Mu Alpha Theta National Convention 2002
Mississippi State University
Matrices Topic Test
Alpha Division

1. Solve for x, y, z, and w: $\begin{bmatrix} 2 & -8 \\ 6 & -7 \end{bmatrix} = \begin{bmatrix} 2x-3 & 3y+1 \\ -5z+1 & -2w-5 \end{bmatrix}$

(a) $\begin{matrix} x=-2.5 \\ y=-3 \\ z=-1 \\ w=1 \end{matrix}$ (b) $\begin{matrix} x=2.5 \\ y=-3 \\ z=-1 \\ w=1 \end{matrix}$ (c) $\begin{matrix} x=-2.5 \\ y=-3 \\ z=-1 \\ w=-1 \end{matrix}$ (d) $\begin{matrix} x=-2.5 \\ y=-3 \\ z=1 \\ w=1 \end{matrix}$ (e) NOTA

2. Evaluate: $\begin{bmatrix} 3x+y & x-2y & 2x \\ 2x-5y & 3x-2y & x+y \end{bmatrix} + \begin{bmatrix} 2x & -3y & 5x+y \\ 3x+2y & x-4y & 2x \end{bmatrix}$

(a) $\begin{bmatrix} 5x+y & x+y & 7x+y \\ 5x-3y & 4x-6y & 3x+y \end{bmatrix}$ (b) $\begin{bmatrix} 5x+y & x-5y & 7x+y \\ 5x+3y & 4x-6y & 3x+y \end{bmatrix}$

(c) $\begin{bmatrix} 5x+y & x-5y & 7x+y \\ 5x-3y & 4x+6y & 3x+y \end{bmatrix}$ (d) $\begin{bmatrix} 5x+y & x-5y & 7x+y \\ 5x-3y & 4x-6y & 3x+y \end{bmatrix}$

(e) NOTA

3. If $A = \begin{bmatrix} 3 & -11 & -1 \\ -6 & 4 & 9 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & -2 & 8 \\ 2 & -4 & -5 \end{bmatrix}$, find $2A - 3B$.

(a) $\begin{bmatrix} -15 & -16 & -26 \\ -18 & 20 & 33 \end{bmatrix}$ (b) $\begin{bmatrix} -15 & -16 & -26 \\ 18 & 20 & 33 \end{bmatrix}$ (c) $\begin{bmatrix} -15 & -16 & 26 \\ -18 & 20 & 33 \end{bmatrix}$

(d) $\begin{bmatrix} -15 & 16 & -26 \\ -18 & 20 & 33 \end{bmatrix}$ (e) NOTA

4. Solve for A: $3A + \begin{bmatrix} 3 & -2 \\ 4 & -7 \\ -2 & 5 \end{bmatrix} = \begin{bmatrix} 6 & 13 \\ -8 & 2 \\ 1 & -1 \end{bmatrix}$.

(a) $\begin{bmatrix} 1 & 5 \\ -4 & 3 \\ 1 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 5 \\ 4 & 3 \\ 1 & -2 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 5 \\ 4 & 3 \\ 1 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & 15 \\ -12 & 9 \\ 3 & -6 \end{bmatrix}$ (e) NOTA

5. Solve for x, y, and z in the matrix equation $4\begin{bmatrix} x & y \\ z & -1 \end{bmatrix} = 2\begin{bmatrix} y & z \\ -x & 1 \end{bmatrix} + 2\begin{bmatrix} 4 & x \\ 5 & -x \end{bmatrix}$.

(a) $\begin{matrix} x=-3 \\ y=2 \\ z=1 \end{matrix}$ (b) $\begin{matrix} x=3 \\ y=2 \\ z=-1 \end{matrix}$ (c) $\begin{matrix} x=3 \\ y=2 \\ z=1 \end{matrix}$ (d) $\begin{matrix} x=-3 \\ y=2 \\ z=-1 \end{matrix}$ (e) NOTA

6. Evaluate: $\begin{bmatrix} 2 & x & 3 \\ -1 & 2 & 7 \\ z & 5 & -2 \end{bmatrix}\begin{bmatrix} 3 & 1 & 5 \\ 3 & -3 & -1 \\ 2 & 0 & y \end{bmatrix}$

(a) $\begin{bmatrix} 3x+12 & 3x+2 & -x+3y+10 \\ 17 & -7 & 7y-7 \\ 3z+11 & z-15 & -2y+5z-5 \end{bmatrix}$ (b) $\begin{bmatrix} 3x+12 & -3x+2 & -x+3y+10 \\ 17 & 7 & 7y-7 \\ 3z+11 & z-15 & -2y+5z-5 \end{bmatrix}$

(c) $\begin{bmatrix} 3x+12 & -3x+2 & -x+3y+10 \\ 17 & -7 & 7y-7 \\ 3z+11 & z-15 & -2y+5z+5 \end{bmatrix}$ (d) $\begin{bmatrix} 3x+12 & -3x+2 & -x+3y+10 \\ 17 & -7 & 7y-7 \\ 3z+11 & z-15 & -2y+5z-5 \end{bmatrix}$

(e) NOTA

7. Find the trace of $A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -2 & 4 \\ 3 & 1 & 3 \end{bmatrix}$.

(a) 15 (b) 4 (c) 3 (d) 2 (e) NOTA

8. If $A = \begin{bmatrix} 1 & 2 \\ x & -1 \end{bmatrix}$, then calculate A^3.

(a) $\begin{bmatrix} 1 & 8 \\ x^3 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2x+1 & 4x+2 \\ 2x^2+x & -2x+1 \end{bmatrix}$ (c) $\begin{bmatrix} 2x+1 & 4x+2 \\ 2x^2+x & 2x+1 \end{bmatrix}$ (d) $\begin{bmatrix} 2x+1 & 4x+2 \\ 2x^2+x & 2x-1 \end{bmatrix}$

(e) NOTA

9. For $x \neq \frac{-2}{3}$, find the inverse of $\begin{bmatrix} 2 & -3 \\ x & 1 \end{bmatrix}$.

(a) $\frac{1}{2-3x}\begin{bmatrix} 1 & 3 \\ -x & 2 \end{bmatrix}$ (b) $\frac{1}{2+3x}\begin{bmatrix} 1 & 3 \\ -x & 2 \end{bmatrix}$ (c) $\frac{1}{2-3x}\begin{bmatrix} 2 & x \\ -3 & 1 \end{bmatrix}$ (d) $\frac{1}{2+3x}\begin{bmatrix} 2 & x \\ -3 & 1 \end{bmatrix}$

(e) NOTA

10. Find x such that the matrix $\begin{bmatrix} 3 & x \\ -2 & -3 \end{bmatrix}$ is equal to its own inverse.

(a) -4 (b) 2 (c) 4 (d) 5 (e) NOTA

11. Find A given that $(2A)^{-1} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(a) $\begin{bmatrix} \frac{1}{2} & 1 \\ \frac{3}{2} & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} 8 & -4 \\ -6 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} \end{bmatrix}$ (e) NOTA

12. Solve for A: $\begin{bmatrix} 1 & -2 \\ 3 & -2 \end{bmatrix} A = \begin{bmatrix} 1 & 3 \\ -2 & 1 \end{bmatrix}$

(a) $\begin{bmatrix} -\frac{3}{2} & -1 \\ -\frac{5}{4} & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{11}{4} & \frac{5}{4} \\ \frac{1}{4} & -\frac{3}{4} \end{bmatrix}$ (c) $\begin{bmatrix} 1 & -\frac{3}{2} \\ -\frac{2}{3} & -\frac{1}{2} \end{bmatrix}$ (d) $\begin{bmatrix} \frac{3}{2} & 1 \\ \frac{5}{4} & 2 \end{bmatrix}$ (e) NOTA

13. Find all x such that the matrix $\begin{bmatrix} x & 3x \\ -2 & x \end{bmatrix}$ is singular.

(a) -6 (b) -6 or 0 (c) 0 (d) 0 or 6 (e) NOTA

14. Evaluate: $\begin{vmatrix} x-3 & 2 \\ 4 & x-1 \end{vmatrix}$

(a) $x^2-4x+11$ (b) x^2-4x-5 (c) $x^2-4x-11$ (d) x^2-4x+5 (e) NOTA

15. Evaluate: $\begin{vmatrix} x & y & 1 \\ 2 & 3 & -1 \\ -2 & -1 & 3 \end{vmatrix}$

(a) $8x-4y+4$ (b) $8x+4y+4$ (c) $10x-4y+4$ (d) $8x-4y-8$ (e) NOTA

16. If $\begin{vmatrix} 2 & -3 \\ x & 4 \end{vmatrix} = \begin{vmatrix} 5 & 4 \\ -1 & 3 \end{vmatrix}$, then calculate the value of x.

(a) $\frac{11}{3}$ (b) $-\frac{11}{3}$ (c) 1 (d) -1 (e) NOTA

17. If A is a 4×4 matrix such that $|A|=-3$, what is the value of $|2A|$.

(a) -6 (b) -48 (c) 6 (d) 48 (e) NOTA

18. Given $x \neq \frac{-3}{2}$. If $A=\begin{bmatrix} 3 & -x \\ 2 & 1 \end{bmatrix}$, then calculate $|A^{-1}|$.

(a) $\frac{1}{3-2x}$ (b) $\frac{1}{3+2x}$ (c) $3-2x$ (d) $3+2x$ (e) NOTA

19. If $A^{-1}=\begin{bmatrix} 2 & 5 \\ -7 & 6 \end{bmatrix}$ and $B^{-1}=\begin{bmatrix} 7 & -3 \\ 2 & 0 \end{bmatrix}$, then calculate $(AB)^{-1}$.

(a) $\begin{bmatrix} 35 & 17 \\ 4 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 24 & -6 \\ -37 & 21 \end{bmatrix}$ (c) $\begin{bmatrix} -35 & 17 \\ 4 & 10 \end{bmatrix}$ (d) $\begin{bmatrix} -24 & -6 \\ -37 & 21 \end{bmatrix}$ (e) NOTA

20. Suppose $|A|=3$ and the adjoint of A is $\begin{bmatrix} 4 & 6 & 7 \\ 1 & 0 & 1 \\ 2 & 3 & 2 \end{bmatrix}$. Find A^{-1}.

(a) $\begin{bmatrix} 12 & 18 & 21 \\ 3 & 0 & 3 \\ 6 & 9 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} -12 & -18 & -21 \\ -3 & 0 & -3 \\ -6 & -9 & -6 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 3 & 2 \\ 0 & -2 & 1 \\ 1 & 0 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} \frac{4}{3} & 2 & \frac{7}{3} \\ \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{2}{3} & 1 & \frac{2}{3} \end{bmatrix}$ (e) NOTA

21. Find the area of the triangle having vertices $(-1,2)$, $(2,2)$, $(-2,4)$.

(a) 6 (b) 3 (c) 2 (d) 0.5 (e) NOTA

22. If $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{v} = -3\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$, and $\mathbf{w} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$, then calculate $3\mathbf{u} - 2\mathbf{v} + \mathbf{w}$.

(a) $13\mathbf{i} + 21\mathbf{j} + 2\mathbf{k}$ (b) $13\mathbf{i} - 21\mathbf{j} + 2\mathbf{k}$ (c) $13\mathbf{i} + 21\mathbf{j} - 2\mathbf{k}$ (d) $13\mathbf{i} - 21\mathbf{j} - 2\mathbf{k}$ (e) NOTA

23. If $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j} + \sqrt{7}\mathbf{k}$, then calculate $\|\mathbf{v}\|$.

(a) $2\sqrt{5}$ (b) $\sqrt{62}$ (c) $4\sqrt{3}$ (d) $\sqrt{54}$ (e) NOTA

24. Find a unit vector in the direction of $\mathbf{v} = 5\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

(a) $\frac{5}{\sqrt{70}}\mathbf{i} + \frac{3}{\sqrt{70}}\mathbf{j} + \frac{6}{\sqrt{70}}\mathbf{k}$ (b) $\frac{5}{\sqrt{70}}\mathbf{i} - \frac{3}{\sqrt{70}}\mathbf{j} + \frac{6}{\sqrt{70}}\mathbf{k}$ (c) $\frac{5}{\sqrt{70}}\mathbf{i} + \frac{3}{\sqrt{70}}\mathbf{j} - \frac{6}{\sqrt{70}}\mathbf{k}$

(d) $\frac{5}{\sqrt{70}}\mathbf{i} - \frac{3}{\sqrt{70}}\mathbf{j} - \frac{6}{\sqrt{70}}\mathbf{k}$ (e) NOTA

25. If $\mathbf{u} = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$, then calculate $\mathbf{u} \bullet \mathbf{v}$.

(a) 16 (b) 4 (c) –16 (d) – 12 (e) NOTA

26. Find the distance between $\mathbf{u} = 2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = -5\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$.

(a) $\sqrt{60}$ (b) $\sqrt{122}$ (c) $\sqrt{34}$ (d) $\sqrt{66}$ (e) NOTA

27. If $\mathbf{v} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$, then what value(s) of *c* makes $\|c\mathbf{v}\| = 1$?

(a) $\frac{1}{\sqrt{14}}$ (b) $-\frac{1}{\sqrt{14}}$ (c) $\pm\frac{1}{\sqrt{14}}$ (d) $\pm\frac{1}{14}$ (e) NOTA

28. Find the angle θ (to the nearest tenth of a degree) between $\mathbf{u} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$.

(a) 84.3° (b) 95.7° (c) 1.5° (d) 1.7° (e) NOTA

29. If $\mathbf{u} = 6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $\mathbf{v} = -2\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, then calculate $\mathbf{v} \times \mathbf{u}$.

(a) $5\mathbf{i} + 14\mathbf{j} + 6\mathbf{k}$ (b) $5\mathbf{i} - 14\mathbf{j} + 6\mathbf{k}$ (c) $5\mathbf{i} + 14\mathbf{j} - 6\mathbf{k}$ (d) $-5\mathbf{i} - 14\mathbf{j} - 6\mathbf{k}$ (e) NOTA

30. Find the area of the parallelogram that has $\mathbf{u} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ as adjacent sides.

(a) 4 (b) $6\sqrt{5}$ (c) $5\sqrt{3}$ (d) $2\sqrt{14}$ (e) NOTA

Work the following tiebreaker on the white portion of the back of the scantron sheet and circle your answer.

Bonus: Find the eigenvalues of $A = \begin{bmatrix} 0 & -3 & 5 \\ -4 & 4 & -10 \\ 0 & 0 & 4 \end{bmatrix}$.

Mu Alpha Theta National Convention 2002
Mississippi State University
Matrices Topic Test
Alpha Division

1. b
2. d
3. a
4. e
5. c
6. d
7. d
8. e
9. b
10. c
11. d
12. a
13. b
14. b
15. a
16. a
17. b
18. b
19. a
20. d
21. b
22. d
23. a
24. c
25. d
26. d
27. c
28. b
29. d
30. b

Tiebreaker: -2,4,6