Mu Alpha Theta Test - 2002

Theta - Individual Test

D 1) $\underline{9}\cdot\underline{10}\cdot\underline{10}\cdot\underline{1}\cdot\underline{1} = 900$

D 2)

$$\begin{array}{r} x^3+4x^2+x+1 \\ x^2-1\ \overline{)\,x^5+4x^4+0x^3-3x^2+ax+1} \\ \ominus x^5 \mp 0x^4 \oplus x^3 \\ \hline 4x^4+x^3-3x^2+0x+1 \\ \ominus 4x^4 \mp 0x^3 \oplus 4x^2 \\ \hline x^3+x^2+0x+1 \\ \ominus x^3 \mp 0x^2 \oplus x \\ \hline x^2+x+1 \\ x^2+0x \oplus 1 \\ \hline x+2 \end{array}$$

E 3) $$\frac{(x+y)^3(z^2-1)}{(x^3+y^3)(z^4-1)(x+y+z)} = \frac{(x+y)^3(z^2-1)}{(x+y)(x^2-xy+y^2)(z^2-1)(z^2+1)(x+y+z)}$$

$$= \frac{(x+y)^2}{(x^2-xy+y^2)(z^2+1)(x+y+z)}$$

E 4). Sum $= \frac{-b}{a} = \frac{-(-1)}{1} = \boxed{1}$ E

Roots are 3, $\sqrt{3}$ Sum = 1.

A 5. $x^2-y^2 = (x-y)(x+y) \Rightarrow (105311)^2-(105305)^2$

$= (105311-105305)(105311+105305) = 6(210616) = 1263696$

B 6. $$\left[\begin{array}{ccc|c} 1 & 2 & 6 & 10 \\ 3 & 4 & 2 & 12 \\ 2 & 3 & 3 & 6 \end{array}\right] \Longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 6 & 10 \\ 0 & -2 & -16 & -18 \\ 0 & -1 & -9 & -14 \end{array}\right] \Longleftrightarrow \left[\begin{array}{ccc|c} 1 & 2 & 6 & 10 \\ 0 & 1 & 8 & 9 \\ 0 & -1 & -9 & -14 \end{array}\right] \Longleftrightarrow \left[\begin{array}{ccc|} 1 & 2 & 6 \\ 0 & 1 & 8 \\ 0 & 0 & -1 \end{array}\right]$$

Solutions are $z = 5,\ y = -31,\ x = 42 \Rightarrow x+y+z = 16$

(D) 7. There are 64 squares of area 1, 96 rectangles of area 3, 64 rectanges of area 5, 32 rectangles of area 7, 36 rectangles of area 9, 48 rectangles of area 15, 24 rectangles of area 21, 16 rectangles of area 35, 16 rectangles of area 25 and 4 rectangles of area 49.

Thus there are 400 rectangles of odd area.

(A) 8. Let $x = \sqrt{2+\sqrt{2+\sqrt{2+\cdots}}} \Rightarrow x^2 = 2 + \sqrt{2+\sqrt{2+\cdots}} = 2 + x \Rightarrow x^2 - x - 2 = 0$
$\Rightarrow (x+1)(x-2) = 0 \Rightarrow x = -1$ or $x = 2$

(E) 9. Let $f(x) = (x-2)^2 + (8-x-3)^2 = (x-2)^2 + (5-x)^2$

$f'(x) = 2(x-2) - 2(5-x) = 4x - 14 = 0 \Rightarrow x = \frac{7}{2}, y = \frac{9}{2}$

Min distance $= \sqrt{(\frac{7}{2}-2)^2 + (\frac{9}{2}-3)^2} = \sqrt{(\frac{3}{2})^2 + (\frac{3}{2})^2}$

$= \sqrt{2(\frac{9}{4})} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$

(B) 10. Total distance $= 100 + 2(\frac{1}{2})100 + 2(\frac{1}{2})^2 100 + 2(\frac{1}{2})^3(100) + \cdots$

$= 100\left(1 + 2(\frac{1}{2}) + 2(\frac{1}{2})^2 + 2(\frac{1}{2})^3 + \cdots\right)$

$= 100\left(1 + 2\left(\frac{1}{2} + (\frac{1}{2})^2 + (\frac{1}{2})^3 + \cdots\right)\right) = 100\left(1 + \frac{2}{2}\left(1 + \frac{1}{2} + \cdots\right.\right.$

$= 100\left(1 + \left(\frac{1}{1-\frac{1}{2}}\right)\right) = 100(1+2) = 300$

(C) 11. Since $\{a_n\}_{n\geq 0}$ is a ~~bounded~~ bounded monotonic function, it converges to say L. Thus $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1} = \ldots$

$\Rightarrow L = \frac{1}{3-L} \Rightarrow 3L - L^2 = 1 \Rightarrow L^2 - 3L + 1 = 0$

$\Rightarrow L = \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$, but $a_n \leq 2 \Rightarrow L = \frac{3-\sqrt{5}}{2}$

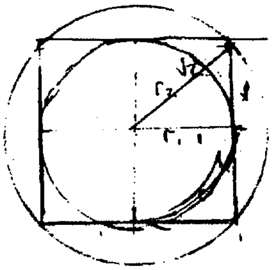
(A) 12. $P(WW + BB + GG) = \frac{4}{18} \cdot \frac{3}{17} + \frac{6}{18} \cdot \frac{5}{17} + \frac{8}{18} \cdot \frac{7}{17}$

$= \frac{12}{306} + \frac{30}{306} + \frac{56}{306} = \frac{98}{306} = \frac{49}{153}$

A 13. $9\cdot3\cdot3\cdot3\cdot3\cdot3 = 9\cdot3^5 = 9(3^5) = 2187$

choose 1 less, 1 more, or keep the same

A 14.



$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{1^2}{\sqrt{2}^2} = \frac{1}{2}$$

C 15. $|x^2-3| = |3x+1| \Rightarrow x^2-3 = \pm(3x+1)$

$\Rightarrow x^2-3 = 3x+1$ or $x^2-3 = -3x-1$

$\Rightarrow x^2-3x-4=0$ or $x^2+3x-2=0$

$(x-4)(x+1)=0$ $\quad x = \frac{-3\pm\sqrt{17}}{2}$

$x=4,\ x=-1$

So $4-1+\left(\frac{-3+\sqrt{17}}{2}\right)+\left(-\frac{3}{2}-\frac{\sqrt{17}}{2}\right) = 0$

E 16. $f(g(x)) = x^5-3x^4+x^2-1 \Rightarrow f(g(2)) = -13$

Let $g(2)=y$, then $f(y) = y^3+3y-5 = -13$

$\Rightarrow y^3+3y+8 = 0$ E(NOTA)

D 17. The numbers can't be more than 10 places long.

Thus we have $9+9^2+9^2\cdot8+9^2\cdot8\cdot7+9^2\cdot8\cdot7\cdot6$
$+9^2\cdot8\cdot7\cdot6\cdot5+9^2\cdot8\cdot7\cdot6\cdot5\cdot4+9^2\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3$
$+9^2\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2+9^2\cdot8\cdot7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1$

$= 8,877,690$

A 18. $R(x) = 12x+2(10-x)-60 = 10x-40$

A 19. $1023+1022+1021+\ldots+2+1$

$= \sum_{k=1}^{1023} k = \frac{(1023)(1024)}{2} = 523,776$

C 20. $P(E+M \text{ in seats 1 and 2}) = \frac{2\cdot1\cdot4\cdot3\cdot2\cdot1}{6\cdot5\cdot4\cdot3\cdot2\cdot1} = \frac{1}{15}$

$P(E+M \text{ next to each other}) = 5\cdot\frac{1}{15} = \frac{1}{3}$

B 21. 3 men work for $\frac{108}{4}$ or \$27 per week.
That is each man works for \$9/week.
Thus, 5 men work for \$45/week
\$135 ÷ \$45 = 3 weeks $\boxed{B}$

B 22.

	r	t	d
down river	$r+c$	$6/(r+c)$	6
lake	r	$\frac{4}{r}$	4
up river	$r-c$	$\frac{6}{r-c}$	6

$\frac{6}{r+c} + \frac{4}{r} = 2$; $\frac{4}{r} + \frac{6}{r-c} = 4$

$\frac{6}{r+c} + \frac{4}{r} = 2$
$- \frac{6}{r-c} + \frac{4}{r} = 4$

$\frac{6}{r+c} - \frac{6}{r-c} = -2$

$6(r-c) - 6(r+c) = -2(r^2-c^2)$
$\cancel{6r} - 6c - \cancel{6r} - 6c = -2r^2 + 2c^2$
$-3c - 3c = -r^2 + c^2$
$-6c = -r^2 + c^2$

$6r + 4r + 4c = 2(r)(r+c)$
$10r + 4c = 2r^2 + 2rc$

$6r + 4r - 4c = 4r(r-c)$
$10r - 4c = 4r^2 - 4rc$

$20r = 6r^2 - 2rc$
$6r^2 - 20r - 2rc = 0$
$2r(3r - 10 - c) = 0$
$c = 3r - 10$

$-6c = -r^2 + c^2$
$-6(3r-10) = -r^2 + (3r-10)^2$
$-18r + 60 \quad -r^2 + 9r^2 - 60r + 100$
$0 = 8r^2 - 42r + 40$
$0 = 2(4r^2 - 21r + 20)$
$(4r-5)(r-4) = 0$
$r = 5/4, 4$

$c = 3r - 10$
$c = 15/4 - 10 = -\frac{25}{4}$
$c = 3(4) - 10 = 2$
B

23) roots are r_1, r_1, r_3

Sum of roots $= -\frac{b}{a} = 0$

$2r_1 + r_3 = 0$

$r_3 = -2r_1$

Suppose $r_1 = 1$, $r_3 = -2$

$(x-1)^2(x+2) = 0$

$(x^2-2x+1)(x+2) = 0$

$x^3 - 3x + 2 = 0$

$c = -3,\ d = 2$ $\nearrow$ $4c^3 + 27d^2 = 4(-27) + 27(4) = 0$

$\boxed{B}$

24. (B)

$10x - (5)(x-2) < 0$

$10x - 5x + 10$

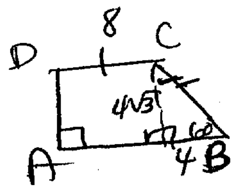
$5x + 10 < 0$

$5x < -10$

$x < -2$

$\boxed{B}$

25. (D)



area $= \frac{1}{2}(4\sqrt{3})(8+8+4)$ $40\sqrt{3}$ $\boxed{D}$

$\frac{1}{2}(4\sqrt{3})(20) =$

26. (D)

$x = \frac{4}{5}(180 - x)$

$5x = 720 - 4x$

$9x = 720°$

$x = 80°$

$\boxed{D}$

27. $\boxed{A}$

$5x - 1 = 3 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\cdots}}}$

Let $y = \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\cdots}}} = $ $y = \frac{2}{1+y}$

$y + y^2 = 2$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

$y = -2, 1$

$\boxed{y = 1}$

$\leftarrow$

$5x - 1 = 3 + 1$

$5x - 1 = 4$

$5x = 5$

$x = 1$

28. $M = kdr^3$ Let $d =$ (density of earth) $r =$ radius of earth

$\boxed{D}$

$\frac{\text{mass of Jupiter}}{\text{mass of Earth}} = \frac{\cancel{k}\left(\frac{5}{22}\cancel{d}\right)(11r)^3}{\cancel{k}\,\cancel{d}\,\cancel{r^3}} = \frac{5}{22}$

$\frac{5}{22}\cdot(11)^3 = \frac{5}{\cancel{22}_2}\cdot 11\,(11)(11)$

$\frac{605}{2}$ D

C

29. r t d

train r h rh

w/ increased speed $\frac{rh}{h-1}$ $h-1$ rh

$\frac{rh}{h-1} = x + r$ (increase orig. speed by x mph)

$\frac{rh}{h-1} - r = x$

$\frac{\cancel{rh - rh} + r}{h-1} = x$

$\frac{r}{h-1} = x$ $\boxed{C}$

B

30. $\frac{1}{1+a^t} + \frac{1}{1+a^{-t}} = \frac{1+a^{-t}+1+a^t}{(1+a^t)(1+a^{-t})}$

$\frac{2+a^{-t}+a^t}{(1+a^t)(1+a^{-t})} = \frac{2+a^{-t}+a^t}{1+a^{-t}+a^t+\cancel{a^0}\,1}$

$= \frac{2+a^{-t}+a^t}{2+a^{-t}+a^t} = 1$

B

p. 6

C 31. $.12x + .32x + .10y + .18y = 133{,}000$

$.44x + .28y = 133{,}000$

$5x + 3y = 1{,}500{,}000$

$\boxed{C}$

$y = 62{,}500$

D 32. $f(x) = \frac{1}{2}x^2 - 3x + 7/2$ $c = 7/2$ $\boxed{D}$

D 33. $C = \begin{bmatrix} 3 & -9 & 4 \\ -\cancel{3} & 12 & -2 \end{bmatrix}$ 2 $\boxed{D}$

$x + y = 4 - 2 = 2$

C 34) $2(3x)(2x-1) + 2(2x+1)(2x-1) + 2(2x+1)3x$

$2[6x^2 \cancel{-3x} + 4x^2 - 1 + 6x^2 \cancel{+3x}]$

$2(12x^2 \cancel{-3x} + 4x^2 - 1) = 2(16x^2 - 1) = 32x^2 - 2$

$\boxed{C}$

B 35. $\sqrt{x-2} = x - 4$

$x - 2 = x^2 - 8x + 16$

$0 = x^2 - 9x + 18$

$0 = (x-6)(x-3)$

$x = 6, 3$ reject 3

6 only $\boxed{B}$

B6

~~36~~. 37. $-\frac{3}{2}x+1=-2x$ $(-2, 4)$

(C) $\frac{1}{2}x+1=0$

$\frac{1}{2}x=-1$

$x=-2$

3~~6~~8. $3x+2=2ky$

[A] $\frac{3x+2}{2k}=y$

$\frac{3}{2k}=\text{slope}=3/2$

$k=1$

38. $\sqrt{y}+\sqrt{y}+\sqrt{y}+\sqrt{y}=2$

[D] $4\sqrt{y}=2$

$\sqrt{y}=2/4$

$y=\frac{4}{16}=1/4$

(D) 39. Eq. of circle: $(x-3)^2+(y-4)^2=25$

radius $=5$

40. $(x+6)-(4x+6)=(4x-2)-(x+6)$

[A] $-3x = 3x-8$

$-6x=-8$

$x=8/6=\frac{4}{3}$

[B] 41. $\frac{1(2)+2(4)+3(3)+\cdots 10(1)}{30}=\frac{157}{30}=5.23$

42. [A] $\sqrt{16}+\sqrt{9}=7$

43. [E]

44. [C]

45. $1.4x = 70$ $\boxed{C}$

$x = 50$

46. C

47. $\frac{2}{3}(6) - 5 + 2(4) + 3(-2) - 5$ $\boxed{B}$

$-1 + -3 = -4$

48. $x, x+2, x+4$ C

$\boxed{C}$

$3x + 6 \le 36$

$3x \le 30$ 13

$x \le 10$

$x + 4 \le 14$

49. $\boxed{A}$

50. $2x - 5y = 22$

$6x - 15y = 22$

$6x - 15y = 66$

$6x - 15y = 22$

$0 = 44$ $\boxed{E}$

impossible