SOLUTIONS___Alpha Equations and Inequalities Topic Test

Note: For each problem, where there is no choice (e), assume (e) none of the above.

1. Solve for k: $4k - 2(k-1) = 12$ a) $\frac{5}{3}$ b) $\frac{7}{3}$ c) 3 d) 5

Answer: d
Solution: $4k - 2k + 2 = 12 \rightarrow 2k = 10 \rightarrow k = 5$

2. If $x : y = 2:1$, find the value of $\frac{x^2 - y^2}{x^2 + y^2}$

a) $\frac{3}{5}$ b) 2 c) $\frac{1}{3}$ d) cannot be determined

Answer: a

Solution: $\frac{x}{y} = \frac{2}{1} = \frac{2a}{1a} \rightarrow \frac{(2a)^2 - a^2}{(2a)^2 + a^2} = \frac{4a^2 - a^2}{4a^2 + a^2} = \frac{3a^2}{5a^2}$

3. Find the solution set for: $7p - 2(p-3) \leq 5(2-p)$

a) ϕ b) $(-\infty, 0.4]$ c) $(-\infty, 1]$ d) $\left(-\infty, \frac{8}{7}\right]$

Answer: b

Solution:
$$\begin{aligned} &7p - 2p + 6 \leq 10 - 5p \\ &5p + 6 \leq 10 - 5p \\ &10p \leq 4 \\ &p \leq 0.4 \end{aligned}$$

4. Express the solution in interval form for $5 \leq 2x - 3 \leq 7$.

a) [1,2] b) $(-\infty, 1] \cup [2, \infty)$ c) $(-\infty, 4] \cup [5, \infty)$ d) [4,5]

Answer: d

Solution:
$$\begin{aligned} &5 \leq 2x - 3 \leq 7 \\ &8 \leq 2x \leq 10 \\ &4 \leq x \leq 5 \end{aligned}$$

5. Solve for r: $\frac{5}{3r} - 10 = \frac{3}{2r}$. a) $-\frac{1}{12}$ b) $-\frac{9}{50}$ c) 60 d) $\frac{1}{60}$

Answer: d

Solution:

$$6r\left(\frac{5}{3r}-10=\frac{3}{2r}\right)$$
$$10-60r=9$$
$$-60r=-1$$
$$r=\frac{1}{60}$$

6. A baseball player threw a ball that traveled according to the equation $h(t)=9.8t+1.1-4.9t^2$ where h = height in meters and t = time in seconds. What is the maximum height reached by the ball?

a) 1 m b) 3.8 m c) 6m d) 15.8m

Answer: c

Solution: $t=\dfrac{-9.8}{2(4.9)}=1 \rightarrow h(1)=9.8+1.1-4.9=6.0m$

7. For what values of x and y is the following true? $(2-i)+4x+yi=6+3i$

a) (1,4) b(- 1, - 4) c) $\left(\frac{3}{2},-1\right)$ d) $\left(-\frac{3}{2},1\right)$

Answer: a

Solution:

$$\begin{array}{ll} 2+4x=6 & -i+yi=3i \\ 4x=4 & (y-1)i=3i \\ x=1 & y-1=3 \\ & y=4 \end{array}$$

8. Solve for the only positive solution to $\left(x^{\frac{1}{2}}\right)\left(x^{\frac{3}{2}}\right)=16$

a) $32\sqrt[3]{2}$ b) 16 c) 8 d) 4

Answer: d

Solution: $x^{\frac{4}{2}}=x^2=16 \rightarrow x=\pm 4$

9. Find the solution set for: $x^2+3x-4\le 0$

a) [- 4, 1] b) [- 1, 4] c) $(-\infty,-4]\cup[1,\infty)$ d) $(-\infty,-1]\cup[4,\infty)$

Answer: a

Solution: $(x+4)(x-1)\le 0$

\+ - +

-4 1 choose the negative section

10. Solve for the solution set: $-3x+\dfrac{-6+3}{-3}>-8x\div\dfrac{4\div 2}{-6\div 3}$

a) $x < \frac{1}{11}$ b) $x > \frac{1}{11}$ c) $x < -\frac{1}{6}$ d) $x > -\frac{1}{6}$

Answer: a

$$-3x+1 > -8x \div -1$$
$$-3x+1 > 8x$$

Solution: $-11x > -1$

$$x < \frac{1}{11}$$

11. Using only one solution, round to the nearest degree to solve for θ given:
$\sin^2\theta - 0.3\sin\theta - 0.4 = 0$ where $0 \le \theta < 360$. a) -1 b) 1 c) 30 d) 53

Answer: d

Solution: Using the quadratic formula, the roots are 0.8 and -0.5. Taking the arcsin of each angle value gives one angle of -30 and the other of 53 degrees. -30 is not on the given interval so choose 53.

12. Solve for f: $A = \frac{24f}{B(p+1)}$.

a) $\frac{24}{A} - Bp - 1$ b) $\frac{AB(p+1)}{24}$ c) $\frac{ABp-1}{24}$ d) $ABp + \frac{1}{24}$

Answer: b

$$B(p+1)\left(A = \frac{24f}{B(p+1)}\right)$$

Solution: $AB(p+1) = 24f$

$$\frac{AB(p+1)}{24} = f$$

13. Find the length of a line segment defined as part of the line 2x - y = 12 between the x and y axes. a) 12 b) 12.490 c) 13.416 d) 18

Answer: c

Solution: The x-intercept for the line is (6,0) and the y-intercept is (0, - 12)

Find the distance between these two points. $\sqrt{6^2+12^2} = \sqrt{180} = 6\sqrt{5} \approx 13.416$

14. Find the value for n if $3+3^2+3^3+\ldots+3^n = 9840$ and $S_n = \frac{a(1-r^n)}{1-r}$.

a) 8 b) 9 c) 10 d) 11

Answer: a

Solution:
$$9840 = \frac{3\left(1-3^n\right)}{1-3}$$
$$-19680 = 3 - 3^{n+1}$$
$$19680 = 3^{n+1}$$
$$3^9 = 3^{n+1}$$
$$n+1 = 9$$
$$n = 8$$

15. The value, in dollars, of a diamond is directly proportional to the square of its mass. If a diamond, worth \$6300 is 200 mg, what is the mass of a diamond worth \$25,200?

a) 450 mg b) 600 mg c) 725 mg d) 800mg

Answer: d

Solution:
$$\frac{6300}{200} = \frac{25200}{m}$$
$$63m = 50400$$
$$m = 800\ mg$$

16. Given the system: $\begin{aligned} 0.25x + 0.4y + 0.2z &= 22 \\ 0.4x + 0.2y + 0.3z &= 28 \\ 0.3x + 0.2y + 0.1z &= 18 \end{aligned}$, find the values for x, y and z.

a) (30, 20, 35) b) (30, 20, 40) c) (40, 15, 30) d) (40, 20, 20)

Answer: c

Solution: On calculator: $\begin{bmatrix} .25 & .4 & .2 \\ .4 & .2 & .3 \\ .3 & .2 & .1 \end{bmatrix}^{-1} \begin{bmatrix} 22 \\ 28 \\ 18 \end{bmatrix} = \begin{bmatrix} 40 \\ 15 \\ 30 \end{bmatrix}$

17. Solve for y: $\frac{y}{y-2} = \frac{y^2+3y}{y^2-4} - \frac{3}{y+2}$

a) $-\frac{3}{2}$ b) - 2 c) 3 d) no real solution

Answer: c

Solution:
$$\left(\frac{y}{y-2} = \frac{y^2+3y}{y^2-4} - \frac{3}{y+2}\right)(y+2)(y-2)$$
$$y(y+2) = y^2+3y-3(y-2)$$
$$y^2+2y = y^2+3y-3y+6$$
$$2y = 6$$
$$y = 3,\ \ y \neq 2 \ or \ -2$$

18. Which points are NOT on the circle defined by the equation $x^2 + y^2 = 25$?

I (0,25) II (-5,0) III (12.5,12.5) IV (3,-4) V $\left(-2\sqrt{2},\ 4\right)$

a) I & III b) III & V c) II & IV d) I, III & V

Answer: d

Solution: Fill the points in for x & y

19. Given: $\begin{cases} y \geq 5 \\ 2 \leq x \leq 7 \\ y - x \leq 40 \end{cases}$ and a profit equation of P = 3x + y, find the coordinates that will maximize the profit. a) (2,42) b) (7,47) c) (2,45) d) (7,49)

Answer: b

Solution: P intersects $2 \leq x \leq 7$ @ (2,42) and (7,47) using both in the profit equation, (7,47) produces a higher profit of 68.

20. Solve for x: $5^{\left(\log_5 x - \log_5 2\right)} = 4$. a) 2 b) 4 c) 6 d) 8

Answer: d

Solution:

$$5^{\left(\log_5 x - \log_5 2\right)} = 4$$

$$5^{\log_5\left(\frac{x}{2}\right)} = 4$$

$$\frac{x}{2} = 4$$

$$x = 8$$

21. The period of the graph of $y = \tan\left(\frac{1}{3}\theta\right)$ is:

a) $\frac{\pi}{3}$ b) 3π c) $\frac{2\pi}{3}$ d) 6π

Answer: b

Solution: $period = \frac{\pi}{B} = \frac{\pi}{\frac{1}{3}} = 3\pi$

22. Express the solution in interval notation: $\frac{3}{x+2} > \frac{2}{x-4}$

a) $(16,\infty)$ b) $(-2,4)\cup(16,\infty)$ c) $\left(-\infty,-\frac{8}{5}\right); x \neq -2$ d) $(-\infty,-2)\cup(4,\infty)$

Answer: b

Solution:

$$\frac{3}{x+2} > \frac{2}{x-4}$$

$$\frac{3}{x+2} - \frac{2}{x-4} > 0$$

$$\frac{3(x-4)-2(x+2)}{(x+2)(x-4)} > 0$$

$$\frac{x-16}{(x+2)(x-4)} > 0$$

-	+	-	+
-2	4	16	

Answer are the parts which are +.

23. Find the values for which $f(x) = g(x)$ given $f(x) = \sqrt{3x} + 1$ and $g(x) = x+1$.

a) -1 b) 0 c) 3 d) $\{0,3\}$

Answer: d

Solution:

$$\sqrt{3x} + 1 = x+1$$

$$\sqrt{3x} = x$$

$$3x = x^2$$

$$x^2 - 3x = 0$$

$$x(x-3) = 0$$

24. How much money, A, does Sasha need to invest today at 9% compounded annually in order to have $5000 in 8 years if the situation is modeled by: $8\log 1.09 + \log A = \log 5000$?

a) $2500 b) $2510 c) $2550 d) $3700

Answer: b

Solution:

$$\log A = \log 5000 - 8\log 1.09$$

$$\log A = \log\left(\frac{5000}{1.09^8}\right)$$

$$A \approx 2510$$

25. Find the sum of the roots given $x^3 - 7x + 6 = 0$. a) - 6 b) 0 c) 6 d) 7

Answer: b

Solution: for all $ax^3 + bx^2 + cx + d = 0$ the sum of the roots = $-\frac{b}{a} = \frac{0}{1} = 0$

26. Solve for x over the Reals: $x^4 + 6x^2 - 40 = 0$

a) ± 2 b) $\pm 2, \pm\sqrt{10}$ c) -10, 4 d) ϕ

Answer: a

Solution: Factor and solve. $(x^2-4)(x^2+10) = 0 \rightarrow x = \pm 2 \text{ or } \pm i\sqrt{10}$

27. A calculator manufacturer predicts that the number, N, of calculators sold when x thousand

of dollars are spent on advertising is given by $N = 2275 + 10000\ \ln(x+1)$. How much advertising money must be spent to sell 62,583 calculators?

a) \$ 9.99 X 10^{499} b) \$ 7803.75 c) \$ 415.05 d) \$ 7.97

Answer: c

Solution:
$$N = 2275 + 10000\ \ln(x+1)$$
$$62583 = 2275 + 10000\ \ln(x+1)$$
$$60308 = 10000\ \ln(x+1)$$
$$6.0308 = \ln(x+1)$$
$$e^{6.0308} = e^{\ln(x+1)}$$
$$416.0477 = x+1$$
$$415.05 = x$$

28. Find the solution set for $-3|x| + 6 \le 12$.

a) $(-\infty, -2] \cup [2, \infty)$ b) [- 2, 2] c) no solution d) all real numbers

Answer: d

Solution:
$$-3|x| + 6 \le 12$$
$$-3|x| \le 6$$ which is true for all values of x.
$$|x| \ge -2$$

29. Find the solution set: $2^{x^2} = 32\left(2^{4x}\right)$. a) 0, 20 b) - 1 c) 5 d) - 1, 5

Answer: d

Solution:
$$2^{x^2} = 32\left(2^{4x}\right)$$
$$2^{x^2} = 2^5\left(2^{4x}\right)$$
$$x^2 = 5 + 4x$$
$$x^2 - 4x - 5 = 0$$
$$(x-5)(x+1) = 0$$

30. Find the solution for $x^3 + 6x^2 - x - 5 < 1$. a) $(-6, 0] \cup (1, 5)$

b) $(-\infty, -6) \cup (0, 5)$ c) $(-6, -1) \cup (1, \infty)$ d) $(-\infty, -6) \cup (-1, 1)$

Answer: d

Solution:
$$x^3 + 6x^2 - x - 6 < 0$$
$$x^2(x+6) - 1(x+6) < 0$$
$$(x+6)(x+1)(x-1) < 0$$

-	+	-	+
-6	-1	1	

choose the negative intervals

31. The equations of the sides of quadrilateral ABCD are: AB: x+6y=15 BC: 4x - y=10 DC: 3x+7y= - 8 AD: x - y= - 6. Which vertex(ices) would give a sum of zero if you added its x and y coordinates?
a) A & B b) B & C c) C & A d) D & A

Answer: c

Solution: When each pair of equations are solved the coordinates in order are (-3,3) , (3,2), (2,-2), & (-5,1), so both A & C are correct.

32. For what value(s) of p does the equation have real and unequal roots?

$$5x^2 - (p-1)x + 1 = 0$$

a) $p > 1+2\sqrt{5}$ b) $p > 1+2\sqrt{5}$ *or* $p < 1-2\sqrt{5}$ c) $p > 3$ d) $p > 1 \pm 2\sqrt{5}$

Answer: b

Solution:
$$(p-1)^2 - 4(5)(1) > 0$$
$$(p-1)^2 > 20$$
$$|p-1| > \sqrt{20}$$
$$p > 1+\sqrt{20} \quad or \quad p < 1-\sqrt{20}$$

33. Given $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, find the value of $\sum_{i=1}^{50} 5i$. a) 255 b) 1275 c) 3275 d) 6375

Answer: d

Solution: $5\left(\frac{50(51)}{2}\right) = 5(25)(51) = 6375$

34. Solve for a, given $(a+6)({}_{10}C_8) - 3(5!) = (a-2)({}_5P_2)$
a) 2 b) 5.2 c) 7 d) 14.08

Answer: a

Solution:
$$(a+6)({}_{10}C_8) - 3(5!) = (a-2)({}_5P_2)$$
$$45(a+6) - 360 = 20(a-2)$$
$$9(a+6) - 72 = 4(a-2)$$
$$9a + 54 - 72 = 4a - 8$$
$$9a - 18 = 4a - 8$$
$$5a = 10$$
$$a = 2$$

35. Determine the exact real root, r, given $r\sqrt[3]{32} - \sqrt[3]{2} = \sqrt[3]{250} - \sqrt[3]{4r^3}$
a) $\sqrt[3]{4}$ b) $\sqrt[3]{7}$ c) $2\sqrt[3]{2}$ d) $7\sqrt[3]{4}$

Answer: a

Solution:

$$r\sqrt[3]{32}-\sqrt[3]{2}=\sqrt[3]{250}-\sqrt[3]{4r^3}$$
$$r\sqrt[3]{8}\sqrt[3]{4}-\sqrt[3]{2}=\sqrt[3]{125}\sqrt[3]{2}-r\sqrt[3]{4}$$
$$2r\sqrt[3]{4}-\sqrt[3]{2}=5\sqrt[3]{2}-r\sqrt[3]{4}$$
$$3r\sqrt[3]{4}=6\sqrt[3]{2}$$
$$r=2\frac{\sqrt[3]{2}}{\sqrt[3]{4}}=\sqrt[3]{4}$$

TIEBREAKER: Solve for x: $\dfrac{11^x\left(7^{2x+3}\right)}{3^{1-x}\left(2^{4x-1}\right)}=5^x$

Answer: -1.807

Solution:

$$11^x\left(7^{2x+3}\right)=5^x\left(3^{1-x}\right)\left(2^{4x-1}\right)$$
$$x\log 11+(2x+3)\log 7=x\log 5+(1-x)\log 3+(4x-1)\log 2$$
$$0.342x+1.690x+2.535=0.477-0.477x+1.204x-0.301$$
$$2.032x+2.535=0.176+0.727x$$
$$1.305x=-2.359$$
$$x=-1.807$$