2002 National Mu Alpha Theta
Sequence & Series Topic Test - Alpha Division

1. What is the common difference in the arithmetic sequence -4, -1, 2, . . ?

(A) -3 (B) 3 (C) 5 (D) 6 (E) NOTA

2. What is the fifth term in the arithmetic sequence for which $a_1 = 1$ and $a_7 = 8$?

(A) $\frac{7}{6}$ (B) $\frac{9}{2}$ (C) $\frac{17}{3}$ (D) $\frac{41}{6}$ (E) NOTA

3. A sequence is explicitly defined by $a_n = 5(-1)^{n+1}$. What is the value of $a_{17} + a_{19}$?

(A) -10 (B) 0 (C) 5 (D) 10 (E) NOTA

4. Evaluate $1^2 - (-2)^2 + 3^2 - (-4)^2 + 5^2 - (-6)^2 = ?$

(A) -91 (B) -21 (C) 17 (D) 91 (E) NOTA

5. What is the fifth term in the geometric sequence when $a_1 = 1$ and $a_7 = 8$?

(A) 2 (B) $2\sqrt{2}$ (C) 4 (D) $4\sqrt{2}$ (E) NOTA

6. What is the next term in the geometric sequence: 1, 8, 64, . . . ?

(A) 2^3 (B) 2^7 (C) 2^8 (D) 2^9 (E) NOTA

7. If a sequence has its general term a_n defined as $a_n = \frac{n-1}{2n-1}$, what is a_{n+2} ?

(A) $\frac{n+1}{2n+3}$ (B) $\frac{n+1}{2n+1}$ (C) $\frac{n+1}{2n-3}$ (D) $\frac{n+3}{2n+1}$ (E) NOTA

8. What is the sum of the first 31 positive integers?

(A) 435 (B) 465 (C) 496 (D) (528) (E) NOTA

9. Find the indicated sum: $\left(\frac{2}{1}-\frac{3}{2}\right)+\left(\frac{3}{2}-\frac{4}{3}\right)+\left(\frac{4}{3}-\frac{5}{4}\right)+\ \ldots\ +\left(\frac{9}{8}-\frac{10}{9}\right)$.

(A) $\frac{1}{2}$ (B) $\frac{19}{36}$ (C) $\frac{8}{9}$ (D) $\frac{16}{9}$ (E) NOTA

For #10 and #11 let $a_n = 3n+1$ and $b_n = 4n-1$.

10. For what value of n does $a_n = b_{n-1}$?

(A) 2 (B) 4 (C) 6 (D) 8 (E) NOTA

11. For what values of n is $|a_n - b_n|$ an odd integer? Assume that k is a positive integer.

(A) n = 2k - 1 (B) n = 1 - 3k (C) n = 2k (D) n = 3k - 1 (E) NOTA

12. A ladder with nine rungs is to be built with the bottom rung 24 inches wide and the top rung 18 inches wide. If the length of the rungs decrease linearly from the bottom to the top, how long is the fifth rung from the bottom?

(A) 19.5 (B) 20.25 (C) 21 (D) 22.5 (E) NOTA

13. A sequence is defined recursively by $\left\{\begin{array}{c} a_1 = 7 \\ a_n = (-1)^n + a_{n-1} \end{array}\right\}$. What is the fifteenth term in the sequence?

(A) 7 (B) 9 (C) 12 (D) 16 (E) NOTA

14. Evaluate the sum: $1+4+7+\ \ldots\ +(3n-2)$ for $n=100$.

(A) 14,652 (B) 14,950 (C) 14,999 (D) 15,251 (E) NOTA

15. A vacuum pump removes 40% of the air in a container with each stroke. What is the minimum number of strokes necessary so that the percent of the original amount of air is less than 8%?

(A) 4 (B) 5 (C) 6 (D) 7 (E) NOTA

16. Evaluate: $\sum_{k=0}^{20}(44+(-4)(k+1))$

(A) -42 (B) -21 (C) 0 (D) 42 (E) NOTA

17. The first few terms of a sequence are: $2-\frac{1}{2}, 2-\frac{3}{8}, 2-\frac{1}{4}, 2-\frac{5}{32}, \ldots$
What is the tenth term in the sequence ?

(A) $\frac{10}{1024}$ (B) $2-\frac{10}{1024}$ (C) $2-\frac{11}{1024}$ (D) $2-\frac{10}{2048}$ (E) NOTA

18. Find the sum of the arithmetic series: $10+10\frac{1}{4}+10\frac{1}{2}+10\frac{3}{4}+\ \ldots\ +20$.

(A) 590 (B) 600 (C) 610 (D) 615 (E) NOTA

19. Evaluate $\sqrt{30+\sqrt{30+\sqrt{30+\ldots}}}$

(A) -5 (B) 6 (C) 25 (D) 36 (E) NOTA

20. For a Halloween exhibit an elementary school visited a pumpkin field. The class was studying the Egyptian pyramids at the time. Some students asked their teacher if he would tell them how many pumpkins they would have to collect in order to make a square pyramid five rows high. The teacher reminded his students that they would have to choose pumpkins that were almost all the same size in order to build a symmetric stable pyramid. The students found many pumpkins that were almost the same size. How many of these would they need to make their pyramid?

(A) 15 (B) 35 (C) 41 (D) 55 (E) NOTA

21. Evaluate: $\sum_{n=0}^{\infty}\left(\frac{1}{3^n}-\frac{3}{5^n}\right)$

(A) $\frac{-37}{12}$ (B) $\frac{-9}{4}$ (C) -1 (D) $\frac{4}{9}$ (E) NOTA

22. What is true about the following sequence: $a_n=\begin{cases} a_1=-4 \\ if\ \ a_{n-1}\ \text{is even},\ a_n=\left(\frac{1}{2}\right)a_{n-1} \\ if\ \ a_{n-1}\ \text{is odd},\ a_n=a_{n-1}+1 \end{cases}$

I The sequence is everywhere increasing.

II The sequence is convergent.

III The sum of the infinite sequence is negative seven.

(A) I only (B) II only (C) III only (D) II and III (E) NOTA

23. The rule for finding a term in a sequence for $n \geq 3$ is to add the two previous terms. What is a_1 when the sequence is: $a_1,\ a_2,\ a_3,\ a_4,\ -2,\ -3,\ -5,\ -8,\ \ldots$?

(A) -1 (B) 0 (C) 1 D) -1 or 1 (E) NOTA

24. What is true about the series: $\sum_{n=1}^{\infty} \frac{(-3)^n}{n(n+1)}$?

I The series is alternating between positive and negative terms

II $\lim_{n\to\infty} a_n \neq 0$

III The series is divergent.

(A) I only (B) II only (C) II and III (D) I, II, and III (E) NOTA

25. Express the following series using sigma notation: $\frac{2}{3\bullet 5}+\frac{4}{5\bullet 7}+\frac{8}{7\bullet 9}+\cdots+\frac{128}{15\bullet 17}$

(A) $\sum_{k=1}^{7} \frac{2^k}{(k+8)(k+10)}$ (B) $\sum_{k=0}^{7} \frac{2^{k+1}}{(2^k-1)(2^k+1)}$

(C) $\sum_{k=1}^{7} \frac{2^k}{(2k+1)(2k+3)}$ (D) $\sum_{k=0}^{7} \frac{2^k}{(2k+1)(2k-1)}$ (E) NOTA

26. A female bee has both a female parent and a male parent whereas a male bee has only a female parent. Consider the "ancestoral" family tree of both a female bee and a male bee. Going back 10 generations, how many more"ancestors" does a female bee have than a male bee?

(A) 143 (B) 144 (C) 145 (D) 233 (E) NOTA

27. Assume $a_n = \left\{ \begin{array}{c} a_1 = x \\ a_2 = y \\ \text{for } n \geq 3,\ a_n = a_{n-1} + a_{n-2} \end{array} \right\}$. Consider any four consecutive terms in the sequence. What is the difference in the product of the first and fourth terms and the product of the second and third terms?

(A) x^2+y^2+xy (B) y^2-x^2+xy (C) 0 (D) $\pm(x^2-y^2+xy)$ (E) NOTA

28. Which of the following assures that $\sum_{n=3}^{\infty} ab^n$ converges?

I $\quad |a| < 1$

II $\quad |b| < 1$

III $\quad a = 0$

(A) I only (B) II only (C) II or III (D) III only (E) NOTA

29. Find the sum of the multiples of 7 between 4000 and 5000?

(A) 91,949 (B) 639,142 (C) 643,643 (D) 647,640 (E) NOTA

30. There is an initial fee of $100 for each of the first two months to join a local health club. To encourage members to continue their membership, during the first year there is a 20% discount after these first two months on the previous two month's membership fees. Earl has been enrolled for 16 months. What is the amount of the last payment that he made?

(A) $20.97 (B) $32.77 (C) $51.20 (D) $64.00 (E) NOTA

31. What is the 48th term in $a_n = \left\{ \begin{array}{c} a_1 = 12 \\ \text{if } a_{n-1} \text{ is even, } a_n = \left(\frac{1}{2}\right) a_{n-1} \\ \text{if } a_{n-1} \text{ is odd, } a_n = 3a_{n-1} + 1 \end{array} \right\}$?

(A) 1 (B) 2 (C) 4 (D) 8 (E) NOTA

32. Which of the following is NOT a term in the arithmetic sequence: 4, 11, 18, 25, 32, 39, . . .

(A) 2226 (B) 2727 (C) 3581 (D) 4267 (E) NOTA

33. A sequence is defined for $n > 2$ by $a_n = a_{n-1} + a_{n-2}$. If $a_7 = 120$, what are NOT possible values for a_1 and a_2 ?

(A) $a_1 = 0,\ a_2 = 15$ (B) $a_1 = 8,\ a_2 = 10$ (C) $a_1 = -8,\ a_2 = 20$ (D) $a_1 = 16,\ a_2 = 5$ (E) NOTA

34. The midpoints of the sides of a 6 by 8 parallelogram are joined to form another parallelogram, and then the midpoints of the sides of the small parallelogram are joined to form an even smaller parallelogram. If this process is continued, find the sum of the perimeters of the first 20 parallelograms formed.

(A) 47.99990845 (B) 95 (C) 95.8125 (D) 95.99581689 (E) NOTA

35. The year 2002 is the first palindrome of the new millennium. What is the sum of the palindrome years of the last millennium?

(A) 12969 (B) 14960 (C) 14990 (D) 15,001 (E) NOTA

36. Assume that x, y, z are the first three terms of a geometric sequence with common ratio r and $x \neq y$. If x, 2y, 3z are the first three terms of an arithmetic sequence, what is r?

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 2 (E) 5

37. Evaluate $\sum_{k=1}^{\infty} \cos^{2k} x$ when $x \neq (2n \pm 1)\pi$ or $x \neq (2n)\pi$.

(A) $\sin^2 x$ (B) $\cos^2 x$ (C) $\sec^2 x$ (D) $\csc^2 x$ (E) NOTA

38. The sum of all four digit numbers that end in four and are NOT divisible by four is closest to which of the following numbers?

(A) 2.4×10^6 (B) $5^2 \times 10^5$ (C) $3^3 \times 10^5$ (D) $6^2 \times 10^6$ (E) NOTA

39. A sequence with $a_1 = 1$ is defined recursively so that each successive term is one more than the reciprocal of the previous term. Determine what value the nth term approaches as n increases.

(A) $\frac{8}{5}$ (B) 1.625 (C) 3.236 (D) $\frac{1+\sqrt{5}}{2}$ (E) NOTA

40. Only one of the female singers: Britney, Celine, and Jennifer, can be a presenter at the next Grammy Awards. To decide which one, they roll, in alphabetical order, a fair die until one of them rolls a six. What is the probability that Celine will do the presentation?

(A) $\frac{30}{91}$ (B) $\frac{1}{3}$ (C) $\frac{7}{18}$ (D) $\frac{1}{2}$ (E) NOTA