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Round 1 Part 1

$\underline{A=1}$ Test $t=1$ $f(1)=\left\{\begin{matrix}1\\0\end{matrix}\right\} > 1 \neq 0$ discontinuous @ $t=1$

since $t-2\neq 0$ $t\neq 2$ but interval $t<1$ so $t\neq 2$ does not apply

Part 2

$\underline{B=2}$ $\Rightarrow v(t)=0$ $t+e^t=0$

$e^t=-t$ one pt of intersection but not on $1\leq t\leq 3$

$\sin(t^2)=0$

$\Rightarrow t^2=\pi, 0, 2\pi \ldots$

$t = 0, \sqrt{\pi}, \sqrt{2\pi}\ldots$ only 2 on interval

Part 3

$$\lim_{b\to\infty}\int_2^b x^{-3}dx \Rightarrow \lim_{b\to\infty} -\frac{1}{2x^2}\Big|_2^b = \lim_{b\to\infty}\left(\frac{-1}{2b^2}+\frac{1}{8}\right) = 0+\frac{1}{8}=\frac{1}{8}$$

$\underline{C=\frac{1}{8}}$ Final : $A\sqrt{BC} = 1\sqrt{2(1/8)} = \boxed{1/2}$

Round 2 Part 1

$$\bar{x} = \frac{1+x+y+x+y}{4} = \frac{1+2x+2y}{4} \qquad \text{Med} = \frac{x+y}{2}$$

$$\bar{x} - \text{med} = \frac{1+2x+2y}{4} - \frac{2x+2y}{4} = \frac{1}{4}$$

$\underline{A=\frac{1}{4}}$

Part 2

Center $= (6,0)$

$2a=12$ $2b=6$
$a=6$ $b=3$

$$\frac{(x-6)^2}{36}+\frac{y^2}{9}=1$$

foci
$a^2=b^2+c^2$
$36-9=c^2$
$27=c^2$
$c=\pm 3\sqrt{3}$

$\underline{B=3\sqrt{3}+6}$

Part 3

$$y' = \frac{-1(x-3)-1(1-x)}{(x-3)^2} = \frac{-x+3-1+x}{(x-3)^2} = 2(x-3)^{-2}$$

$$y'' = -4(x-3)^{-3} = \frac{-4}{(x-3)^3}$$

$+$ | $-$ at 3 $(-\infty, 3)$

$\underline{C=3}$

Final: $4A+B-2C$

$4(\frac{1}{4})+3\sqrt{3}+6-2(3) = \boxed{1+3\sqrt{3}}$

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Round 3

Part 1

Apply l'Hôpital's Rule

$\underline{A = \frac{1}{2}}$

$$\lim_{h\to 0} \frac{\frac{1}{2+h} - 0}{1} = \frac{1}{2}$$

Part 2

$x^3 - 3x + 2 = x + 2$

$x^3 - 4x = 0$

$\underline{B = 8}$

$x(x+2)(x-2) = 0$

$-2, 0, 2$

$$\int_{-2}^{0} (x^3 - 3x + 2 - x - 2)\,dx + \int_{0}^{2} (x + 2 - x^3 - 3x + 2)\,dx$$

$$= \left(\frac{x^4}{4} - 2x^2\right)\Big|_{-2}^{0} + \left(2x^2 - \frac{x^4}{4}\right)\Big|_{0}^{2}$$

$$= -4 + 8 + 8 - 4 = 8$$

Part 3

$$\frac{1}{2}(x^2 - y^2)^{-1/2}(2x - 2yy') = 1 + y' \quad @ (5,4)$$

$$\frac{x}{\sqrt{x^2 - y^2}} - \frac{y}{\sqrt{x^2-y^2}}y' = 1 + y'$$

$$\frac{5}{3} - \frac{4}{3}y' = 1 + y'$$

$$\frac{2}{3} = \frac{7}{3}y'$$

$$2/7 = y'$$

$$y - 4 = \frac{2}{7}(x - 5)$$

$$y = \frac{2}{7}x - \frac{10}{7} + 4$$

$$y = 2/7x + \frac{18}{7}$$

$$C = \frac{2}{7} + \frac{18}{7} = \frac{20}{7}$$

Final: $\frac{AB}{C} = \frac{1}{2}(8) \cdot \frac{7}{20} = \boxed{\frac{7}{5}}$

Round 4

Part 1

$\underline{A = 6}$

$$\frac{\cancel{x-10}}{\cancel{(x-10)}(x+5)} = 2^{-7}$$

$x + 5 = 128$

$x = 123$

Part 2

$$y' = \frac{1(x+3) - 1(x+2)}{(x+3)^2} = \frac{x+3-x-2}{(x+3)^2} = \frac{1}{(x+3)^2} \text{ or } (x+3)^{-2}$$

$$y'' = -2(x+3)^{-3} = \frac{-2}{(x+3)^3}$$

$\underline{B = -2}$

Part 3

$y' = \cos x - \sin x = 0$ or ~~undefined~~

$\cos x = \sin x \qquad x = \frac{\pi}{4}, \cancel{5\pi/4}$

$\underline{C = \frac{\pi}{4}}$

Final: $\frac{C}{\pi}(A+B) = \frac{1}{4}(6 + {}^{-}2) = \boxed{1}$

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Round 5 Part 1

$\vec{V} = \langle 0+2, 4-3, 8-5 \rangle = \langle 2,1,3 \rangle$ $\|\vec{v}\| = \sqrt{2^2+1^2+3^2}$

unit vector $= \dfrac{\vec{V}}{\|\vec{V}\|} = \left\langle \dfrac{2}{\sqrt{14}}, \dfrac{1}{\sqrt{14}}, \dfrac{3}{\sqrt{14}} \right\rangle$ $\underline{A = 14-6 = 8}$

Part 2

$\lim_{x\to\infty} \left(1+\frac{3}{x}\right)^x$

$\ln y = \lim_{x\to\infty} x \ln\left(1+\frac{3}{x}\right) = \lim_{x\to\infty} \dfrac{\ln(1+3/x)}{\frac{1}{x}} = \lim_{x\to\infty} \left(\dfrac{1}{1+3/x}\right)\left(\dfrac{-3x^{-2}}{-x^{-2}}\right)$

$\underline{B = e^3}$ $= \lim_{x\to\infty} \left(\dfrac{3}{1+3/x}\right) = 3$

so $\ln y = 3$

$y = e^3$

Part 3

$f(0) = \sqrt{1+0} = 1$

$f'(0) = \frac{1}{2}(1+0)^{-1/2} = \frac{1}{2}$

$f''(0) = -\frac{1}{4}(1+0)^{-3/2} = -\frac{1}{4}$

$f'''(0) = \frac{3}{8}(1+0)^{-5/2} = \frac{3}{8}$

$\underline{C = \frac{1}{16}}$ $1 + \frac{1}{2}x + \dfrac{\left(-\frac{1}{4}\right)}{2}x^2 + \dfrac{\left(\frac{3}{8}\right)}{6}x^3 = 1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{1}{16}x^3$

Final: $CA - B = \frac{1}{16}(8) - e^3 = \boxed{\frac{1}{2} - e^3}$

Round 6 Part 1

x	y	$\Delta y = \frac{dy}{dx}(\Delta x)$
2	1	$3(0.1) = .3$
2.1	1.3	$3.4(0.1) = 0.34$
2.2	1.64	

$\underline{A = 1.64}$

Part 2

$V = \pi \int_{-1}^{1} x^2\,dy = 2\pi \int_0^1 (4-y^2)\,dy$ circle $x^2+y^2 = 4$ $\underline{B = \frac{22\pi}{3}}$

Part 3

$y = x^2+1$

$\frac{dy}{dt} = 2x\frac{dx}{dt} + \; = 2(1)\left(\frac{3}{2}\right) + 0 =$

$s =$ distance from origin $= \sqrt{x^2+y^2}$

$s' = \dfrac{1\left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)}{2(x^2+y^2)^{1/2}} = \dfrac{1\left(\frac{3}{2}\right) + 2(3)}{\sqrt{1+4}}$

$= \dfrac{15}{2\sqrt{5}} = \underline{\dfrac{3\sqrt{5}}{2} = C}$

Final: $\left(\frac{B}{\pi}\right)\left(\frac{C}{\sqrt{5}}\right) + A = 11 + 1.64 = \boxed{12.64}$

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Round 7

$1 + \frac{2}{4}(x-1) + \frac{3}{9}(x-1)^2 + \frac{4}{16}(x-1)^3 + \dots$

Part 1 nth term $\frac{(x-1)^{n-1}}{n-1}$

$1 + \frac{1}{2}(x-1) + \frac{1}{3}(x-1)^2$

$\lim_{n\to\infty} \left| \frac{(x-1)^n}{n} \cdot \frac{n-1}{(x-1)^{n-1}} \right| < 1$

$\lim_{n\to\infty} |x| \cdot \left| \frac{n-1}{n} \right| < 1$

$\underline{A=0}$

$\Rightarrow |x| < 1 \qquad -1 < x < 1$ Check Endpoints

Part 2 arc length $\int_1^4 \sqrt{1 + \left(x - \frac{1}{4x}\right)^2}\, dx$ $\qquad y = \frac{1}{2}x^2 - \frac{1}{4}\ln x$

$y' = x - \frac{1}{4x}$

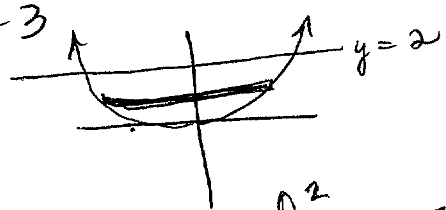
$= \int_1^4 x + \frac{1}{4x}\, dx = \frac{x^2}{2} + \frac{1}{4}\ln x \Big|_1^4 = 8 + \frac{1}{4}\ln 4 - \frac{1}{2}$

$\underline{B = \frac{15}{2}}$

$= \frac{15}{4} + \frac{1}{4}\ln 4 = \frac{15 + \ln 4}{4}$

or $\frac{15}{2} + \ln 2$

Part 3



$x^2 = 4y$

$x = \sqrt{4y}$ $\qquad$ side of sq. $= 2 \cdot 2\sqrt{y}$

$4\sqrt{y}$

$\int_0^2 (4\sqrt{y})^2 dy = 16\left(\frac{y^2}{2}\right)\Big|_0^2 = 8(4) = \underline{32 = C}$

Final: $(A+B)\left(\frac{C}{5}\right) = \frac{15}{2} \cdot \frac{32}{5} = 3(16) = \boxed{48}$

Round 8 Part 1

$a(t) = (2, e^{-t})$

$v(t) = (2t + c_1, -e^{-t} + c_2) = (0,0) \Rightarrow c_1 = 0;\ c_2 = +1$

position$(t) = (t^2 + c_3, +e^{-t} + t + c_4) = (3,3) \Rightarrow c_3 = 3;\ c_4 = 2$

so at $t = 2$ $\quad (7, e^{-2} + 4)$

$\underline{A = 4}$

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Round 8

Part 2

$a(t) = 4\pi \cos t \Rightarrow v(t) = 4\pi \sin t + C_1 \quad v=0, t=0$

$0 = 0 + C_1 \Rightarrow C_1 = 0$

$p(t) = -4\pi \cos t + C_2$

$t=0 \quad -4\pi + C_2 = 0$

$C_2 = 4\pi$

$p(t) = -4\pi \cos t + 4\pi$

$\text{avg. vel.} = \dfrac{p(\pi) - p(0)}{\pi - 0} = \dfrac{4\pi + 4\pi}{\pi} = \underline{8 = B}$

Part 3

$a_1 = 10 \qquad \frac{10}{2} = 5 = a_2$ odd

10, 5, 16

$a_3 = 3(5)+1 = 16$ even

$a_4 = \frac{16}{2} = 8$

$a_5 = \frac{8}{2} = 4$

$a_6 = \frac{4}{2} = 2$

$a_7 = \frac{2}{2} = 1$

$a_8 = 3(1)+1 = 4$

$a_9 = \frac{4}{2} = 2$

$a_{10} = \frac{2}{2} = 1$

$\underline{C = 1}$

Final: $A^C \cdot B$

$4^1(8)$

$= \boxed{32}$

Round 9

Part 1

m = rainy mornings 9 clear so $9 + m$ = total mornings

a = " afternoons 12 clear $12 + a$ = " afternoons

$m + a = 11$ — total rainy

$m - a - 3 = 0$

~~$9 + m$ = total mornings~~ $\rightarrow$ $m - a = 3$

$m + a = 11$

$2m = 14$

$m = 7$

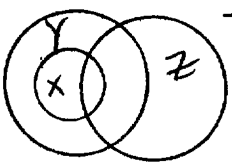
$9 + 7 = 16$ = total days

$-\ 11$ rainy

$\underline{A = 5}$ clear all day

Part 2

$\underline{B = 2}$



Part 3 $a + b + c + d = 72$

$a + 5 = b - 5 = 5c = \frac{d}{5} = 10$

$a = 5 \quad b = 15 \quad c = 2 \quad d = 50$

$\underline{C} = 5 + 1 + 5 + 2 + 5 + 0 = \underline{18}$

Final: $\dfrac{C}{B} + A = \dfrac{18}{2} + 5 = \boxed{14}$

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Round 10 Part 1

2)
$$\begin{array}{r|rrrr} & 1 & 4 & -7 & -10 \\ & & 2 & 12 & -5 \\ \hline & 1 & 6 & 5 & \boxed{0} \end{array}$$

$(x-2)(x+5)(x+1)$

$x = 2, -5, -1$

Second part $\Rightarrow$ $(x-3) = 2$ or -5 or -1

so $x = 5, -2, 2$

$A = 225$

Part 2 The parity of white marbles is maintained throughout the iterative process. We start with an even number of white and must end that way. It cannot be zero since no provision is made to remove the last 2 whites. So only 2 can be left at end and they must be white. $\Rightarrow$ p(2 remaining white) = 1

$B = 1$

Part 3 $d = 2N \Rightarrow N < 9687$

$d = 1$ so $\dfrac{a6bc}{1e3fg}$ Keep working to eliminate & reason.

$$\frac{7692}{15384}$$

$C = 4$

Final $C - 2B + \sqrt{A} = 4 - 2(1) + \sqrt{225}$

$2 + 15 = \boxed{17}$