Solution Key
Sequence & Series Topic Test - Alpha Division

1. $d = a_n - a_{n-1}$ $\quad d = -1-(-4) = 3$ B

2. $a_n = a_1 + (n-1)d$ $\quad a_5 = 1 + 4 \cdot \frac{7}{6} = \frac{34}{6} \sim \frac{17}{3}$ C
$8 = 1 + (n-1)d$
$8 = 1 + 6d$
$\frac{7}{6} = d$

3. $a_{17} = 5(-1)^{18} = 5$
$a_{19} = 5(-1)^{20} = 5$
$\frac{a_{19}}{a_{17}+a_{19}} = \frac{5}{10}$ D

4. $1^2-(-2)^2+3^2-(-4)^2+5^2-(-6)^2 =$
$1 - 4 + 9 - 16 + 25 - 36 = -21$ B

5. $a_n = a_1 \cdot r^{n-1}$
$8 = 1 \cdot r^6$ $\quad \therefore a_5 = (\sqrt{2})^4 = 4$ C
$\sqrt[6]{8} = r$
$\sqrt{2} = r$

6. $a_n = 2^0, 2^3, 2^6, \ldots$ with ratio $r = 2^3$. $a_4 = 2^6 \cdot 2^3 \sim 2^9$ D

7. $a_{n+2} = \frac{(n+2)-1}{2(n+2)-1} = \frac{n+1}{2n+3}$ A

8. $S_n = \frac{n}{2}[a_1 + a_n] = \frac{31}{2}[1+31] = 496$ C

9. Remove ( ) and the sum reduces to $2 - \frac{10}{9} = \frac{8}{9}$ C

10. $a_n = b_{n-1}$
$3n+1 = 4(n-1)-1$ or
$3n+1 = 4n-5$
$6 = n$

$a_1 = 4 \quad b_1 = 3$
$a_2 = 7 \quad b_2 = 7$
$a_3 = 10 \quad b_3 = 11$
$a_4 = 13 \quad b_4 = 15$
$a_5 = 16 \quad b_5 = 19$
$a_6 = 19 \quad b_6 = 23$
C

11. By examining the table if n is odd, difference is odd. $a_n = 3n+1$ is odd when n is even and is even when n is odd. For a $b_n = 4n-1$ is always odd. For a difference to be odd, they must have differing parity. $\therefore a_n$ is even when n is odd. A

12. $a_1 = 24$ and $a_9 = 18$
$a_n = a_1 + (n-1)d$ $\quad \therefore a_5 = 24 + 4(\frac{-3}{4}) = 21$ C
$18 = 24 + 8d$
$-6 = 8d$
$-\frac{3}{4} = d$

13. $a_1 = 7$
$a_2 = (-1)^2 + 7 = 8$
$a_3 = (-1)^3 + 8 = 7$
$a_4 = (-1)^4 + 7 = 8$
This pattern continues. Since $n = 15$ is odd, $a_{15} = 7$. A

14. S_n is an arith series whose terms diff by 3. $n = 100 \rightarrow 3n-2 = 298$
$\therefore S_{100} = \frac{100}{2}(1+298) = 14,950$ B.

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15. $A_n = A_0(.6)^{n-1} < .08\,A_0$

$(n-1)\log(.6) < \log .08$

$n < \dfrac{\log(.08)+\log(.6)}{\log(.6)}$

$n > 5.9444$

If $n=6$, then the minimum number of strokes $(n-1)$ is 5. B

16. This is sum of arith seq. of 21 terms

$R_0 = 40,\ R_{20} = -40$

$S_{21} = \frac{21}{2}[40-40]$

$= 0$ C.

17. All the terms are written as

$2 - \dfrac{n+1}{\text{power of } 2}$ where power $= n+1$

$\therefore a_{10} = 2 - \dfrac{11}{2^{10}}$ or $2 - \dfrac{11}{1024}$ C

18. $d = \frac{1}{4}$ $a_n = 10 + \frac{(n-1)}{4}$

$20 = 10 + \frac{(n-1)}{4}$

$41 = n$

$S_{41} = \frac{41}{2}[10+20]$ D

$= 615$

19. $\sqrt{30+x} = x$

$30 + x = x^2$

$0 = x^2 - x - 30$

$(x-6)(x+5)$ B.

$\therefore x = 6$ or -5. Sum is $+$.

20 Pyramid = sum $1+4+9+16+25$ or 55 D

21. $\sum \frac{1}{3^n} - 3\sum\frac{1}{5^n} = \dfrac{1}{1-\frac{1}{3}} - 3\cdot\dfrac{1}{1-\frac{1}{5}} = -\dfrac{9}{4}$ B

22. When written out

$a_n = -4, -2, -1, 0, 0, 0, \ldots$

$S_n = -7$ D

23. Working backwards,

$a_4 - 2 = -3 \Rightarrow a_4 = -1$

$a_3 + a_4 = -2 \Rightarrow a_3 = -1$

$a_2 + a_3 = -1 \Rightarrow a_2 = 0$

$a_1 + a_2 = -1 \Rightarrow a_1 = -1$ A

24. Looking at the series

$S_1 = +\frac{3}{2}$

$S_2 = -\frac{3}{2} + \frac{9}{6} = 0$

$S_3 = -\frac{3}{2} + \frac{9}{6} - \frac{27}{12} = -\frac{9}{4}$

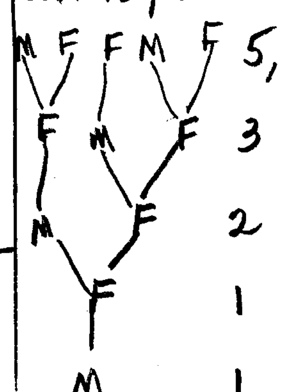
$S_4 = -\frac{3}{2} + \frac{9}{6} - \frac{27}{12} + \frac{81}{20} = \frac{9}{5} \ldots$

a_n alternates in sign, $|a_n| \to \infty$

$\therefore$ all three statements are true D

25. Check by looking at answers. numerators are 2^1 to 2^7; denominators are odd integers of form $2n \pm 1$, $2n \pm 3$. Only C satisfies all conditions. C

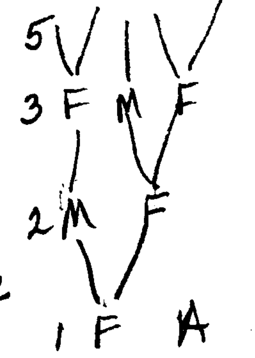
26. By writing for 10 generations



5, 8, 13, 21, 34, 55, 89 for male

F has $144 - 1 = 143$ more bees in ancestoral tree

8, 13, 34, 55, 89, 14



A

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27. $x, y, x+y, x+2y, 2x+3y, 3x+5y$, etc.
$a_1 - a_4$: $x^2+2xy-xy-y^2 = x^2-y^2+xy$
$a_2 - a_5$: $2xy+3y^2-x^2-3xy-2y^2 = y^2-x^2-xy$
$\therefore \pm(x^2-y^2+xy)$ D

28. If $a=0$, then $\Sigma = 0$. If $|b|<1$ then $a\Sigma b^n$ has a sum. C

29. The smallest is 4004 and largest is 4998. Terms differ by 7.
$4998 = 4004 + 7(n-1)$
$142 = n-1$
$143 = n$
$S_{143} = \frac{143}{2}(4004+4998)$
$= 643,643$ C

30. Earl pays \$100 for each of the first two months. By the end of the first year he has had 5 reductions of 20% so that his last fee is $100(.8)^5 = 32.768$ or \$32.77. He will continue paying this for the next 4 months. B

31. By writing out the terms:
12, 6, 3, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, ...
There is a repeat starting with a_8.
$\therefore$ 48 − 7 = 41 is the term in the sequence that repeats. $41/3 \Rightarrow$ rem 2.
$\therefore$ the 48th term is the 2d term in the repeat. This term is 2. B

32. a_n is arithmetic with $a_1 = 4$ and $d = 7$. By checking the terms $(2226-4)/7$ is not an integer
or $2226 = 4 + (n-1)7$
$317\frac{3}{7} \neq n$
The other numbers are all 4 more than a multiple of 7 A

33. By expressing each term
$a_1, a_2, a_1+a_2, a_1+2a_2, 2a_1+3a_2, 3a_1+\sqrt{a_2}$,
$a_7 = 5a_1 + 8a_2 \Rightarrow 120 = \sqrt{a_1} + 8a_2$
$\therefore a_2 = \frac{-\sqrt{}}{8}a_1 + 15$

possible ans are E

a_1	0	8	16	−8
a_2	15	10	5	20

34. By investigation there are 2 sets of parallelograms being formed whose perimeters are: and
I: $28, 14, 7, \frac{7}{2}, \frac{7}{4}, \ldots$
II: $20, 10, 5, \frac{5}{2}, \frac{5}{4}, \ldots$
I sum of 10 terms is $\frac{28(1-.5^9)}{1-.5}$
II sum of 10 terms is $\frac{20(1-.5^9)}{1-.5}$
or $(56+40)(1-.5^9) = 95.8125$ C

35. In the last millennium the first was 1001 and last was 1991. $S_{10} = \frac{10}{2}(1001+1991)$ since $d = 110$
$= 14960$ B.

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36. $x, y, z \Rightarrow y = rx,\ z = r^2x$

$x, 2y, 3z \Rightarrow 2y - x = 3z - 2y$

by subst. $2rx - x = 3r^2x - 2rx$

$0 = x(3r^2 - 4r - 1)$

$\therefore x = 0,\ r = \frac{1}{3}$ or $-\frac{1}{4}$ B.

37. sum $= \cos^2 x + \cos^4 x + \cos^6 x + \dots$

where $a_1 = \cos^2 x$ and $r = \cos^2 x$

Since $|\cos^{2k} x| < 1$, sum $= \dfrac{\cos^2 x}{1 - \cos^2 x}$

or $\cot^2 x$. E

38. The first is 1014. Since 1024 is divisible by 4, the second is 1034.

$\therefore d = 20$. The last is 9994

$\therefore 9994 = 1014 + (n-1)20$

$450 = n$

$S_n = \frac{450}{2}[1014 + 9994]$

$= 2,476,800$ This number is closest to $5^2 \times 10^5 = 2,500,000$. B

39. The sequence is:

$1,\ 1+1,\ 1+\frac{1}{2},\ 1+\frac{2}{3},\ 1+\frac{3}{5},\ 1+\frac{5}{8}, \dots$

$\frac{1}{1}, \frac{2}{1}, \frac{3}{2}, \frac{5}{3}, \frac{8}{5}, \frac{13}{8}, \dots$

where the term is $\frac{a_{n+1}}{a_n}$ where a_n is the Fibonacci sequence. These terms oscillate in value and approach $\phi = 1.618\dots$ D

40. Since Celine goes second on each round, the prob she wins on first round is $\frac{5}{6} \cdot \frac{1}{6} = P(C_1)$

If she misses on the first round the prob she wins on the second round is $P(C_1) + P(C_2) = \frac{5}{6} \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$

This continues until she wins on the nth round.

$\frac{5}{6}\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^7\left(\frac{1}{6}\right) + \dots$

The ratio $\left(\frac{5}{6}\right)^3$ represents the three misses, including her own on the previous round, that have occurred since her last chance

$$P(C) \text{ wins} = \sum_{k=0}^{\infty} \left(\frac{5}{6}\right)^{3k+1}\left(\frac{1}{6}\right)$$

or $\dfrac{\left(\frac{5}{6}\right)\left(\frac{1}{6}\right)}{1 - \left(\frac{5}{6}\right)^3} = \dfrac{30}{91}$