1. $(3-4i)+(-5+7i)=$

Solution:

$(3-4i)+(-5+7i)=-2+3i$

Answer:

B

Alpha - Complex Numbers - Solution

2. $(8-6i)-(2i-7)=$

Solution:

$(8-6i)+(-2i+7) = 15-8i$

Answer:

A

3. $(5+3i) - [(-1+2i) + (7-5i)] =$

Solution:

$$(5+3i) - [(-1+2i)+(7-5i)] = (5+3i) - (6-3i)$$
$$= (5+3i) + (-6+3i)$$
$$= -1+6i$$

Answer:

D

4. $(4+2i)(2-3i) =$

Solution:

$(4+2i)(2-3i) = 8 - 12i + 4i - 6i^2$

$= 8 - 8i + 6$

$= 14 - 8i$

Answer:

B

5. $(2-i)(-3+2i)(5-4i) =$

Solution:

$$\begin{aligned}(2-i)(-3+2i)(5-4i) &= (-6+4i+3i+2)(5-4i)\\ &= (-4+7i)(5-4i)\\ &= -20+16i+35i+28\\ &= 8+51i\end{aligned}$$

Answer:

B

6. $\dfrac{3-2i}{2-3i} =$

Solution:

$$\frac{3-2i}{2-3i}\left(\frac{2+3i}{2+3i}\right) = \frac{6+9i-4i+6}{4+6i-6i+9}$$

$$= \frac{12+5i}{13}$$

$$= \frac{12}{13} + \frac{5}{13}i$$

Answer:

E

7. $|\sqrt{5} - 3i| =$

Solution:

$$|\sqrt{5} - 3i| = \sqrt{(\sqrt{5})^2 + (-3)^2}$$
$$= \sqrt{5 + 9}$$
$$= \sqrt{14}$$

Answer:

B

8. Find real numbers x and y such that

$3x + 2iy - ix + 5y = 7+5i$

Solution:

$3x + 2iy - ix + 5y = (3x+5y) + (-x+2y)i = 7+5i$

So,

$$\begin{cases} 3x+5y = 7 \\ -x+2y = 5 \end{cases}$$

$$\begin{cases} 3x+5y = 7 \\ -3x+6y = 15 \end{cases}$$

$11y = 22$

$y = 2$

$3x + 5(2) = 7$

$3x = -3$

$x = -1$

$x = -1, \; y = 2$

Answer:

D

9. $|(x+2) + i(y-1)| = 4$ represents the equation of a circle with

Solution:

$|(x+2) + i(y-1)| = 4$

$\sqrt{(x+2)^2 + (y-1)^2} = 4$

$(x+2)^2 + (y-1)^2 = 16$

center $(-2, 1)$ and radius 4

Answer:

B

10. Express $-\sqrt{6}-\sqrt{2}\,i$ in polar form.

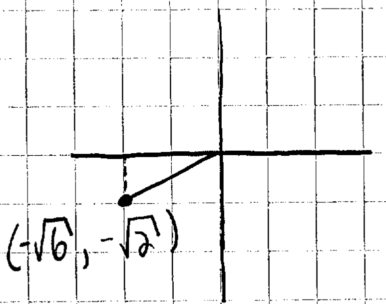
Solution:

$r = \sqrt{(-\sqrt{6})^2+(-\sqrt{2})^2}$

$= \sqrt{6+2}$

$= \sqrt{8}$

$= 2\sqrt{2}$



$y = r\sin\theta \qquad \theta \text{ in III}$

$-\sqrt{2} = 2\sqrt{2}\sin\theta \qquad \theta \text{ in III}$

$-\frac{1}{2} = \sin\theta \qquad \theta \text{ in III}$

$\theta = 210^\circ$

So, $-\sqrt{6}-\sqrt{2}\,i = 2\sqrt{2}\left(\cos\frac{2\pi}{6} + i\sin\frac{2\pi}{6}\right)$

Answer:

C

11. Express

$$3\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\cdot 7\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$$

in polar form.

Solution:

$$21\left[\cos\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)+i\sin\left(\frac{\pi}{2}+\frac{3\pi}{4}\right)\right]=$$

$$21\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$$

Answer:

A

12. Express

$$5\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right) \div 2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$$

in polar form

Solution:

$$\frac{5}{2}\left[\cos\left(\frac{3\pi}{4} - \frac{2\pi}{3}\right) + i\sin\left(\frac{3\pi}{4} - \frac{2\pi}{3}\right)\right] =$$

$$\frac{5}{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$$

Answer:

D

13. Express

$[2(\cos 15^\circ + i\sin 15^\circ)]^7 \div [4(\cos 45^\circ + i\sin 45^\circ)]^3$

in rectangular form

Solution

$128(\cos 105^\circ + i\sin 105^\circ) \div [64(\cos 135^\circ + i\sin 135^\circ)] =$

$2[\cos(-30^\circ) + i\sin(-30^\circ)] =$

$2\left(\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = \sqrt{3} - i$

Answer:

B

14. Express $(\sqrt{3}-i)^5$ in polar form.

Solution

$$(\sqrt{3}-i) = 2(\cos 30^\circ - i\sin 30^\circ)$$

$$(\sqrt{3}-i)^5 = [2(\cos 30^\circ - i\sin 30^\circ)]^5$$

$$= 32(\cos 150^\circ - i\sin 150^\circ)$$

$$= 32\left(\cos \frac{5\pi}{6} - i\sin \frac{5\pi}{6}\right)$$

$$= 32\left[\cos\left(-\frac{5\pi}{6}\right) + i\sin\left(-\frac{5\pi}{6}\right)\right]$$

Answer:

D

15. Which of the following is NOT a fourth root of $(-2\sqrt{3}-2i)$?

Solution:

$$-2\sqrt{3}-2i = 4\left[\cos\left(\frac{7\pi}{6}+2k\pi\right)+i\sin\left(\frac{7\pi}{6}+2k\pi\right)\right]$$

$$(-2\sqrt{3}-2i)^{\frac{1}{4}} = 4^{\frac{1}{4}}\left[\cos\left(\frac{\frac{7\pi}{6}+2k\pi}{4}\right)+i\sin\left(\frac{\frac{7\pi}{6}+2k\pi}{4}\right)\right]$$

$k=0$, $\sqrt{2}\left(\cos\frac{7\pi}{24}+i\sin\frac{7\pi}{24}\right)$

$k=1$, $\sqrt{2}\left(\cos\frac{19\pi}{24}+i\sin\frac{19\pi}{24}\right)$

$k=2$, $\sqrt{2}\left(\cos\frac{31\pi}{24}+i\sin\frac{31\pi}{24}\right)$

$k=3$, $\sqrt{2}\left(\cos\frac{43\pi}{24}+i\sin\frac{43\pi}{24}\right)$

Answer:

E

16. Form the cubic equation with zeros 2 and $1 \pm i$.

Solution:

$(x-2)[x-(1+i)][x-(1-i)] =$

$(x-2)(x-1-i)(x-1+i) =$

$(x^2 - x - xi - 2x + 2 + 2i)(x-1+i) =$

$\cancel{x^3} - \cancel{x^2} + \cancel{x^2 i} - \cancel{x^2} + \cancel{x} - \cancel{xi} - \cancel{x^2 i} + \cancel{xi} + \cancel{x} - \cancel{2x^2} + \cancel{2x} - \cancel{2xi} +$

$\cancel{2x} - 2 + \cancel{2i} + \cancel{2xi} - \cancel{2i} - 2 =$

$x^3 - 4x^2 + 6x - 4$

Answer:

B

17. For the simplest equation with 3 as a doubb root and 2i as another root.

Solution:

$(x-3)(x-3)(x-2i)(x+2i)=0$

$(x^2-6x+9)(x^2+4)=0$

$\cancel{x^4}+\cancel{4x^2}-\cancel{6x^3}-\cancel{24}x+\cancel{9x^2}+36=0$

$x^4-6x^3+13x^2-24x+36$

Answer:

C

18. If $z = 2+3i$ and $w = -3+5i$, calculate $z \cdot w$.

Solution:

$$
\begin{aligned}
z \cdot w &= 2(-3) + (3)(5) \\
&= -6 + 15 \\
&= 9
\end{aligned}
$$

or

$$
\begin{aligned}
z \cdot w &= \mathrm{Re}(\bar{z} w) \\
&= \mathrm{Re}[(2-3i)(-3+5i)] \\
&= \mathrm{Re}(-6 + 10i + 9i + 15) \\
&= \mathrm{Re}(9 + 19i) \\
&= 9
\end{aligned}
$$

Answer:

A

19. If $z = 4+7i$ and $w = -3-2i$, calculate $z \times w$.

Solution:

$$z \times w = 4(-2) - 7(-3)$$
$$= -8 + 21$$
$$= 13$$

or

$$z \times w = \text{Im}(\bar{z} w)$$
$$= \text{Im}[(4-7i)(-3-2i)]$$
$$= \text{Im}(-12 - 8i + 21i - 14)$$
$$= \text{Im}(-26 + 13i)$$
$$= 13$$

Answer:

B

20. Let θ be the acute angle between $z = 2+2i$ and $w = \sqrt{3} - i$.

Solution:

$$\cos\theta = \frac{z \cdot w}{|z||w|}$$

$$= \frac{2\sqrt{3} - 2}{(\sqrt{8})(2)}$$

$$= \frac{2\sqrt{3} - 2}{4\sqrt{2}}$$

$$= \frac{\sqrt{3} - 1}{2\sqrt{2}} \left(\frac{\sqrt{2}}{\sqrt{2}}\right)$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$

Answer:

C

For 21-25, $z = 2+3i$, $w = \sqrt{3} - i$, and $u = -2+5i$.

21. $z^2 - 4z + 3 =$

Solution:

$(2+3i)^2 - 4(2+3i) + 3 = 4 + 12i - 9 - 8 - 12i + 3$

$= -10$

Answer:

0

22 $\bar{z}(w-\bar{u}) =$

Solution:

$(2-3i)[\sqrt{3}-i-(-2-5i)] =$

$(2-3i)(\sqrt{3}-i+2+5i) =$

$(2-3i)(\sqrt{3}+2+4i) =$

$2\sqrt{3}+4+8i-3i\sqrt{3}-6i+12 =$

$(2\sqrt{3}+16)+i(2-3\sqrt{3})$

Answer:

E

23. $|z-w+u| =$

Solution

$|2+3i - \sqrt{3}+i - 2+5i| = |-\sqrt{3}+9i|$

$= \sqrt{(-\sqrt{3})^2+9^2}$

$= \sqrt{3+81}$

$= \sqrt{84}$

$= 2\sqrt{21}$

Answer:

B

24. $\mathrm{Re}(z^2 - 3w - u) =$

Solution:

$\mathrm{Re}(\cancel{4} + 12i - 9i - 3\cancel{\sqrt{3}} + 3i + \cancel{2} - 5i) =$

$\mathrm{Re}[(-3 - 3\sqrt{3}) + 4i] =$

$-3 - 3\sqrt{3}$

Answer:

D

25. $\text{Im}\left(\frac{zu}{w}\right) =$

Solution:

$$\text{Im}\left[\frac{(2+3i)(-2+5i)}{\sqrt{3}-i}\right] = \text{Im}\left(\frac{-4+10i-6i-15}{\sqrt{3}-i}\right)$$

$$= \text{Im}\left(\frac{-19+4i}{\sqrt{3}-i}\right)$$

$$= \text{Im}\left[\frac{(-19+4i)(\sqrt{3}+i)}{4}\right]$$

$$= \text{Im}\left(\frac{-19\sqrt{3}-19i+4i\sqrt{3}-4}{4}\right)$$

$$= \frac{-19+4\sqrt{3}}{4}$$

Answer:

A

26. $\dfrac{i^4 + 3i^9 + i^{16}}{2 - i^5 + i^{10} + i^{15}} =$

Solution:

$$\frac{1 + 3i + 1}{2 - i - 1 - i} = \frac{2 + 3i}{1 - 2i}\left(\frac{1+2i}{1+2i}\right)$$

$$= \frac{2 + 4i + 3i - 6}{1 + 4}$$

$$= \frac{-4}{5} + \frac{7i}{5}$$

Answer:

B

27. $\dfrac{2+2i}{-1+3i} + \dfrac{1-2i}{2-3i} =$

solution $\dfrac{2+2i}{-1+3i}\left(\dfrac{-1-3i}{-1-3i}\right) + \dfrac{1-2i}{2-3i}\left(\dfrac{2+3i}{2+3i}\right) =$

$\dfrac{-2-6i-2i+6}{10} + \dfrac{2+3i-4i+6}{13} =$

$\dfrac{4-8i}{10} + \dfrac{8-i}{13} =$

$\dfrac{2-4i}{5}\left(\dfrac{13}{13}\right) + \dfrac{8-i}{13}\left(\dfrac{5}{5}\right) =$

$\dfrac{26-52i+40-5i}{65} = \dfrac{66}{65} - \dfrac{57}{65}i$

Answer:

A

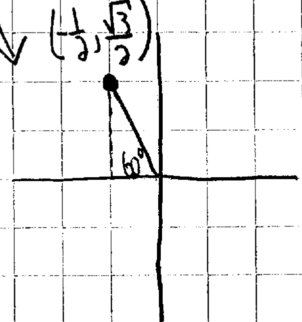
28. Express the roots of $x^2+x+1=0$ in polar form.

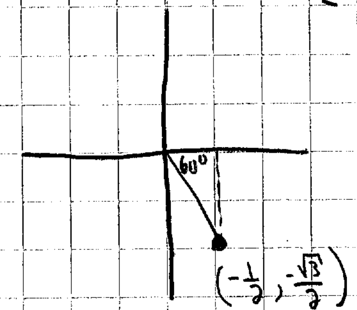
Solution:

$$x = \frac{-1 \pm \sqrt{1^2-4(1)(1)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{-3}}{2}$$

$$= -\frac{1}{2} + \frac{\sqrt{3}}{2} i \quad \text{or} \quad -\frac{1}{2} - \frac{\sqrt{3}}{2} i$$





So

$$-\frac{1}{2} + \frac{\sqrt{3}}{2} i = \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$$

Answer: B

and

$$-\frac{1}{2} - \frac{\sqrt{3}}{2} i = \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$$

29. Find all complex cube roots of 27 in rectangular form.

Solution:

$x^3 = 27$

$x^3 - 27 = 0$

$(x-3)(x^2+3x+9) = 0$

$x - 3 = 0 \qquad x^2 + 3x + 9 = 0$

$x = 3 \qquad x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(9)}}{2(1)}$

$= \dfrac{-3 \pm \sqrt{-27}}{2}$

$= \dfrac{-3 \pm 3i\sqrt{3}}{2}$

$= -\dfrac{3}{2} \pm \dfrac{3\sqrt{3}}{2} i$

Solution:

C

30. Find the area of a triangle having vertices at $(-4-i)$, $(1+2i)$, and $(4-3i)$.

Solution:

choose sign which makes positive

$$\text{Area} = \pm\frac{1}{2}\begin{vmatrix} -4 & -1 & 1 \\ 1 & 2 & 1 \\ 4 & -3 & 1 \end{vmatrix}$$

$$= \pm\frac{1}{2}\left[-4(5) - (-1)(-3) + 1(-11)\right]$$

$$= \pm\frac{1}{2}(-20-3-11)$$

$$= \pm\frac{1}{2}(-34)$$

$$= -\frac{1}{2}(-34)$$

$$= 17$$

Answer:

B

Bonus: Given that $3-2i$ is a zero of

$h(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$,

find the remaining zeros.

Solution:

$3+2i$ is also a zero

$[x-(3-2i)]\,[x-(3+2i)] =$

$(x-3+2i)(x-3-2i) =$

$\cancel{x^2} - 3x - 2\cancel{x}i - 3\cancel{x} + 9 + \cancel{6}i + 2\cancel{x}i - \cancel{6}i + 4 =$

$x^2 - 6x + 13$

$$
\begin{array}{r|l}
 & x^2 - 3x - 10 \\
\hline
x^2 - 6x + 13 & x^4 - 9x^3 + 21x^2 + 21x - 130 \\
 & -\underline{x^4 - 6x^3 + 13x^2} \\
 & -3x^3 + 8x^2 + 21x \\
 & -\underline{-3x^3 + 18x^2 - 39x} \\
 & -10x^2 + 60x - 130 \\
 & -\underline{-10x^2 + 60x - 130} \\
 & 0
\end{array}
$$

$x^2 - 3x - 10 = 0$

$x = -2, 5$

Answer:

D