Alpha - Logs/Exponents Solution Key

1. $(x^{-1}-y^{-1})^{-1} = \left(\frac{1}{x}-\frac{1}{y}\right)^{-1} = \left(\frac{y-x}{xy}\right)^{-1} = \frac{xy}{y-x}$ $\boxed{d}$

2. $\log_2(x^2+2x) = 3$ $\boxed{b}$

$2^3 = x^2+2x$

$8 = x^2+2x$

$0 = x^2+2x-8$

$0 = (x+4)(x-2)$

$x = \cancel{-4}, 2$

3. $\log_b 250 = \log_b \frac{1000}{4} = \log_b 1000 - \log_b 4$

$= \log_b 10^3 - \log_b 2^2$

$= 3\log_b 10 - 2\log_b 2 = 3(1.4307) - 2(.4307)$

$= 3.4307$ a

4. $x^{1/6} = \frac{7}{3-\sqrt{2}} - 8^{1/6}$ c

$x^{1/6} = 3$, $x = 729$

5. $n = 2-(-2)^{(2--2)} = 2-(-2)^4 = 2-16 = -14$ $\boxed{b}$

6. $\log_4 8 + \log_8 4 + \log_2 8 = \log_2 x$ a

$\frac{3}{2} + \frac{2}{3} + 3 = \log_2 x$

$\frac{31}{6} = \log_2 x$

$2^{31/6} = x$ $\quad x = 2^5 \cdot 2^{1/6} = 32\sqrt[6]{2}$

7. $\log \frac{x+1}{3x+1} = 2$ d

$10^2 = \frac{x+1}{3x+1}$

$100 = \frac{x+1}{3x+1}$

$300x+100 = x+1$

$299x = -99$

$x = -\frac{99}{299}$

8. Factor: $(e^x-2)(e^{2x}-e^x+2)=0$

$e^x = 2$

$x = \ln 2$ a

9. $3^{6x} = 3^{2x-4}$ $\qquad 3^{-y} = 3^{3}$ $\qquad \log_{10} 2z = 1$

$6x = 2x - 4$ $\qquad y = -3$ $\qquad 2z = 10$

$4x = -4$ $\qquad z = 5$

$x = -1$

$x + y + z = -1 + -3 + 5 = 1$ $\boxed{d}$

10. $\left[\frac{1}{2}a^{-2} \cdot \frac{1}{a^{-1}}\right]^{-1} = \left[\frac{a}{2a^2}\right]^{-1} = \left(\frac{1}{2a}\right)^{-1} = 2a$ $\boxed{b}$

11. $b^{b/2} = 3^{b}$ $\qquad$ a

$\frac{b}{2} \log b = b \log 3$

$\log b = (\cancel{b} \log 3) \cdot \frac{2}{\cancel{b}}$

$\log b = 2 \log 3$

$\log b = \log 9$

$b = 9$

12. $(3^x - 3^{-x})^2 = 9^x - 2 + 9^{-x} = 9^x + 9^{-x} - 2 = 34 - 2 = 32$ $\boxed{b}$

$(3^x - 3^{-x})^2 = 32$

$3^x - 3^{-x} = \sqrt{32} = 4\sqrt{2}$

13. $\log\left(x^{\log x^2}\right) = 18$

$10^{18} = x^{\log x^2}$ $\qquad \boxed{d}$

$\log 10^{18} = \log x^2 (\log x)$

$18 = (2 \log x)(\log x)$

$9 = \log^2 x$

$\pm 3 = \log x$

$x = 10^{\pm 3}$

14. $4^{-x^2} = 2^{1-3x}$; $\qquad 2^{-2x^2} = 2^{1-3x}$; $\qquad -2x^2 = 1 - 3x$ $\qquad \boxed{e}$

$0 = 2x^2 - 3x + 1$

$0 = (2x - 1)(x - 1)$

$x = \frac{1}{2}, 1$

15. $f'(x) = 3^{-x^2-1/2}(-2x)\ln 3 = 0$

$x=0$

$f'(x)$ $\xleftarrow{\quad + \quad | \quad - \quad}$ 0

maximum occurs at $x=0$ $\boxed{a}$

$f(x) = 3^{-1/2} = \frac{1}{\sqrt{3}}$

$\left(\frac{1}{\sqrt{3}}\right)^3 = \frac{1}{3\sqrt{3}} = \frac{1\cdot\sqrt{3}}{3\sqrt{3}\cdot\sqrt{3}} = \frac{\sqrt{3}}{9}$

16. $5^{2x} - 3 = 5^x + 9$ $\boxed{d}$

$5^{2x} - 5^x - 12 = 0$

$(5^x+3)(5^x-4) = 0$

$\cancel{5^x = -3}$ $\quad 5^x = 4$ $\quad y = 5^x + 9 = 4 + 9 = 13$

17. $\ln\left(\frac{e}{\sqrt[5]{5}}\right)^5 + \ln x = 3$

$\ln\left[\left(\frac{e^5}{5}\right)x\right] = 3$ $\boxed{a}$

$e^3 = \frac{xe^5}{5}$

$5e^3 = xe^5$

$\frac{5e^3}{e^5} = x$

$\frac{5}{e^2} = x$

18. $4^{x-1} - 4^x = -24$

$4^{x-1}(1-4) = -24$

$(4^{x-1})(-3) = -24$

$4^{x-1} = 8$

$2^{2x-2} = 2^3$

$2x-2 = 3$

$2x = 5$

$x = 5/2$

$(2x)^{2x} = 5^5 = 3125$ $\boxed{d}$

19. $$\frac{\left(1+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)^{-1}}{\left(1-\frac{1}{x}\right)^{-2}} = \frac{\left(\frac{x+1}{x}\right)\cdot\left(\frac{x^2-1}{x}\right)^{-1}}{\left(\frac{x-1}{x}\right)^{-2}} = \frac{\left(\frac{x+1}{x}\right)\cdot\left(\frac{x-1}{x}\right)^{2}}{\left(\frac{x^2-1}{x}\right)}$$

$$= \frac{\cancel{x+1}}{x}\cdot\frac{\cancel{(x-1)}(x-1)}{\cancel{x}\cdot x}\cdot\frac{\cancel{x}}{\cancel{(x-1)}\cancel{(x+1)}}$$

$$= \frac{x-1}{x^2} \quad \boxed{c}$$

20. $\boxed{c}$

21. $\log 27 - \log\sqrt{3} = \log\sqrt{x}$

$\log\frac{27}{\sqrt{3}} = \log\sqrt{x}$ $\quad\boxed{b}$

$\frac{27}{\sqrt{3}} = \sqrt{x}$

$\sqrt{3x} = 27$

$3x = 729$

$x = 243$

22. $\log(x^2-15x)=2$ $\quad\boxed{c}$

$100 = x^2-15x$

$0 = x^2-15x-100$

$0 = (x-20)(x+5)$

$x = 20, \cancel{-5}$

23. $\boxed{a}$

24. $A = P\left(1+\frac{r}{n}\right)^{nt}$

$2100 = 700\left(1+\frac{.07}{4}\right)^{4t}$

$3 = (1.0175)^{4t}$ $\quad\boxed{b}$

$\ln 3 = 4t\ln(1.0175)$

$\frac{\ln 3}{\ln 1.0175} = 4t$

$63.325 = 4t$

$t \doteq 15.8$

25.)

$37 = \frac{250}{5+44.99e^{-.0208t}}$

$185 + 1664.63e^{-.0208t} = 250$

$1664.63e^{-.0208t} = 65$

$e^{-.0208t} = .0390477$

$-.0208t(\ln e) = \ln(.0390477)$

$t = 155.9 \doteq 156$ yrs

$\boxed{e}$

$$\begin{array}{r} 1971 \\ 156 \\ \hline 2127 \end{array}$$

Tie breaker:

$y-15 = Ce^{kt}$

when $t=0$, $y=68$

$53 = C$

$y-15 = 53e^{kt}$

when $t=1$, $y=52$

$37 = 53e^{k}$

$-.359374 = k$

$y-15 = 53e^{-.359374t}$

when $t=5$, $y = 23.78$

$24°F$