Inner School Test Part A: Answers & Selected solutions

(Numbers not listed here have hand-written scanned solutions.)

9. Wherever the function f(x) intersects the line y = x, it also intersects its inverse. Following this logic through gives us the following solutions **(0,0)**, **(-2,-2)**, **(1,1)**.

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10. Answer: 741
           100a + 10b + c
   abc
   a + b + c = 12
   a - b = b - c
   a > b + c
   a + b + c = 12
   -a + 2b - / + c = 0/3b = 12
   b = 4
   a + c = 8
   8 - c > 4 + c
   4 > 2c
   c < 2
   abc is odd numbered so the digit "c" must be odd.
   Hence c = 1, a = 7 and b = 4
                                         NUMBER 741
15. 16
16. 498,500
17. 1.8 (velocity of the bird)
18. 15
19.36
20. \frac{2}{195}
21. 1760 yds
22.
23. π
24. Al- Khowarizmi
29. 2 at 8 points and 7 at 12 points
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30.
$$x = \frac{1}{2}\sqrt{2-\sqrt{2}} x' - \frac{1}{2}\sqrt{2+\sqrt{2}} y'$$
$$y = \frac{1}{2}\sqrt{2+\sqrt{2}} x' + \frac{1}{2}\sqrt{2-\sqrt{2}} y'$$

31.
$$33\frac{1}{3}$$
 min or $\frac{5}{9}$ hr

32. Alf and Bert are guilty. If Bert is innocent, then Cash is innocent and Alf is guilty; but Alf never works alone. Therefore, no one is guilty. Therefore, Bert cannot be innocent; he must be guilty. If Bert is guilty, then Cash is innocent and Alf is guilty.

33. 8, 16

Player Weight Number			Cumulative
	1		I
1	İ	x .	х
2	Ì	x+2	x+1
3		(x+1)+3	x+2
4		(x+2)+4	x+3
5		(x+3)+5	x+4

Fifth player weighs 8 more kilograms than the first! When difference is 2 lbs ->5th player weighs 16 more kilograms.

34. 7

Let n be the last number on the board. The largest possible average is obtained of the digit 1 is erased; the average is then

(2+3+...+n)/(n-1)=((n+1)n/2-1)(n-1)=(n+2)/2The smallest average possible is obtained when n is erased the average then: n(n-1)/2(n-1) = n/21+2+.....n-1/n-1 Thus n/2 <= 35 7/17 <= n+2/2 n <= 70 14/17 <= n+2 68 14/17 <= n <= 70 14/17 Therefore n = 69 pr 70. Since 35 7/17 is the average of (n-1) integers (35 7/17)(n-1) must be an integer and n is 69. If x is the number erased. 1/2(69)(70)-x/68 = 357/1769 ??? 35 - x = 35 (7/17) 68x = 7 35. $12\pi \frac{m^2}{m}$ sec $A = pi r^2$ $r = x \tan 30 \text{ degrees} = x / 3^{.5}$ $= pi x^2 / 3$ dx / dt = 3 m/s

dA / dt = dA / dx * dx / dt= (2pi * x / 3) * 3m/s= 2pi * x m/sat x = 6 dA/dt = 2 pi (6) m^2/s = 12 pi m^2/s 36. \$215.54 million $P(5) = 30 + 6 \log(5 + 2)/\log 2$ = 46.84 million dollars $80 = 30 + 6 \log 2 (x + 2)$ $50 / 6 = \log 2 (x + 2)$ $2^{(50/6)} = (x + 2)$ x = 320.54 million dollars. therefore: increase in spending = 320.54 - 5 = 215.54 million 37. $c_1 = 81 \text{ km/hr}$ $c_2 = 135 \text{ km/hr}$ Let x = rate of car 1 (m/s)Let y = rate of car 2 (m/s) 1800m = 30x + 30y1800-30x = 30y7200-120x = 120y1800 + 120x = 120ySolve for Car 1: 7200-120x = 1800+120x5400 = 240xx = 22.5 m/sConvert to km/hr x = 22.5m/sec *60sec/min*60min/hr = 81 km/hr 1000 m/km Solve for Car 2: Substitute x = 22.5 m/s 1800+120x = 120yy = 37.5 m/sConvert to km/hr x =37.5m/sec *60sec/min*60min/hr = 135 km/hr 1000 m/km 38. 21.998 0.5*b*h = 11b = length ACb*h = 22 h = length CBb = 22/h c = length AB $c^2 = b^2 + h^2$ $100 = b^2 + h^2$ $100 = (22/h)^2 + h^2$ $100 = 22^2/h^2 + h^2$ $100h^2 = 22^2 + h^4$ $h^{4-100h^{2}+284} = 0$ To solve, use quadratic formula: $h^2 = 100 + [(100)^2 - 4(484)] = 9.74$ 2

 $h^2 = \frac{100 - [(100)^2 - 4(484)]}{2} = 2.258$ Since b = 22/h, then b = 9.74 <----- Therefore, use h= 2.258

39. 76 km/hr

100 - 3t = 88 100 - 88 = 3t t = 12/3t = 4 km/h per person

When six persons are on board, the van travels at 100 - 6t = 100 - 6(4) = 76 km/h

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40. 15.1 sec
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\begin{array}{l} t0 = 2s \mbox{ for speeder} \\ ds = 40 \mbox{ m/s to } + 40 \mbox{ m/s t} \\ dp = 3.0 \mbox{ m/s}^2 \ (t)^2 \\ ds = dp \\ 40 \ (2) + 40t = 3.0t^2 \\ 3.0t^2 - 40t - 80 = 0 \\ t = 40 \ +/- \ (1600 + 4(3)(80))^{-}.5 = 15.1, \ -1.77 \\ \mbox{ time always (t)} \\ \mbox{ It will take the police } 15.1 \ s \ to \ catch \ the speeder. \end{array}
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