SOLUTIONS—Alpha Equations and Inequalities Topic Test

Note: For each problem, where there is no choice (e), assume (e) none of the above.

1. Solve for $k$: $4k - 2(k - 1) = 12$
   a) $\frac{5}{3}$  b) $\frac{7}{3}$  c) 3  d) 5

Answer: d
Solution: $4k - 2k + 2 = 12 \rightarrow 2k = 10 \rightarrow k = 5$

2. If $x:y = 2:1$, find the value of $\frac{x^2 - y^2}{x^2 + y^2}$
   a) $\frac{3}{5}$  b) 2  c) $\frac{1}{3}$  d) cannot be determined

Answer: a
Solution: $\frac{x}{y} = \frac{2}{1} = \frac{2a}{1a} \rightarrow \frac{(2a)^2 - a^2}{(2a)^2 + a^2} = \frac{4a^2 - a^2}{4a^2 + a^2} = \frac{3a^2}{5a^2}$

3. Find the solution set for: $7p - 2(p - 3) \leq 5(2 - p)$
   a) $\emptyset$  b) $(-\infty, 0.4]$  c) $(-\infty, 1]$  d) $(-\infty, \frac{8}{7}]$

Answer: b
Solution: $7p - 2p + 6 \leq 10 - 5p$

   $5p + 6 \leq 10 - 5p$

   $10p \leq 4$

   $p \leq 0.4$

4. Express the solution in interval form for $5 \leq 2x - 3 \leq 7$.
   a) $[1,2]$  b) $(-\infty, 1] \cup [2, \infty)$  c) $(-\infty, 4] \cup [5, \infty)$  d) $[4,5]$

Answer: d
Solution: $8 \leq 2x \leq 10$

   $4 \leq x \leq 5$

5. Solve for $r$: $\frac{5}{3r} - 10 = \frac{3}{2r}$
   a) $-\frac{1}{12}$  b) $-\frac{9}{50}$  c) 60  d) $\frac{1}{60}$

Answer: d
6. A baseball player threw a ball that traveled according to the equation 
\[ h(t) = 9.8t + 1.1 - 4.9t^2 \] 
where \( h \) = height in meters and \( t \) = time in seconds. What is the maximum height reached by the ball?

a) 1 m  
 b) 3.8 m  
 c) 6 m  
 d) 15.8 m

Answer: c

Solution: 
\[ t = \frac{-9.8}{2(4.9)} = 1 \rightarrow h(1) = 9.8 + 1.1 - 4.9 = 6.0 \text{m} \]

7. For what values of \( x \) and \( y \) is the following true? 
\[ (2-i) + 4x + yi = 6 + 3i \]

a) (1,4)  
 b) (-1, -4)  
 c) \( \frac{3}{2}, -1 \)  
 d) \( \frac{-3}{2}, 1 \)

Answer: a

Solution: 
\[ 2 + 4x = 6 \quad -i + yi = 3i \]
\[ 4x = 4 \quad (y-1)i = 3i \]
\[ x = 1 \quad y-1 = 3 \]
\[ y = 4 \]

8. Solve for the only positive solution to
\[ \begin{bmatrix} x^2 \\ x^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix} \]

a) \( 32 \sqrt{2} \)  
 b) 16  
 c) 8  
 d) 4

Answer: d

Solution: 
\[ x^\frac{4}{2} = x^2 = 16 \rightarrow x = \pm 4 \]

9. Find the solution set for: 
\[ x^2 + 3x - 4 \leq 0 \]

a) \([-4, 1]\)  
 b) \([-1, 4]\)  
 c) \((-\infty, -4] \cup [1, \infty)\)  
 d) \((-\infty, -1] \cup [4, \infty)\)

Answer: a

Solution: 
\[ (x+4)(x-1) \leq 0 \quad + - + \]
\[ x = -4 \quad 1 \quad \text{choose the negative section} \]

10. Solve for the solution set: 
\[ -3x + \frac{-6+3}{-3} > -8x + \frac{4+2}{-6+3} \]
a) \(x < \frac{1}{11}\)  
b) \(x > \frac{1}{11}\)  
c) \(x < -\frac{1}{6}\)  
d) \(x > -\frac{1}{6}\)

Answer: a
\[-3x + 1 > -8x - 1\]
\[-3x + 1 > 8x\]
Solution: 
\[-11x > -1\]
\[x < \frac{1}{11}\]

11. Using only one solution, round to the nearest degree to solve for \(\theta\) given:
\[\sin^2 \theta - 0.3 \sin \theta - 0.4 = 0\] where \(0 \leq \theta < 360\).
a) -1  
b) 1  
c) 30  
d) 53
Answer: d
Solution: Using the quadratic formula, the roots are 0.8 and -0.5. Taking the arcsin of each angle value gives one angle of -30 and the other of 53 degrees. -30 is not on the given interval so choose 53.

12. Solve for \(f\):
\[A = \frac{24f}{B(p+1)}\]
a) \(\frac{24}{A} - Bp - 1\)  
b) \(\frac{AB(p+1)}{24}\)  
c) \(\frac{ABp - 1}{24}\)  
d) \(ABp + \frac{1}{24}\)
Answer: b
\[B(p+1) \left( A = \frac{24f}{B(p+1)} \right)\]
Solution: 
\[\frac{AB(p+1)}{24} = f\]

13. Find the length of a line segment defined as part of the line \(2x - y = 12\) between the x and y axes.
a) 12  
b) 12.490  
c) 13.416  
d) 18
Answer: c
Solution: The x-intercept for the line is (6,0) and the y-intercept is (0, -12)
Find the distance between these two points. \(\sqrt{6^2 + 12^2} = \sqrt{180} = 6\sqrt{5} \approx 13.416\)

14. Find the value for \(n\) if \(3 + 3^2 + 3^3 + ... + 3^n = 9840\) and \(S_n = \frac{a(1-r^n)}{1-r}\).
a) 8  
b) 9  
c) 10  
d) 11
Answer: a
\[
9840 = \frac{3(1-3^n)}{1-3}
\]
\[-19680 = 3 - 3^{n+1}
\]
Solution: $19680 = 3^{n+1}$
\[3^n = 3^{n+1}
\]
\[n+1 = 9
\]
\[n = 8
\]

15. The value, in dollars, of a diamond is directly proportional to the square of its mass. If a diamond, worth $6300 is 200 mg, what is the mass of a diamond worth $25,200?

a) 450 mg       b) 600 mg          c) 725 mg          d) 800 mg

Answer: d

Solution: \[
\frac{6300}{200} = \frac{25200}{m}
\]
Solution: \[63m = 50400
\]
\[m = 800 \text{ mg}
\]

\[0.25x + 0.4y + 0.2z = 22
\]

16. Given the system: \[0.4x + 0.2y + 0.3z = 28, 0.3x + 0.2y + 0.1z = 18
\]

a) (30, 20, 35)    b) (30, 20, 40)    c) (40, 15, 30)    d) (40, 20, 20)

Answer: c

Solution: On calculator: \[
\begin{bmatrix}
.25 & .4 & .2 \\
.4 & .2 & .3 \\
.3 & .2 & .1
\end{bmatrix}
^{-1}
\begin{bmatrix}
22 \\
28 \\
18
\end{bmatrix}
= \begin{bmatrix}
40 \\
15 \\
30
\end{bmatrix}
\]

17. Solve for \(y\): \[\frac{y}{y-2} = \frac{y^2+3y}{y^2-4} - \frac{3}{y+2}
\]

a) \(-\frac{3}{2}
\) b) -2    c) 3    d) no real solution

Answer: c

\[
\left(\frac{y}{y-2} = \frac{y^2+3y}{y^2-4} - \frac{3}{y+2}\right)\left(y+2\right)\left(y-2\right)
\]
\[y(y+2) = y^2+3y-3(y-2)
\]

Solution: \[y^2 + 2y = y^2 + 3y - 3y + 6
\]
\[2y = 6
\]
\[y = 3, \ y \neq 2 \text{ or } -2
\]
18. Which points are NOT on the circle defined by the equation $x^2 + y^2 = 25$?
   I (0,25)   II (-5,0)   III (12.5,12.5)   IV (3,-4)   V ($-2\sqrt{2}$, 4)
   a) I & III  b) III & V  c) II & IV  d) I, III & V
   Answer: d
   Solution: Fill the points in for x & y

19. Given:
   \begin{align*}
   y &\geq 5 \\
   2 \leq x \leq 7 & \text{ and a profit equation of } P = 3x + y, find the coordinates that will maximize the profit.
   \end{align*}
   a) (2,42) b) (7,47) c) (2,45) d) (7,49)
   Answer: b
   Solution: P intersects $2 \leq x \leq 7$ @ (2,42) and (7,47) using both in the profit equation, (7,47) produces a higher profit of 68.

20. Solve for x: $5^{\log_5(x - \log_5 2)} = 4$.
   a) 2 b) 4 c) 6 d) 8
   Answer: d
   Solution:
   \begin{align*}
   5^{\log_5(x - \log_5 2)} &= 4 \\
   5^{\log_5\left(\frac{x}{2}\right)} &= 4 \\
   \frac{x}{2} &= 4 \\
   x &= 8
   \end{align*}

21. The period of the graph of $y = \tan\left(\frac{1}{3}\theta\right)$ is:
   a) $\frac{\pi}{3}$ b) 3$\pi$ c) $\frac{2\pi}{3}$ d) 6$\pi$
   Answer: b
   Solution: $period = \frac{\pi}{B} = \frac{\pi}{\frac{1}{3}} = 3\pi$

22. Express the solution in interval notation: $\frac{3}{x+2} > \frac{2}{x-4}$
   a) (16, $\infty$) b) (-2,4) $\cup$ (16, $\infty$) c) $(-\infty, -\frac{8}{5})$; x $\neq$ -2 d) $(-\infty, -2) \cup (4, \infty)$
   Answer: b
Solution:

\[
\frac{3}{x+2} > \frac{2}{x-4} \\
\frac{3}{x+2} - \frac{2}{x-4} > 0 \\
\frac{3(x-4) - 2(x+2)}{(x+2)(x-4)} > 0 \\
\frac{x-16}{(x+2)(x-4)} > 0 \\
\]

-2 4 16 + + Answer are the parts which are +.

23. Find the values for which \( f(x) = g(x) \) given \( f(x) = \sqrt{3x} + 1 \) and \( g(x) = x + 1 \).
   a) -1  b) 0  c) 3  d) \{0,3\}
Answer:  d
Solution:
\[
\sqrt{3x} + 1 = x + 1 \\
\sqrt{3x} = x \\
3x = x^2 \\
x^2 - 3x = 0 \\
x(x - 3) = 0 \\
\]

24. How much money, \( A \), does Sasha need to invest today at 9% compounded annually in order to have $5000 in 8 years if the situation is modeled by: \( 8 \log 1.09 + \log A = \log 5000 \)?
   a) $2500  b) $2510  c) $2550  d) $3700
Answer:  b
Solution:
\[
\log A = \log 5000 - 8 \log 1.09 \\
\]
\[
A \approx 2510 \\
\]

25. Find the sum of the roots given \( x^3 - 7x + 6 = 0 \).
   a) -6  b) 0  c) 6  d) 7
Answer:  b
Solution:  for all \( ax^3 + bx^2 + cx + d = 0 \) the sum of the roots = \( -\frac{b}{a} = 0 \) = 0

26. Solve for \( x \) over the Reals: \( x^4 + 6x^2 - 40 = 0 \)
   a) \( \pm 2 \)  b) \( \pm 2, \pm \sqrt{10} \)  c) -10, 4  d) \( \phi \)
Answer:  a
Solution:  Factor and solve. \( (x^2 - 4)(x^2 + 10) = 0 \) \( \rightarrow x = \pm 2 or \pm i \sqrt{10} \)

27. A calculator manufacturer predicts that the number, \( N \), of calculators sold when \( x \) thousand
of dollars are spent on advertising is given by \( N = 2275 + 10000 \ln(x+1) \). How much advertising money must be spent to sell 62,583 calculators?

a) $9.99 \times 10^5$

b) $7803.75$

c) $415.05$

d) $7.97$

Answer: c

\[ 62583 = 2275 + 10000 \ln(x+1) \]
\[ 60308 = 10000 \ln(x+1) \]
\[ 60308 = \ln(x+1) \]
\[ e^{6.0308} = e^{\ln(x+1)} \]
\[ 416.0477 = x + 1 \]
\[ 415.05 = x \]

28. Find the solution set for \( -3|x| + 6 \leq 12 \).

a) \((-\infty, -2] \cup [2, \infty)\)

b) \([-2, 2]\)

c) no solution

d) all real numbers

Answer: d

\[-3|x| + 6 \leq 12 \]
\[-3|x| \leq 6 \quad \text{which is true for all values of } x. \]
\[-|x| \geq -2 \]

29. Find the solution set:
\[ 2x^2 = 32 \left(2^{4x}\right) \]

a) 0, 20

b) -1

c) 5

d) -1, 5

Answer: d

\[ 2x^2 = 32 \left(2^{4x}\right) \]
\[ 2x^2 = 2^5 \left(2^{4x}\right) \]

Solution:
\[ x^2 = 5 + 4x \]
\[ x^2 - 4x - 5 = 0 \]
\[ (x-5)(x+1) = 0 \]

30. Find the solution for \( x^3 + 6x^2 - x - 5 < 1 \).

a) \((-6, 0] \cup (1,5)\)

b) \((-\infty, -6) \cup (0,5)\)

c) \((-6, -1) \cup (1, \infty)\)

d) \((-\infty, -6) \cup (-1,1)\)

Answer: d

\[ x^3 + 6x^2 - x - 6 < 0 \]

Solution:
\[ x^2(x+6) - 1(x+6) < 0 \]
\[ (x+6)(x+1)(x-1) < 0 \]

\[
\begin{array}{ccccccc}
-6 & + & -1 & - & +
\end{array}
\]

choose the negative intervals
31. The equations of the sides of quadrilateral ABCD are: AB: x+6y=15  BC: 4x - y=10  
   DC: 3x+7y=-8  AD: x - y= -6. Which vertex(ices) would give a sum of zero if you 
   added its x and y coordinates? 
   a) A & B  b) B & C  c) C & A  d) D & A 
   Answer: c 
   Solution: When each pair of equations are solved the coordinates in order are (-3,3), (3,2), 
   (2,-2), & (-5,1), so both A & C are correct.

32. For what value(s) of p does the equation have real and unequal roots? 
   \[ 5x^2 -(p-1)x +1 = 0 \]
   a) \( p > 1 + 2\sqrt{5} \)  b) \( p > 1 + 2\sqrt{5} \) or \( p < 1 - 2\sqrt{5} \)  c) \( p > 3 \)  d) \( p > 1\pm 2\sqrt{5} \)
   Answer: b 
   Solution: 
   \[(p-1)^2 - 4(5)(1) > 0 \]
   \[(p-1)^2 > 20 \]
   \[|p-1| > \sqrt{20} \]
   \[p > 1 + \sqrt{20} \] or \( p < 1 - \sqrt{20} \)

33. Given \( \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \), find the value of \( \sum_{i=1}^{50} 5i \). a) 255  b) 1275  c) 3275  d) 6375 
   Answer: d 
   Solution: 
   \[ 5 \left( \frac{50(51)}{2} \right) = 5(25)(51) = 6375 \]

34. Solve for a, given \( (a+6)\binom{10}{6} - 3(5!) = (a-2)\binom{7}{2} \)
   a) 2  b) 5.2  c) 7  d) 14.08 
   Answer: a 
   \[ (a+6)\binom{10}{6} - 3(5!) = (a-2)\binom{7}{2} \]
   \[ 45(a+6) - 360 = 20(a-2) \]
   \[ 9(a+6) - 72 = 4(a-2) \]
   Solution: \[ 9a + 54 - 72 = 4a - 8 \]
   \[ 9a - 18 = 4a - 8 \]
   \[ 5a = 10 \]
   \[ a = 2 \]

35. Determine the exact real root, r, given \( r \sqrt{32} - \sqrt{2} = \sqrt{250} - \sqrt{4r^3} \)
   a) \( \sqrt[3]{4} \)  b) \( \sqrt[3]{7} \)  c) \( 2\sqrt{2} \)  d) \( 7\sqrt[3]{4} \)
   Answer: a
Solution: \[ r \sqrt[3]{32} - \sqrt[3]{2} = \sqrt[3]{250} - \sqrt[3]{4r^3} \]
\[ r \sqrt[3]{8} \sqrt[4]{4} - \sqrt[2]{2} = \sqrt[3]{125} \sqrt[2]{2} - r \sqrt[2]{4} \]

TIEBREAKER: Solve for \( x \):
\[
\frac{11^x \left( 7^{2x+3} \right)}{3^{1-x} \left( 2^{4x-1} \right)} = 5^x
\]
Answer: \(-1.807\)