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Round 1

Part 1 $a_2 = \frac{10}{2} = 5$; $a_3 = 3(5)+1 = 16$; $a_4 = \frac{16}{2} = 8$

$a_5 = 3(8)+1 = 25$; $a_6 = 3(25)+1 = 76$; $a_7 = \frac{76}{2} = 38$;

$\underline{A = 29}$ $a_8 = \frac{38}{2} = 19$; $a_9 = 3(19)+1 = 58$; $a_{10} = \frac{58}{2} = 29$

Part 2 $r = \frac{\sqrt{6}}{6}$ or $\frac{1}{\sqrt{6}}$ $S = \frac{a}{1-r} = \frac{6}{1-1/\sqrt{6}} = \frac{6\sqrt{6}}{\sqrt{6}-1} \cdot \frac{\sqrt{6}+1}{\sqrt{6}+1}$

$\underline{B = 5}$

$= \frac{36+6\sqrt{6}}{6-1} = \frac{36+6\sqrt{6}}{5}$

Part 3

$y = 4\log_4 x + 1$ $4\log_4(4)+1 = y$

$\underline{C = 20}$ $y = 2\log_4 x + 3$ $4+1 = y$

$0 = 2\log_4 x - 2$ $\underset{\sim}{y=5}$

$\log_4 x = 1 \Rightarrow \underset{\sim}{4^1 = x}$

Final: $|A-B-C| = |29-5-20| = \boxed{4}$

Round 2

Part 1 Odd function $\Rightarrow$ if $P(1) = 2$ then $P(-1) = -2$

so $P(-3) \Rightarrow -5$

$\underline{A = -2}$ $P(-5) = -1$ given

$P(-1) =$ opposite $P(1) \Rightarrow -2$

Part 2 $\cos\theta = -3/5$ QII $\Rightarrow \sin\theta = \frac{4}{5}$

$\underline{B = \frac{1}{25}}$ $\left(-\frac{3}{5} + \frac{4}{5}\right)^2 = \frac{1}{25}$

Part 3 To be a factor $\Rightarrow$ polynomial division with no remainder

Use synthetic division

$$\begin{array}{r|rrrr} 6 & 1 & -K & -21 & -18 \\ & \downarrow & 6 & 36-6K & 90-36K \\ \hline & 1 & 6-K & 15-6K & \end{array}$$

$\underline{C = 2}$ $\Rightarrow 90-36K-18 = 0$

$72-36K = 0$

$36K = 72$

$K = 2$

Final $B^{(A+C)} = \left(\frac{1}{25}\right)^{(-2+2)} = \boxed{1}$

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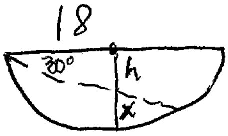
Round 3

Part 1 $\bar{x} = \dfrac{1+x+y+x+y}{4} = \dfrac{1+2x+2y}{4}$

$A = \frac{1}{4}$ Med $= \dfrac{x+y}{2}$ Difference $\dfrac{1+2x+2y-2x-2y}{4} = \frac{1}{4}$

Part 2



$\tan 30^\circ = \dfrac{h}{18}$

$18\left(\frac{1}{\sqrt{3}}\right) = h = \frac{18}{\sqrt{3}} = 6\sqrt{3}$

$B = 18 - 6\sqrt{3}$

$18 - h = x \Rightarrow 18 - 6\sqrt{3} = x$

Part 3

center $= (6,0)$ $2a = 12$, $a = 6$ $2b = 6$, $b = 3$

$C = 6 + 3\sqrt{3}$

$\dfrac{(x-6)^2}{36} + \dfrac{y^2}{9} = 1$ Foci $a^2 = b^2 + c^2$

$36 - 9 = c^2$

$\pm 3\sqrt{3} = \sqrt{27} = c$

Final

$2AB + C$

$2\left(\frac{1}{4}\right)(18 - 6\sqrt{3}) + 6 + 3\sqrt{3}$

$9 - 3\sqrt{3} + 6 + 3\sqrt{3}$

$\boxed{15}$

Round 4

Part 1 (subtract) $x^2 + 1 - 6x - 8 - 9 = 0$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ or -2

$12y + 12 = 0$

$y = -1 \Rightarrow$

$(8,-1)$ or $(2,-1)$

$A = (8)(-1)(-2)(-1) = -16$

Part 2

$4 = 1 + A + B = 3$ $-6 = -1 + A - B - 3$

$A + B = 6$ $A - B = -2$

$2A = 4 \rightarrow A = 2$

$2 + B = 6$

$B = 4$

$B = 2 + 4 = 6$

Part 3

$3^{4\cos^2 x}\, 3^{2\cos x} = 3^{6\left(\frac{1}{3}\right)}\, 3^{2\cos x} \Rightarrow 4\cos^2 x + \cancel{2\cos x} = 2 + \cancel{2\cos x}$

$2\cos^2 x = 1$

$\cos x = \pm\sqrt{\frac{1}{2}} \Rightarrow 45^\circ$ QI

$C = 45$

Final: $A + B + C = -16 + 6 + 45 = \boxed{35}$

Round 5

Part 1 Expand by C_3

$x\begin{vmatrix} 3 & 1 \\ -1 & -2 \end{vmatrix} + 3\begin{vmatrix} 4 & x^2 \\ 3 & 1 \end{vmatrix} = 8$

$A = -1$

$-5x + 12 - 9x^2 = 8$

$9x^2 + 5x - 4 = 0$

$(9x - 4)(x + 1) = 0$

$x = \frac{4}{9}$ or -1

Round 5

Part 2

$B = \frac{-1}{5}$

$y = \frac{3}{4}\sin(2(0) - 3\pi/2) = \frac{3}{4}\sin\left(-\frac{3\pi}{2}\right) = \frac{3}{4}$

$\cot\beta = 3/4 \underset{QII}{\Rightarrow} \cos\beta = -\frac{3}{5} \quad \sin\beta = \frac{4}{5}$

$B = 2\sin\beta + 3\cos\beta = 2\left(\frac{4}{5}\right) + 3\left(-\frac{3}{5}\right) = -\frac{1}{5}$

Part 3

$\Rightarrow {}_nC_4 = {}_nC_5 \Rightarrow n = 9 \quad {}_9C_2\, x^{9-2}(-y)^2 = 36x^7y^2$

$C = \frac{36}{9} = 4$

Final: $\text{int}(4^{-1} - 1/5) = \text{int}\left(\frac{1}{4} - \frac{1}{5}\right) = \text{int}\left(\frac{5-4}{20}\right) = \text{int}\left(\frac{1}{20}\right) = \boxed{0}$

Round 6

Part 1

$A = 8$

$\frac{(x-2)^2}{x+1} = 4 \quad 4x + 4 = x^2 - 4x + 4$

$0 = x^2 - 8x$

$0 = x(x-8) \quad x \neq 0, \ x = 8$

Part 2 least occurs when $x = -4 \Rightarrow$ smallest $|4+x| = 0$

$|5+y| \leq 100 \quad -100 \leq y+5 \leq 100 \quad -105 \leq y \leq 95$

$B = -105$

Part 3

Total possible $= 5^3 = 125$ choose different $\underline{5} \cdot \underline{4} \cdot \underline{3}$

Probability $= \frac{\cancel{5} \cdot 4 \cdot 3}{\cancel{125}} = \frac{12}{25} = C$

Final $= \left(\frac{12}{25}\right)(-105)\left(\frac{1}{8}\right) = \boxed{\frac{-63}{10}}$

Round 7

Part 1 $\frac{1}{x} = x - 1 \quad 1 = x^2 - x \quad x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

$x = \frac{1 \pm \sqrt{5}}{2}$

$A = 1$

$= \frac{1 + \sqrt{5} + 1 - \sqrt{5}}{2}$

Part 2 work backwards

$d + 0 = 1 \Rightarrow d = 1$

$c + d = 0 \Rightarrow c = -1$

$B = -3$

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Round 7 Part 3 $d \Rightarrow 2N \Rightarrow N < 9687$

$d = 1$ so $\begin{array}{r} a6bc \\ \hline 1e3fg \end{array}$

$\underline{C = 4}$ Keep working to Eliminate $\begin{array}{r} 7692 \\ \hline 15384 \end{array}$

Final $|A+B| + C = |1-3| + 4 = \boxed{6}$

Round 8 Part 1 $\tan 15^\circ = \frac{h}{50}$

$\tan 15 =$

$\tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - 1/\sqrt{3}}{1 + 1/\sqrt{3}} = \frac{\sqrt{3}-1}{\sqrt{3}+1}$

$= \frac{3+1-2\sqrt{3}}{3-1} = \frac{4-2\sqrt{3}}{2} = 2-\sqrt{3}$

so $(2-\sqrt{3})50 = h$ $\underline{A = 100 - 50\sqrt{3}}$

Part 2 $a+b+c+d = 72$

$a+5 = b-5 = 5c = \frac{d}{5} = 10$

$\Rightarrow a = 5 \quad b = 15 \quad c = 2 \quad d = 50$

$\underline{B} = 5+1+5+2+5+0 = \underline{18}$

Part 3 $x^2(1+x) \Rightarrow$ perfect square since x^2 already is then when does $(1+x)$ = perfect square?

$(1+x)$ occurs every 1 less than a perfect square $\{1, 4, 9 \ldots 100\}$

$\{0, 3, 8 \ldots 99\}$ $\underline{C = 9}$

Final: $A\left(\frac{B}{C}\right) = (100 - 50\sqrt{3})\left(\frac{18}{9}\right) = \boxed{200 - 100\sqrt{3}}$

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Round 9 Part 1

11 R

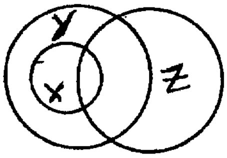
9 clear mornings $9+m$ = total mornings
12 clear afternoons $12+a$ = total afternoons

$\underline{A} = 9+7-11$ $\quad m+a=11$
or $12+4-11 = 16-11 = \underline{5}$

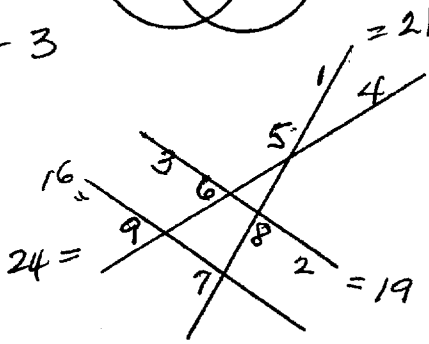
$m-a-3=0$
$m-a=3$
$m+a=11$
$2m=14$
$m=7$ $\quad a=4$

Part 2

$\underline{B=2}$



Part 3



$\underline{C} = 5+6+8 = \underline{19}$

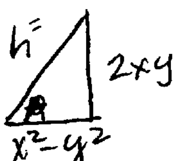
Final

$6A - 4B + C$

$6(5) - 4(2) + 19 = \boxed{41}$

Round 10

Part 1



$h^2 = (x^2-y^2)^2 + (2xy)^2$
$= x^4 - 2x^2y^2 + y^4 + 4x^2y^2$
$= x^4 + 2x^2y^2 + y^4$
$h^2 = (x^2+y^2)^2$ $\quad h = x^2+y^2$

so $\sin A = \dfrac{2xy}{x^2+y^2}$

$\underline{A=4}$

Part 2

$$\begin{array}{r|rrrr} 2 & 1 & 4 & -7 & -10 \\ & & 2 & 12 & 10 \\ \hline & 1 & 6 & 5 & \underline{0} \end{array}$$

$(x-2)(x^2+6x+5) = (x-2)(x+5)(x+1)$

Second part $\Rightarrow (x-3) = 2, -5, -1 \Rightarrow 5, -2, 2$

$\underline{B=225}$

Part 3 The parity of the white marbles is maintained. We start with an even number of white and must end that way. It can not be zero since no provision is made to remove the last 2 whites. So only 2 can be left at end and they must be white. $\quad \underline{C=1}$

Final $\dfrac{\sqrt{B}}{A+C} = \dfrac{15}{4+1} = \boxed{3}$