Mu Alpha Theta National Convention
Mississippi State University 2002
ALPHA DIVISION--Complex Numbers

. i is the imaginary unit. When possible, express answers in $a + bi$ form.

1. $(3-4i)+(7i-5)=$

   a. $10-9i$
   b. $-2+3i$
   c. $-9+10i$
   d. $-2-3i$
   e NOTA

2. $(8-6i)-(2i-7)=$

   a. $15-8i$
   b. $1-8i$
   c. $6+i$
   d. $1+9i$
   e. NOTA

3. $(5+3i)-\left[(-1+2i)+(7-5i)\right]=$

   a. -1
   b. 11
   c. $11+6i$
   d. $-1+6i$
   e. NOTA

4. $(4+2i)(2-3i)=$

   a. $2-8i$
   b. $14-8i$
   c. $2+8i$
   d. $14+16i$
   e. NOTA

5. $(2-i)(-3+2i)(5-4i)=$

   a. $-48+51i$
   b. $8+51i$
   c. $8+41i$
   d. $-48+41i$
   e. NOTA

6. $\dfrac{3-2i}{2-3i} =$

   a. $\dfrac{5}{13}i$
   b. $-\dfrac{12}{5}-i$
   c. $\dfrac{12}{13}-\dfrac{5}{13}i$
   d. $\dfrac{12}{13}+i$
   e. NOTA

7. $\left|\sqrt{5}-3i\right| =$

   a. 14
   b. $\sqrt{14}$
   c. $\sqrt{8}$
   d. 8
   e. NOTA

8. Find all real numbers x and y such that $3x+2iy-ix+5y=7+5i$.

   a. $x=1, y=2$
   b. $x=-1, y=-2$
   c. $x=1, y=-2$
   d. $x=-1, y=2$
   e. NOTA

9. $\left|(x+2)+i(y-1)\right| = 4$ represents the equation of a circle with

   a. center $(2,-1)$ and radius 4
   b. center $(-2,1)$ and radius 4
   c. center $(2,-1)$ and radius 16
   d. center $(-2,1)$ and radius 16
   e. NOTA

10. Express $-\sqrt{6}-i\sqrt{2}$ in polar form.

a. $2\sqrt{2}\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$

b. $2\sqrt{2}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

c. $2\sqrt{2}\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$

d. $2\sqrt{2}\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$

e. NOTA

11. Express $3\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)\cdot 7\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)$ in polar form.

a. $21\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$

b. $21\left(\cos\frac{3\pi^2}{8}+i\sin\frac{3\pi^2}{8}\right)$

c. $21(\cos\pi+i\sin\pi)$

d. $10\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$

e. NOTA

12. Express $5\left(\cos\frac{3\pi}{4}+i\sin\frac{3\pi}{4}\right)\div\left[2\left(\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}\right)\right]$ in polar form.

a. $\frac{5}{2}(\cos\pi+i\sin\pi)$

b. $3(\cos\pi+i\sin\pi)$

c. $3\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

d. $\frac{5}{2}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

e. NOTA

13. Express $\left[2\left(\cos 15^\circ + i\sin 15^\circ\right)\right]^7 \div \left[4\left(\cos 45^\circ + i\sin 45^\circ\right)\right]^3$ in rectangular form.

a. $-\sqrt{3}+i$
b. $\sqrt{3}-i$
c. $-\sqrt{3}-i$
d. $\sqrt{3}+i$
e. NOTA

14. Express $\left(\sqrt{3}-i\right)^5$ in polar form.

a. $32\left(\cos\frac{11\pi}{6}+i\sin\frac{11\pi}{6}\right)$
b. $32\left(\cos\frac{5\pi}{6}+i\sin\frac{5\pi}{6}\right)$
c. $32\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$
d. $32\left(\cos\frac{7\pi}{6}+i\sin\frac{7\pi}{6}\right)$
e. NOTA

15. Which of the following is NOT a fourth root of $-2\sqrt{3}-2i$?

a. $\sqrt{2}\left(\cos\frac{31\pi}{24}+i\sin\frac{31\pi}{24}\right)$
b. $\sqrt{2}\left(\cos\frac{7\pi}{24}+i\sin\frac{7\pi}{24}\right)$
c. $\sqrt{2}\left(\cos\frac{43\pi}{24}+i\sin\frac{43\pi}{24}\right)$
d. $\sqrt{2}\left(\cos\frac{19\pi}{24}+i\sin\frac{19\pi}{24}\right)$
e. NOTA

16. Form the cubic equation with zeros 2 and $1 \pm i$.

a. $x^3 - 4x^2 + 6x + 4$
b. $x^3 - 4x^2 + 6x - 4$
c. $x^3 - 4x^2 + 4x + 4$
d. $x^3 - 4x^2 + 4x - 4$
e. NOTA

17. Form the simplest equation with 3 as a double root and $2i$ as another root.

a. $x^4 - 6x^3 + 5x^2 + 24x - 36$
b. $x^4 + 13x^2 + 36$
c. $x^4 - 6x^3 + 13x^2 - 24x + 36$
d. $x^4 - 6x^3 + 13x^2 - 24x - 36$
e. NOTA

18. If $z = 2 + 3i$ and $w = -3 + 5i$, calculate $z \cdot w$.

a. 9
b. $\sqrt{29}$
c. 19
d. $-21 + i$
e. NOTA

19. If $z = 4 + 7i$ and $w = -3 - 2i$, calculate $z \times w$.
a. $\sqrt{74}$
b. 13
c. - 26
d. $2 - 29i$
e. NOTA

20. Let θ be the angle between $z = 2 + 2i$ and $w = \sqrt{3} - i$. Calculate $\cos\theta$.
a. $\dfrac{-\sqrt{6} + \sqrt{2}}{4}$
b. $\dfrac{-\sqrt{6} - \sqrt{2}}{4}$

c. $\dfrac{\sqrt{6} - \sqrt{2}}{4}$
d. $\dfrac{\sqrt{6} + \sqrt{2}}{4}$
e. NOTA

NOTE: For 21 - 25, $z = 2 + 3i$, $w = \sqrt{3} - i$ and $u = -2 + 5i$.

21. $z^2 - 4z + 3 =$

a. 8
b. $-10 + 24i$
c. $8 + 24i$
d. - 10
e. NOTA

22. $\bar{z}(w - \bar{u}) =$

a. $2\sqrt{3} + 3i\sqrt{3}$
b. $\left(2\sqrt{3} + 16\right) - \left(2 - 3\sqrt{3}\right)i$
c. $\left(2\sqrt{3} + 16\right) + \left(2 + 3\sqrt{3}\right)i$
d. $\left(2\sqrt{3} - 8\right) + \left(2 - 3\sqrt{3}\right)i$
e. NOTA

23. $|z - w + u| =$

a. $2\sqrt{13}$
b. $2\sqrt{21}$
c. $4\sqrt{13}$
d. $4\sqrt{21}$
e. NOTA

24. $\mathrm{Re}\left(z^2 - 3w - u\right) =$

a. $15 - 3\sqrt{3}$
b. $-7 - 3\sqrt{3}$
c. $11 - 3\sqrt{3}$
d. $-3 - 3\sqrt{3}$
e. NOTA

25. $\mathrm{Im}\left(\frac{zu}{w}\right) =$

a. $\frac{-19+4\sqrt{3}}{4}$

b. $\frac{-19+4\sqrt{3}}{2}$

c. $\frac{19+4\sqrt{3}}{4}$

d. $\frac{19+4\sqrt{3}}{2}$

e. NOTA

26. $\frac{i^4+3i^9+i^{16}}{2-i^5+i^{10}+i^{15}} =$

a. $\frac{8}{5}+\frac{7}{5}i$

b. $-\frac{4}{5}+\frac{7}{5}i$

c. $\frac{4}{3}-\frac{7}{3}i$

d. $\frac{8}{3}+\frac{7}{3}i$

e. NOTA

27. $\frac{2+2i}{-1+3i}+\frac{1-2i}{2-3i} =$

a. $\frac{66}{65}-\frac{57}{65}i$

b. $-\frac{72}{65}-\frac{57}{65}i$

c. $\frac{66}{65}+\frac{57}{65}i$

d. $-\frac{72}{65}+\frac{57}{65}i$

e. NOTA

28. Express the roots of $x^2 + x + 1 = 0$ in polar form.

a. $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ and $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$
b. $\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}$ and $\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$
c. $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ and $\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3}$
d. $\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}$ and $\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}$
e. NOTA

29. Find all complex cube roots of 27 in rectangular form

a. $3, -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}$
b. $3, -\frac{3}{2} \pm \frac{9\sqrt{3}}{2}i$
c. $3, -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$
d. $-3, -\frac{3}{2} \pm \frac{3\sqrt{3}}{2}i$
e. NOTA

30. Find the area of a triangle having vertices at $-4 - i$, $1 + 2i$ and $4 - 3i$.

a. 14
b. 17
c. 9
d. 34
e. NOTA

Work the bonus in the white portion on the back of the scantron sheet.

Bonus: Given that $3 - 2i$ is a zero of $f(x) = x^4 - 9x^3 + 21x^2 + 21x - 130$, find the remaining zeros.

Alpha Division
Complex Numbers
Answer Key

1. b
2. a
3. d
4. b
5. b
6. e
7. b
8. d
9. b
10. c
11. a
12. d
13. b
14. d
15. e
16. b
17. c
18. a
19. b
20. c
21. d
22. e
23. b
24. d
25. a
26. b
27. a
28. b
29. c
30. b

BONUS: -2, 5, 3+2i