2002 National Alpha FUNctions Test Solutions

1. $a(-2) = 2(-2)^3 - 3(-2)^2 + 156(-2) - 321 = -16 - 12 - 312 - 321 = -661$ $\boxed{C}$

2. $b(3e-2) = 2(3e-2)^2 - 4(3e-2) + 1 \Rightarrow$ calculator $\Rightarrow 52.14$ $\boxed{A}$

3. The vertex of the parabola will give us the maximum height, so we must find $\left(\frac{-b}{2a}, c\left(\frac{-b}{2a}\right)\right) \Rightarrow (3.375, 1.796875)$. Thus, our maximum height, rounded to four decimal places, is 1.797 $\boxed{D}$

4. The sum of the roots of any equation is $-\frac{b}{a}$. The product of the roots of this equation is given by $\frac{g}{a}$. Therefore, A = 5/8, and B = 5. So, we have $\dfrac{100\left(\frac{5}{8}\right)^2 \pi}{5!} = \dfrac{\frac{625\pi}{16}}{120} = 1.022653...$ $\boxed{E}$

5. 3x – 5 = 2a(x – 3) + b(x – 2)

   3x – 5 = 2ax + bx –6a –2b

$$\begin{cases} 2a + b = 3 \\ -6a - 2b = -5 \end{cases}$$ Solving this system, a = -1/2, $-a^2 = -\frac{1}{4}$ $\boxed{C}$

6. Odd functions obey f(-x) = -f(x). Also, odd functions are symmetric with respect to the origin. Finally, odd polynomial functions have all odd exponents, and even polynomial functions have all even exponents. f(x) is even, g(x) is odd, h(x) is nothing, and j(x) is odd. $\boxed{D}$

7. Switch x and y to find the inverse.

$$y = \frac{2x-3}{4x+2} + \frac{16x+8}{4x+2} \Rightarrow y = \frac{18x+5}{4x+2}$$

$$x = \frac{18y+5}{4y+2}$$

$$4xy + 2x = 18y + 5$$

$$y(4x - 18) = -2x + 5$$

$$y = k^{-1}(x) = \frac{-2x+5}{4x-18} \boxed{D}$$

8. Solve the system using matrices on the graphing calculator:

$$\begin{bmatrix} \frac{1}{15} & \frac{1}{12} & 0 \\ \frac{3}{10} & 0 & \frac{4}{15} \\ 0 & -\frac{1}{7} & \frac{3}{14} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{53}{15} \\ -\frac{1}{7} \end{bmatrix}.$$ (p, q, r) = (10, 4, 2).

$$\frac{(4 \cdot 4\sqrt{\pi})(100!)(-4)}{100!} = -64\sqrt{\pi} = -113.43... \boxed{B}$$

9. Discriminant:

$25-4(a)(-6-a)<0$

$4a^2+24a+25<0$ Now find the critical numbers

$$a=\frac{-24\pm\sqrt{576-4(4)(25)}}{8}$$

$$a=-3\pm\frac{\sqrt{11}}{2}$$

Testing on both sides of the two numbers to find where our quadratic is less than zero, we find $(-3-\frac{\sqrt{11}}{2},-3+\frac{\sqrt{11}}{2})$ $\boxed{A}$

10. A one-to-one function has the property that it's inverse is also a function. Therefore, one-to-one functions pass the vertical (VLT) *and* horizontal line test (HLT). I. The absolute value would flip the negative part of the "log" above the x-axis, and it would not pass the HLT. For II and IV, parabolas only pass one of the line tests, so they are not one-to-one. III. The square root would eliminate the part of the graph below the x-axis, so it would pass both tests. V. This function is the step function, and fails to pass the HLT. VI and VII. Both of these are exponential functions, which pass both tests. So we get III, VI, VII or $\boxed{D}$

11. We get $(-i)(-i)(1)(i^{\frac{1}{2}})=-(-1)^{\frac{1}{4}}$ $\boxed{B}$

12. (-4) Complete the square: $(x-4)^2+(y+5)^2=12$. The radius-squared is 12, so this is FALSE.

(3) Complete the square: $\frac{(x-3)^2}{9}+\frac{(y+75)^2}{169}=1$. So

$a=3, b=13;$ Area is $ab\pi$, so we get 39π TRUE.

(-5) Complete the square: $\frac{(x-3)^2}{4}+\frac{(y-2)^2}{9}=0$. (3, 2) = TRUE.

(1) Any three *NONCOLINNEAR* points. FALSE.

So, -4 + 1 = -3 $\boxed{D}$

13. We get a cylinder. $r(x)=6\pi x^2$. $\frac{6\pi^3+27\pi}{.4}$ $\boxed{C}$

14. Range is the set of y-values. y = 0 when x = 1. $[0,+\infty)\Rightarrow\boxed{A}$

15. Domain of t(u(x)) is domain of t(x) union domain of t(u(x)). The domain of u(x) is $x \neq 0$. The composition, t(u(x)), is $t(u(x)) = \dfrac{\frac{1}{x^2}+2}{\frac{1}{x}-3}$. The denominator cannot equal zero again, so we get $x \neq \frac{1}{3}$. Our domain is $(-\infty,0)\cup(0,\frac{1}{3})\cup(\frac{1}{3},+\infty) \Rightarrow \boxed{E}$

16. We factor by grouping on the top, and factor the difference of squares on the bottom: $v(x) = \dfrac{(x^2-3)(x-3)}{(x+3)(x-3)}$, and see that there is a removable discontinuity at x = 3. So, really, we want to find the asymptotes of $\dfrac{x^2-3}{x+3}$. There is a vertical asymptote at x = -3, and we long divide to find there is a slant asymptote: y = x – 3 $\boxed{B}$

17. The area between w(q) and y(q) is a triangle with base length 12 and height 6, so the area is 36 (a = 36). The sum of the reciprocals of any polynomial is – (linear coefficient)/(constant), so b = 1. $36^2\pi e = 11067.49$ $\boxed{B}$

18.
$$\begin{aligned}
&x(y-3) = (y-3)^2 + 5(y-3) + 6\\
&x(y) = y^2 - y\\
&y(3x) = 3(3x)^3 - 2(3x) + 5\\
&y(x) = 81x^3 - 6x + 5\\
&\left(81x^3 - 6x + 5\right)\left(y^2 - y\right)\\
&= 81x^3y^2 - 81x^3y - 6xy^2 + 6xy + 5y^2 - 5y \ \boxed{E}
\end{aligned}$$

19. Flip over the y-axis gives z(-x). Moving z(-x) 3 units down, we have z(-x) – 3. $\boxed{C}$

20. If you plug in the point (4, 1), you get 4 = A + B + C $\boxed{B}$

21. Eighth term of $(2a+3b)^{10}$

${}_{10}C_3\,(2a)^3(3b)^7 = 2099520a^3b^7$ $\boxed{D}$

22. If B(x) is an odd function, then B(-x) = -B(x). So, B(-3) = -5, B(-5) = -1, and B(-1) = -2 $\boxed{B}$

23. An invertible function *means* that its inverse is a function. A one-to-one function has *at most* one intersection point with any horizontal line drawn (think of a cubic function). All functions are relations. $\boxed{B}$

24. If we graph $C(x)$ on a calculator, we can pinpoint the roots, and we determine that a = -6.38983, b = -.146577, c = 3.20307. So abc to eight decimal places, is 3.0000021. The product is –K/a or 9/3 or 3. $\boxed{A}$

25. We only need to fence three sides. Let x be the width of the two sides. Then, the length of the fence is 300-2x. If we let an area function a(x) be a function of area, we have $a(x) = x(300-2x) = -2x^2 + 300x$. If we want to enclose a maximum area, we find the vertex of this parabola, which will give us the maximum area. (-b/2a, a(-b/2a)) is (75, 11250). So, $\frac{11250}{\pi e\sqrt{2}} = 931.52211...$ $\boxed{A}$

26. What you have to find out is the derivative of a constant is zero. However, this is easy to discover, since –6 is the same thing as $-6x^0$. So, $F'(x) = 12x^3 - 10x + 10$. So $G(x) = 12x^3 - 10x + 10$, and $G(-5)$ is –1440 $\boxed{A}$

27. We have x < 14.777…. Natural numbers are integers greater than zero (the counting numbers), and we have 14 of them in this solution set. So, the number of distinguishable permutations of "fourteen" is $\frac{8!}{2!} = 20160 = \boxed{C}$.

28. Complete the square to get $x = (y-3)^2 - 7$, so the vertex is (-7, 3). Since this is a y-squared parabola, we have y = 3 $\boxed{B}$

29. Functions with leading terms with odd exponents always have to have at LEAST one real root, because complex roots appear in pairs. $\boxed{A}$

30. If we complete the square, we find that the point (1, 1) is a major axis endpoint, which would be the closest distance between (1, 1) and (1, 8). That distance is 7 $\boxed{C}$

Tie Breaker

$K(x) = 4x^{-2} - x^{\frac{1}{2}}$. $L(x) = -8x^{-3} - \frac{1}{2}x^{-\frac{1}{2}}$. $L(9) = -\frac{259}{1458}$