Inner School Test Part B

This test can be worked on by any student member of your delegation. No sponsor or adult is to assist with any part of this test. This test must be completed and submitted while all students remain in the assigned room. Once a student leaves the room, he or she will not be allowed to return. Only one answer sheet per school will be accepted. Be sure to put your school name on the answer sheet.

1. Write the number 55 using five 4's.

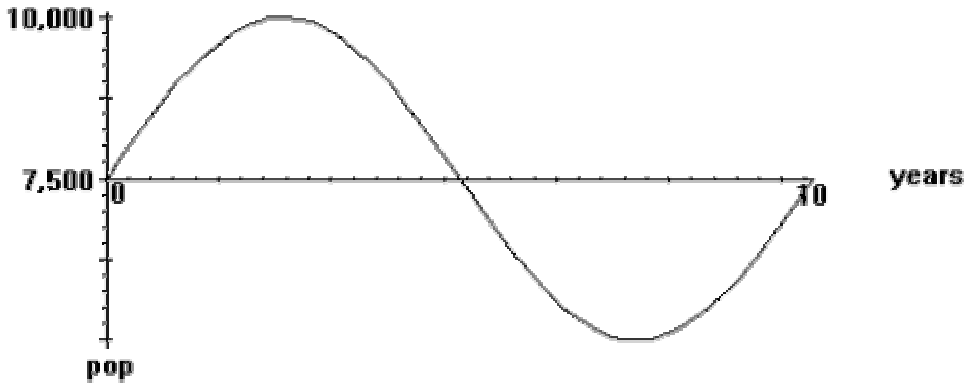
2. Find 3 integers in an arithmetic progression whose product is a prime number.

3. What is wrong with the following proof?

$$5 > 3$$
$$5\left(\log_{10}\frac{1}{2}\right) > 3\left(\log_{10}\frac{1}{2}\right)$$
$$\log_{10}\left(\frac{1}{2}\right)^5 > \log_{10}\left(\frac{1}{2}\right)^3$$
$$\log_{10}\frac{1}{32} > \log_{10}\frac{1}{8}$$
$$\frac{1}{32} > \frac{1}{8}$$

4. A plane climbs 30 meters for every 100 meters of run. If the speed of the plane is 225 kilometers per hour, find how high the plane will be in 6 seconds?
5. Express the function $f(x) = |x| + |x-2|$ without using absolute value signs.
6. Find an expression for all positive integers that meet all 3 criteria
   a. leave a remainder of 3 when divided by 4
   b. leave a remainder of 5 when divided by 6
   c. leave a remainder of 7 when divided by 8

7. The population of a certain species in the wild varies according to a sine curve over a period of 10 years. Using the diagram, determine at what time will the population be at a minimum in the 10 year period.



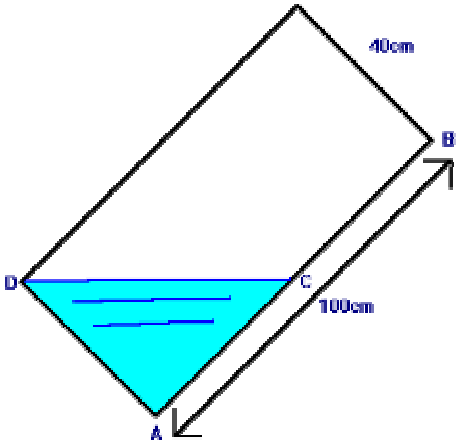
8. A parabola with vertex (2,0) and axis of symmetry parallel to the y-axis, passes through (3,1) and (-3,t). Find the value of t.

9. An IBM Computer Manufacturer finds that when x millions of dollars are spent on research, the profit of (x), in millions of dollars, is p(x) = 20 + 5 log (x+3). How much would you say should be spent on research to make a profit of 40 million dollars? 120 Million dollars?

10. You and your best friend Sparky are playing a game that involves flipping a coin. Sparky has just flipped ten heads in a row. You, being a bright person, are quite suspicious, but Sparky assures you that the coin is a fair coin. If Sparky isn't lying, what is the probability that the next toss of the coin will be heads?

11. If Hypatia randomly takes a coin from her purse, its expected value is 15 cents. If she had another dime, the expected value would be only 14 cents. What coins does she have in her purse ?

12. Faye was babysitting her two year old brother Frank. When Faye wasn't watching, Frank had taken the aquarium and tipped it on its side, so the water level was as shown. When Faye found him, she quickly grabbed the tank and returned it to a horizontal position. What was the depth of the aquarium in cm if the dimensions of the tank are 100cm long, 60cm wide and 40cm high. (C is the midpoint of AB)



13. Find all real numbers x, such that $x^{x^5-13x^3+36x}=1$.
14. Consider the integral $\int_{\sqrt{2}}^{5}\left(4x^2-3\right)^{19}dx$. Determine new upper and lower limits of integration using the substitution $u=4x^2-3$.

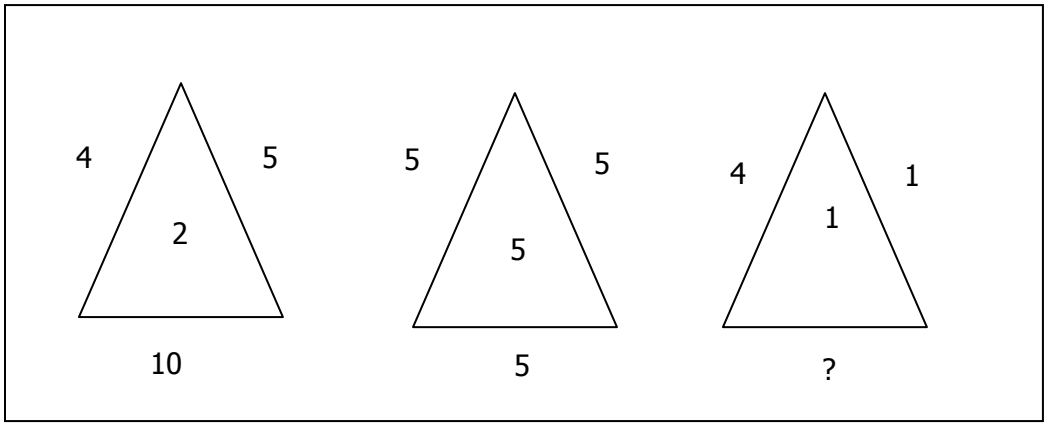
15. Factor fully: $x^6+5x^5-6x^4-x^2-5x+6$.

16. Find the horizontal asymptotes (if any) and state any restrictions for $f(x)=\dfrac{ax^3}{bx^3+cx+d}$

17. Determine the open interval where the graph of $f(x) = x^4 - 4x^3 - 4x^2 + 1$ is concave downward.

18. While driving his car, Bob observes that the odometer reading forms a palindrome. It displays 13,931. He keeps driving. Two hours later, he looks at the odometer agai and, to his surprise, it displays a different palindrome. What is the most likely speed that Bob is traveling?

19. Uncover the pattern and use it to complete the third triangle.



20. Determine a and b so that - 1 will be a double root of the function: $d^4 + ad^3 + (a-b)d^2 + bd + 1$.

21. A high school guidance counselor surveyed 100 freshmen to see their interest in taking the 3 languages offered at the high shoo: French, Spanish and Russian. Since the survey only provided the information listed below, he did not know how many were not interested in any language choice. Determine how many this was.
    - 5 expressed an interest in all 2
    - 8 expressed interest in French and Russian
    - 19 in Spanish and Russian
    - 15 in Spanish and French
    - 31 in Spanish
    - 29 in French
    - 26 in Russian

22. Find (a, b, c, d) if [a, b, c, d]= $\frac{5}{13}$ and a, b, c, and d are positive integers such that

$$[a,b,c,d] = \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}}$$

23. Sergi earns 1% interest a month on every dollar he saves from working. He receives interest on the interest, paid at the end of each month. His deposits, made on the first of each month beginning with March and ending in August were respectively $77, $51, $37, $86, $39, and $35. On August 1, how much money will he have altogether including interest?

24. The braking distance needed to stop a car is proportional to the square of the car's speed. It takes 50 meters for a car traveling 88 kph. Find the rate of change for the braking distance between 88 kph and 100 kph.

25. If $a^x = b$ and $b^y = a$, how are x and y related?

26. Three different planes contain the points (1, 2, 3) and (4, 6, 8). Use this information to determine the intersection of the 3 planes.

27. The whispering gallery in the Capitol's Statuary Hall is an elliptical chamber 46 ft wide and 96 ft long. A politician noted this feature because the desk of the opposing party's floor leader was at one focus. How far from that desk should the politician stand to overhear the floor leader's conversation?

28. Given the non-linear sequence 3, 5, 9, 17, 33, 65, 129, 257, 513, ... Find the next term and an explicit formula for the nth term.

29. Suppose you can get as many $0.22 stamps and $0.03 stamps as you want. You could make $0.24 using 8 $0.03 stamps, but you could not make $0.23 postage exactly. What is the largest value that cannot be made with these stamps?

30. Two sides of a nonisosceles triangle measure 6 cm and 11 cm. If the third length, in cm, of the third side is also an integer, how many possible lengths can the third side be?