2002 Mu Alpha Theta National Tournament
Mu Level Individual Test

1) How many six digit numbers (leading digit cannot be zero) are there such that any two adjacent digits have a difference of no more than one? Consider that 0 can be adjacent to 9.

A) 2430 B) 2187 C) 320 D) 288 E) NOTA

2) Evaluate $\int_0^1 \sqrt{(1-x)(1+x)}dx$.

A) π B) π^2 C) $\frac{\pi}{4}$ D) $\frac{\pi}{2}$ E) NOTA

3) Find the derivative with respect to x of $\sin x \tan x + \cos x - \sec x$.

A) $\cos^2 x$ B) $\tan^2 x$ C) $\sec^2 x$ D) $\sin^2 x$ E) NOTA

4) Find $\frac{d}{dx}\left[\cos(2x)\right]$.

A) $2\cos(2x)$ B) $-\sin(2x)$ C) sin(x)cos(x) D) -4sin(x)cos(x)
E) NOTA

5) Evaluate $\int x^2 e^{x+1} dx$.

A) $\frac{x^3}{3}e^{x+1} + C$ B) $e^{x+1}(x+1)^2 + C$ C) $e^{x-1}x^2 + C$
D) $e \cdot e^x (x-1)^2 + e^{x+1} + C$ E) NOTA

6) Compute the ratio of the area to the perimeter of a triangle with sides measuring 10, 12, and 14.

A) $\frac{2\sqrt{6}}{3}$ B) $24\sqrt{2}$ C) $3\sqrt{3}$ D) $2\sqrt{6}$ E) NOTA

7) Two marbles are drawn from a vase which contains 4 white marbles, 6 blue marbles, and 8 green marbles. What is the probability that if I pick two marbles, they will be the same color?

A) $\frac{49}{153}$ B) $\frac{58}{153}$ C) $\frac{15}{153}$ D) $\frac{53}{156}$ E) NOTA

8) Find the second derivative of $e^x \sin x \cos x$ at $\frac{\pi}{2}$.

A) $-e^{\frac{\pi}{2}}$ B) $e^{\frac{\pi}{2}}$ C) $2e^{\frac{\pi}{2}}$ D) $-2e^{\frac{\pi}{2}}$ E) NOTA

9) Evaluate $\lim_{x\to 0}\frac{\sin(2x)}{x}$.

A) 0 B) $\frac{1}{2}$ C) 1 D) 2 E) NOTA

10) $e^{i\pi} =$

A) 1 B) -1 C) e D) $-e^{\pi}$ E) NOTA

11) Evaluate: $\int_{-1}^{2}\frac{t^2}{\sqrt{t+2}}dt$

A) $\frac{16}{15}$ B) $\frac{26}{15}$ C) $\frac{46}{15}$ D) $\frac{22}{5}$ E) NOTA

12) Find $\int \sin^3 x\cos x - \sin x\cos^3 x dx$.

A) $\frac{\sin^2 x\cos^2 x}{2}+C$ B) $\frac{\sin x\cos x}{2}+C$ C) $x\sin^2 x + x\cos^2 x + C$
D) $\sin^4(\cos(x))$+C E) NOTA

13) Find a complex number Z such that Z = arccos (2)

A) $\ln(2-\sqrt{3})i$ B) $\ln(2+\sqrt{3})i$ C) $\ln(2+\sqrt{3})$
D) $-\ln(2+\sqrt{3})i$ E) NOTA

14) Given that a certain cylinder has a volume V, what is the minimum surface area that the cylinder can have?

A) $2\pi\sqrt[3]{\frac{V^2}{4\pi}}$ B) $3\sqrt[3]{2\pi V^2}$ C) 2r D) $\sqrt[3]{\frac{V}{2\pi}}$ E) NOTA

15) Find the remainder when $x^5+4x^4-3x^2+1$ is divided by x^2-1.

A) $x-1$ B) -1 C) 3x D) x E) NOTA

16) What is the minimum distance from (2,3) to a point on the line y = -x+8.

A) $\sqrt{2}$ B) $\frac{\sqrt{2}}{2}$ C) $\frac{2\sqrt{3}}{3}$ D) $\frac{\sqrt{3}}{3}$ E) NOTA

17) Find the middle entry of the 100^{th} row of Pascal's Triangle.

A) $\frac{100!}{51!49!}$ B) $\frac{100!}{(50!)^2}$ C) 101 D) 1189 E) NOTA

18) Find the sum of the values for the solutions for x, y, and z:

x+2y+6z=10
3x+4y+2z=12
2x+3y+3z=6

A) 2 B) 16 C) 8 D) -4 E) NOTA

19) How many rectangles can be drawn on an 8x8 chess board which have odd area?

A) 100 B) 200 C) 300 D) 400 E) NOTA

20) Evaluate $\sum_{k=0}^{\infty} \frac{3^{k+1}}{k!}$.

A) $\frac{\pi^2}{6}$ B) sin(3) C) 335 sin(3) D) $4e^3$ E) NOTA

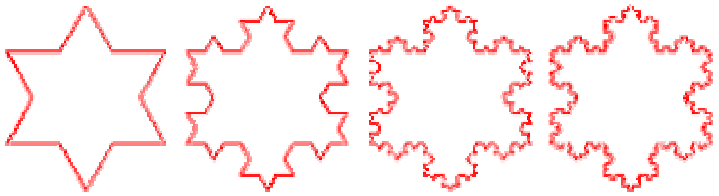
21) Evaluate $\prod_{1}^{200} \left(\frac{n}{n+1}\right)^2$.

A) 199/200 B) ½ C) 1/40000 D) 234/501 E) NOTA

22) Determine the convergence or divergence of $\prod_{1}^{\infty} \frac{n^2+1}{n^2}$.

A) Converges B) Diverges
C) It cannot be determined whether it converges or diverges

23) To construct a Koch Snowflake, first you draw an Equilateral Triangle. Then you trisect each of the sides, and construct on the middle segment another equilateral triangle. If this is repeated infinitely, the resulting shape will be a Koch Snowflake (see picture). If a Koch Snowflake is created on an equilateral triangle with sides of length 2, what is it's area?



A) $4\pi/3$ B) $\frac{11\sqrt{3}}{5}$ C) $\frac{6\sqrt{3}}{5}$ D) $2\sqrt{3}$ E) NOTA

24) Find the volume of the solid obtained by revolving about the line y = -1 the region between the graph of the equation y = x+1 and the line y = -1 on the interval [0,1].

A) $\frac{10}{3}\pi$ B) π C) $\frac{19}{3}\pi$ D) $\pi\left(2\sqrt{2}-1\right)$ E) NOTA

25) Evaluate $\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$.

A) $\sqrt{15}$ B) $\frac{1}{6}$ C) $\frac{\sqrt{15}-3}{2}$ D) $\frac{\sqrt{15}+3}{2}$ E) NOTA

26. Evaluate: $\sqrt{2+\sqrt{2+\sqrt{2+\ldots.}}}$

A) 2 B) $2\sqrt{2}$ C) $\sqrt{3}$ D) $2\sqrt{3}$ E) NOTA

27. A person rolls six 4-sided dice each of which has sides labeled A, B, C, and D. What is the probability that there is a tie for the letter which occurs the most?

A) $\frac{195}{512}$ B) $\frac{15}{512}$ C) $\frac{45}{512}$ D) $\frac{135}{512}$ E) NOTA

28. $\log_{4n} 40\sqrt{3} = \log_{3n} 45$. Find n^3.

A) 4 B) 12 C) 15 D) 60 E) 75

29. Find $\cot\left[\frac{1}{2}Arc\sin\left(\frac{-3}{5}\right)\right]$

A) -3 B) 3 C) 1/3 D) -1/3 E) NOTA

30. The trigonometric form of $2i$ is :

A) $2\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$ *B*) $2\left(\cos\pi+i\sin\pi\right)$

C) $2\left(\sin\frac{3\pi}{2}+i\cos\frac{3\pi}{2}\right)$ D) $-2(\cos\pi+i\sin\pi)$ E) NOTA

31. Which of the following cannot be expressed as the difference of the squares of two nonconsecutive integers?

A. 43 B. 44 C. 45 D. 48 E. All of these

32. If p and q are distinct primes each of which is greater than 3, let *d* denote the number of positive integers n such that p^13*q^5 divided by n is a positive integer. Which of the following is *d*?

A. 84 B. 65 C. 48 D.110 E. NOTA

33. The sum of the solutions to 1000x^2-500x+73737=0 is :

A. 1/2 B. 10 C. -1/2 D. 1000 E. NOTA

34. A debating team with 4 members is to be chosen from among 20 students. Find the number of distinct possible teams. (Of course two teams are considered the same if they have the same members, even if the members were chosen in different orders).

A. 116280 B). 2000 C. 3000 D. 4845 E. 5000

35. What is the sum of all three solutions to x^3-x^2-4x-6=0.

A) 2 B) -1 C) 3 D) -3 E) NOTA

36. What is the magnitude of the difference between 105311^2 and 105305^2 .

A) 1263696 B) 1243692 C) 210622 D) 210610 E) NOTA

37. If x and y satisfy the equation y=(300-x)(x+100) then the largest possible value of y is closest to

A) 5000 B)10,000 C) 25,000 D) 50,000 E) 100,000

38. If the point (0,y) is on the line passing through (20,30) and (50,100) then y is:

A) -50/3 B) 230/3 C) 650/3 D) -10 E) NOTA

39. 1+2+3…+999+1000 =

A) 50,000 B) 100,000 C) 500,500 D) 1,500,000 E) 5,000,000

40. Harry, Burly, and Joe are to receive a combined total of five pennies from Mr. Wilson's will. Assuming each person receives at least zero pennies and no more than five pennies, the number of possible ways in which the treasure can be divided is closest to which number?

A) 10 B) 20 C) 30 D) 40 E) 50

41. Ten balls, numbered 1 through 10, are placed in a bag. Draw one ball at random from the bag, put it back, and then draw a second time. What is the probability that 4 divides the product of your two selections?

A) ½ B) ¼ C) 9/20 D) 13/24 E) NOTA

42. An equilateral triangle is inscribed in a circle which is then inscribed in a square. What is the ratio of the area of the triangle to the area of the square?

A) $\frac{\sqrt{3}}{4}$ B) $\frac{3\sqrt{3}}{16}$ C) $\frac{5\sqrt{3}}{16}$ D) $\frac{3\sqrt{3}}{10}$ E) NOTA

43. How many 5 digit palindromes are there? (Palindromes are numbers which read the same forwards and backwards).

A) 1000 B) 100 C) 2187 D) 900 E) NOTA

44. Suppose a sequence is defined as follows. $a_1 = 2, \quad a_{n+1} = \frac{1}{3 - a_n}$

It can be shown that the sequence is decreasing and $0 \le a_n \le 2$ for all n.

Find the limit of the sequence.

A) $\frac{3+\sqrt{5}}{2}$ B) $\frac{1}{3}$ C) $\frac{3-\sqrt{5}}{2}$ D) 1 E) NOTA

45. How many positive integers are there whose digits are all different?

A) 623,529 B) 9,864,100 C) 986,409 D) 8,877,690
E) NOTA

46. A ball is dropped from a height of 100 feet onto a hard level surface.
Suppose that each time it bounces, it rebounds to half of its previous height.
If the ball continues to bounce indefinitely, find the total distance that it travels.

A) 200 ft B) 300 ft C) 150 feet D) 400 ft E) NOTA

47. For what value of the constant *c* will there be no inverse for the matrix

$$\begin{bmatrix} 1 & 4 & c \\ 2 & -1 & 7 \\ 3 & -2 & 11 \end{bmatrix}$$

A) -2 B) -1 C) 0 D) 1 E) 2

48. Let $x = \sqrt{2+\sqrt{2}} - \sqrt{2-\sqrt{2}}$. Then $384x^2 - x^8 =$

A) 442 B) 444 C) 448 D) 452 E) NOTA

49. For what positive constant *c* will the function $f(x) = x^2 + cx + 2$ have minimum value of -1 ?

A) 2 B) 6 C) $\sqrt{6}$ D) $3\sqrt{2}$ E) $2\sqrt{3}$

50. If $x = \log_3 5$ and $y = \log_3 7$, then $\log_3 \frac{45}{7} =$

A) $\frac{2x}{y}$ B) $\frac{3x}{y}$ C) $\frac{x+2}{y}$ D) x-y+2 E) 2(x-y)

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1. B
2. C
3. E (0)
4. D
5. D
6. A
7. A
8. D
9. D
10. B
11. B
12. A
13. B
14. B
15. E (x+2)
16. E (3sqrt(2)/2)
17. B
18. B
19. D
20. E (3e^3)
21. E (1/40401)
22. A
23. E
24. C
25. C
26. A
27. A
28. E
29. A
30. A
31. A
32. A
33. A
34. D
35. E
36. A
37. D
38. A
39. C
40. B
41. C
42. B
43. D
44. C
45. D
46. B
47. B
48. C
49. E
50. D