c}$	6
lake	r	$\frac{4}{r}$	4

42.)

$\frac{6}{r+c} + \frac{4}{r} = 2$

$6r + 4r + 4c = 2r(r+c)$

$10r + 4c = 2r^2 + 2rc$

$10r = 2r^2 + 2rc - 4c$

$\frac{6}{r-c} + \frac{4}{r} = 4$

$6r + 4r - 4c = 4r(r-c)$

$10r - 4c = 4r^2 - 4rc$

$10r = 4r^2 - 4rc + 4c$

$20r = 6r^2 - 2rc$

$6r^2 - 20r - 2rc = 0$

$2r(3r - 10 - c) = 0$

$\cancel{r=0}$ or $c = 3r - 10$ $\boxed{B}$

$\frac{6}{r+3r-10} + \frac{4}{r} = 2$

solve for r: $r = 5/4, 4$

$c = 3(5/4) - 10$ or $c = 3(4) - 10$

$c = \cancel{15/4 - 10}$ $c = 2$

43.) roots are r_1, r_1, r_3

Sum of roots $= -b/a = 0$

$2r_1 + r_3 = 0$

$r_3 = -2r_1$

Let $r_1 = 1$, $r_3 = -2$

$(x-1)^2(x+2) = (x^2-2x+1)(x+2) = 0$

$x^3 - 3x + 2 = 0$ $\boxed{B}$

$c = -3$, $d = 2$

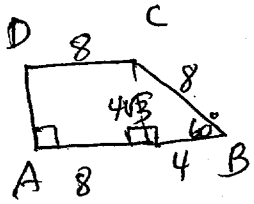
$4c^3 + 27d^2 = 4(-27) + 27(4) = 0$

44.) $10x - 5(x-2) < 0$ $\boxed{B}$

$10x - 5x + 10$

$5x + 10 < 0$

$x < -2$

45.) 

$A = \frac{1}{2}(4\sqrt{3})[8+12] = \frac{1}{2}(4\sqrt{3})(20) = 40\sqrt{3}$ $\boxed{D}$

46.) $x = \frac{4}{5}(180 - x)$ $\boxed{D}$

$5x = 720 - 4x$

$9x = 720$

$x = 80$

Let $y = \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\cdots}}}$ Then $y = \frac{2}{1+y}$

$y + y^2 = 2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2$, $y = 1$

47.) $5x - 1 = 3 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\cdots}}}$

$5x - 1 = 3 + 1$

$5x - 1 = 4$

$5x = 5$ $\boxed{A}$

$x = 1$

48) $m = k d r^3$

$$\frac{m_{Jupiter}}{m_{Earth}} = \frac{\cancel{k}\left(\frac{5}{22}\cancel{d}\right)(11r)^3}{\cancel{k}(\cancel{d})\, r^3} = \frac{\frac{5}{22}(11)^3 \cancel{r^3}}{\cancel{r^3}} = \frac{5}{\cancel{22}_2} \cdot (\cancel{11} \cdot 11 \cdot 11)$$

$$\frac{55(11)}{2} = 302.5$$

or

$\frac{605}{2}$ $\boxed{D}$

49.)

	r	t	d
train	r	h	rh
w/ increased speed	$\frac{rh}{h-1}$	$h-1$	rh

$$\frac{rh}{h-1} = x + r$$ (increase orig. speed by x)

solve for x

C

$$\frac{rh}{h-1} - r = x$$

$$\frac{rh - r(h-1)}{h-1} = x$$

$$\frac{\cancel{rh - rh} + r}{h-1} = x$$

$$\frac{r}{h-1} = x$$

50. $$\frac{1}{1+a^t} + \frac{1}{1+a^{-t}} = \frac{1+a^{-t}+1+a^t}{(1+a^t)(1+a^{-t})}$$

$$= \frac{2+a^{-t}+a^t}{1+a^{-t}+a^t+1} = \frac{2+a^{-t}+a^t}{2+a^{-t}+a^t}$$

$$= \boxed{1}$$

B