Solution Key

2002 MU ALPHA THETA NATIONAL CONVENTION
ALPHA DIVISION TRIGONOMETRY TOPIC TEST

Each question in this test has five available answer choices. Choice *(e)* for all questions is NOTA, which indicates "None of These Answers." Scoring for the test is 4 points for each correct answer, 1 point for each unanswered question, and 0 points for each incorrect answer.

1. Convert from polar to rectangular coordinates: $(4, \frac{2\pi}{5})$

D (a) (4.19, 0.30)
(b) (4, 120)
(c) (-0.5, 0.87)
(d) (1.24, 3.80)
(e) NOTA

$x = r\cos\theta = 4\cos\frac{2\pi}{5} = 1.236067977$

$y = r\sin\theta = 4\sin\frac{2\pi}{5} = 3.804226065$

2. Determine $\sin\frac{8\pi}{9}$.

calculator must be in radian mode

A (a) 0.3420
(b) 0.0487
(c) -0.9397
(d) -0.3639
(e) NOTA

3. If $\tan\theta = -\frac{8}{15}$ where θ lies in QII, then determine $\sec\theta$.

C (a) $\frac{17}{15}$
(b) $\frac{16}{17}$
(c) $-\frac{17}{15}$
(d) $-\frac{16}{17}$
(e) NOTA

$\tan^2\theta + 1 = \sec^2\theta$

$(-\frac{8}{15})^2 + 1 = \sec^2\theta$

$\frac{64}{225} + \frac{225}{225} = \sec^2\theta$

$\frac{289}{225} = \sec^2\theta$

$\sqrt{\frac{289}{225}} = \sec^2\theta$

form

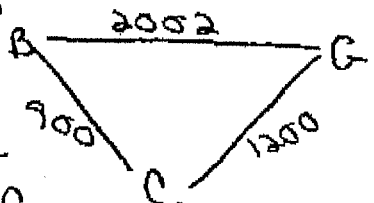
$\pm\frac{17}{15} = \sec\theta$

use neg. since QII.

4. A student attending the national Mu Alpha Theta convention determined that the cafeteria, boys' dormitory and girls' dormitory are noncollinear and form a triangle. The girls' dormitory is 900 feet from the cafeteria and the boys' dormitory is slightly farter at 1200 feet. If the distance between the dormitories is 2002 feet, then determine the angle formed at the cafeteria.

C (a) 125°
(b) 140°
(c) 144°
(d) 158°
(e) NOTA

B = Boy's dorm
G = Girl's dorm
C = Cafeteria



$2002^2 = 900^2 + 1200^2 - 2(900)(1200)\cos C$

$\frac{2002^2 - 900^2 - 1200^2}{-2(900)(1200)} = \cos C$

$-0.8139 = \cos C$

$\arccos -0.8139 = m\angle C$

$144.477 = m\angle C$

TRIGONOMETRY
TOPIC TEST

5. Find the exact value of $\cot(\arcsin\frac{5}{13})$

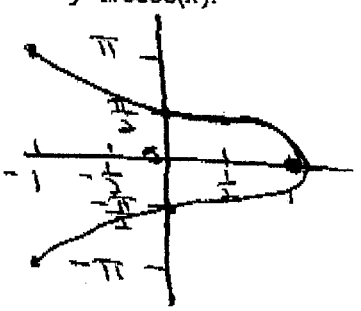
A (a) $\frac{12}{5}$
(b) $\frac{6}{12}$
(c) $\frac{12}{13}$
(d) $\frac{13}{12}$
(e) NOTA

$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{\frac{12}{13}}{\frac{5}{13}} = \frac{12}{5}$

$\sin^2\theta + \cos^2\theta = 1$
$(\frac{5}{13})^2 + \cos^2\theta = 1$
$\frac{25}{169} + \cos^2\theta = 1$
$\cos^2\theta = 1 - \frac{25}{169}$
$\cos^2\theta = \frac{144}{169}$
$\cos\theta = \pm\frac{12}{13}$
use pos

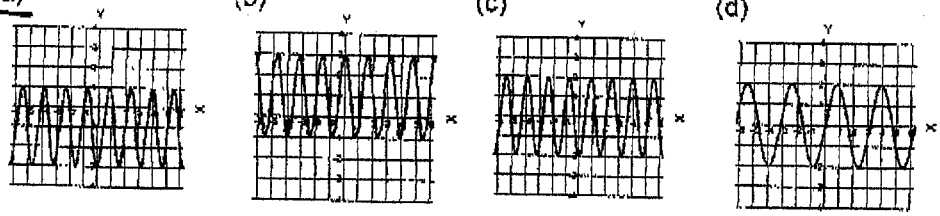
6. State the range for the function y=arccos(x).

C (a) $(0,2\pi)$
(b) $(-\frac{\pi}{2},\frac{\pi}{2})$
(c) $[0,\pi]$
(d) $[-\frac{\pi}{2},\frac{\pi}{2}]$
(e) NOTA



7. The graph of the function $y = -2\sin(4x+\frac{\pi}{2})-1$ appears as:

A (a) (b) (c) (d) (e) NOTA



amplitude $= |-2| = 2$
period $= \frac{2\pi}{4} = \frac{\pi}{2}$
phase shift $= \frac{-\frac{\pi}{2}}{4} = -\frac{\pi}{8}$
vertical shift $= -1$

8. State the amplitude of the function in #7.

B (a) 4 (b) 2 (c) 1 (d) 0.5 (e) NOTA

9. *Determine the exact value of* $\cos\frac{5\pi}{12}$ Use $\frac{2\pi}{6}$ and half-angle identity

D (a) $\frac{-\sqrt{2}-\sqrt{6}}{4}$ (b) $\frac{\sqrt{2}+\sqrt{6}}{4}$ (c) $\frac{\sqrt{2}-\sqrt{6}}{4}$ (d) $\frac{\sqrt{6}-\sqrt{2}}{4}$ (e) NOTA

$\frac{\sqrt{2-\sqrt{3}}}{2}$

or $\cos(\frac{\pi}{6}+\frac{\pi}{4})$

10. Ship A is 72 miles from a lighthouse on the shore. Its bearing from the lighthouse is N 15° E. Ship B is 81miles from the same lighthouse. Its bearing from the lighthouse is N 52° E. Find the distance between the two ships to the nearest mile.

D (a) 9 miles (b) 67 miles (c) 54 miles (d) 49 miles (e) NOTA

9. $\cos\frac{5\pi}{12} = \cos\frac{10\pi}{24}$

$$\cos\frac{10\pi}{24} = \sqrt{\frac{1+\cos\frac{5\pi}{6}}{2}}$$

$$= \sqrt{\frac{1+\left(\frac{-\sqrt{3}}{2}\right)}{2}} = \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{2\left(1-\frac{\sqrt{3}}{2}\right)}{2\cdot 2}}$$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

OR $\cos\frac{5\pi}{12} = \cos\left(\frac{9\pi}{12}+\frac{\pi}{4}\right)$

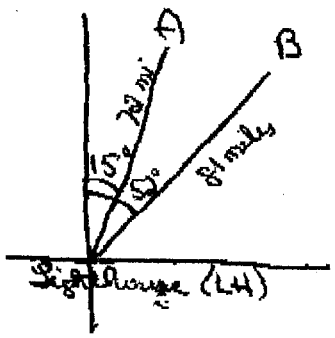
$$\cos\left(\frac{9\pi}{12}+\frac{\pi}{4}\right) = \cos\frac{9\pi}{12}\cdot\sin\frac{\pi}{4} - \sin\frac{9\pi}{12}\cdot\cos\frac{\pi}{4}$$

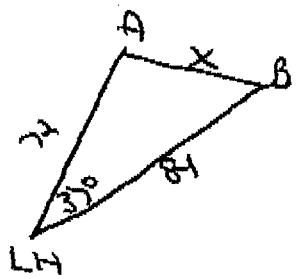
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$$

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

10.





$$x^2 = 72^2 + 81^2 - 2(72)(81)(\cos 37^\circ)$$

$$= 5184 + 6561 - (11664)(0.7986)$$

$$= 2430.1296$$

$$x = \sqrt{2430.1296}$$

$$= 49 \text{ miles}$$

TRIGONOMETRY
TOPIC TEST

11. Find all solutions over the interval $[0, 2\pi)$: $\cos x + \sin 2x = 0$.

C

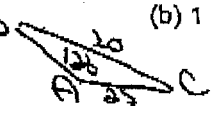
(a) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{6}, \frac{5\pi}{6}$ (b) $0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$ (c) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}$ (d) $0, \pi, \frac{\pi}{3}, \frac{2\pi}{3}$ (e) NOTA

$\cos x + 2\sin x \cos x = 0$
$\cos x(1 + 2\sin x) = 0$
$\cos x = 0$ or $1 + 2\sin x = 0$
$\cos x = 0$: $x = \frac{\pi}{2}, \frac{3\pi}{2}$
$1 + 2\sin x = 0$: $2\sin x = -1$, $\sin x = -\frac{1}{2}$, $x = \frac{7\pi}{6}, \frac{11\pi}{6}$

12. Determine the number of solutions in ΔABC if $a = 20, b = 25, \alpha = 126°$

A

(a) 0 (b) 1 (c) 2 (d) 3 (e) NOTA

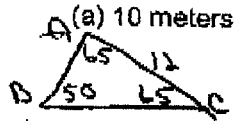


$\angle A$ is largest $\angle$ in Δ. However, the longest side is not opposite it.

13. Determine the length of $\overline{AB}$ in ΔABC if $\alpha = 65°, \beta = 50°, b = 12$ meters

B

(a) 10 meters (b) 14 meters (c) 2 meters (d) 8 meters (e) NOTA



$m\angle C = 180 - (65 + 50) = 65$

$\frac{\sin 65}{AB} = \frac{\sin 50}{12}$ $AB = \frac{12\sin 65}{\sin 50}$

14. A triangular lot has sides of length 400 feet, 175 feet, and 325 feet. Find the EXACT area of the lot.

A

(a) $3750\sqrt{55}$ sq.ft. (b) $625\sqrt{110}$ sq.ft. (c) $250\sqrt{165}$ sq.ft. (d) $25\sqrt{6}$ sq.ft. (e) NOTA

$s = \frac{400+175+325}{2} = 450$

Area $= \sqrt{450(450-400)(450-175)(450-325)}$
$= \sqrt{450(50)(275)(125)}$ =

15. Find the area of the triangle using the given information to two decimal places.
$\beta = 65°, a = 8.4m, c = 12.6m$

D

(a) 105 m^2 (b) 52.9 m^2 (c) 22.3 m^2 (d) 47.9 m^2 (e) NOTA

16. Convert $(-\sqrt{3}, 1)$ to EXACT polar coordinates. Express θ in radian measure.

B

(a) $\left(2, \frac{4\pi}{6}\right)$ (b) $\left(2, \frac{5\pi}{6}\right)$ (c) $\left(2, \frac{\pi}{3}\right)$ (d) $\left(2, \frac{2\pi}{3}\right)$ (e) NOTA

$r = \sqrt{x^2 + y^2} = \sqrt{(-\sqrt{3})^2 + 1^2}$ $r = \sqrt{3+1}$ $r = 2$

$\theta = \pi + \arctan -\frac{1}{\sqrt{3}}$
$= \pi + \left(-\frac{\pi}{6}\right)$

17. Solve for all values of $\cos 2x + 5\cos x = 6$ (k is an integer in answer choices)

C

(a) $0 + k\pi$ (b) $\frac{\pi}{2} + (2k+1)\pi$ (c) $0 + 2k\pi$ (d) $0 + \frac{k\pi}{2}$ (e) NOTA

18. Find the EXACT value for $\sin 2x$ if $\cos x = -\frac{9}{41}$ and x lies in QIII.

C

Pythagorean Triple

(a) $-\frac{720}{1681}$ (b) $\frac{360}{1681}$ (c) $\frac{720}{1681}$ (d) $-\frac{360}{1681}$ (e) NOTA

$\sin 2x = 2\sin x \cos x$
$= 2\left(-\frac{40}{41}\right)\left(-\frac{9}{41}\right)$

19. Find the EXACT value for $\tan 165°$.

B

(a) $2\sqrt{3}$ (b) $\sqrt{3} - 2$ (c) $2 - \sqrt{3}$ (d) $\frac{2\sqrt{3}}{3}$ (e) NOTA

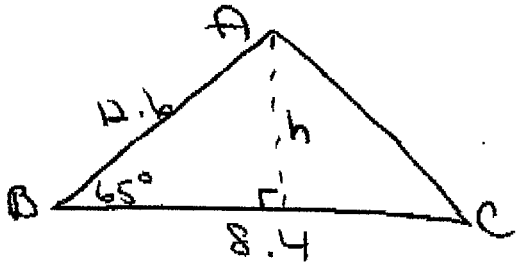
$\tan 165° = \tan(120 + 45)$ $\tan 120 = -\sqrt{3}$ $\tan 45 = 1$
$= \frac{\tan 120 + \tan 45}{1 - (\tan 120)(\tan 45)}$
$= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$
$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{(1 - \sqrt{3})}{1 - \sqrt{3}} = \frac{1 - 2\sqrt{3} + 3}{1 - 3} = \frac{4 - 2\sqrt{3}}{-2} = \sqrt{3} - 2$

14 $\text{Area} = \sqrt{450(50)(275)(125)}$

$= \sqrt{(225\cdot 2)(25\cdot 2)(25\cdot 11)(25\cdot 5)}$

$= \sqrt{225\cdot 25\cdot 25\cdot 25\cdot 4\cdot 11\cdot 5}$

15



$\text{Area} = \frac{1}{2}(8.4)[\sin 65(12.6)]$

17. $\cos 2x + 5\cos x = 6$

$2\cos^2 x - 1 + 5\cos x - 6 = 0$

$2\cos^2 x + 5\cos x - 7 = 0$

$(2\cos x + 7)(\cos x - 1) = 0$

$2\cos x + 7 = 0$

$\cos x = -\frac{7}{2}$

extraneous

$\cos x - 1 = 0$

$\cos x = 1$

$x = \cos^{-1} 1$

$x = 0°$

20. $\sin\left[\arccos\frac{1}{2} + \arcsin(-1)\right]$

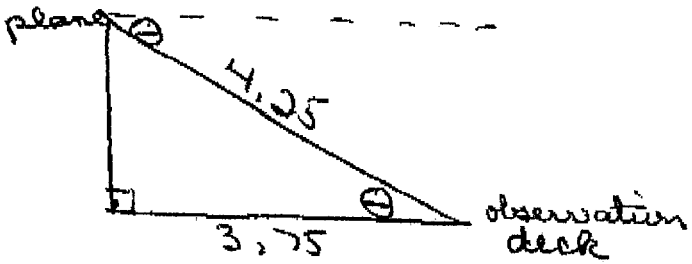
$= \sin(\arccos\frac{1}{2})\cdot\cos(\arcsin(-1)) + \cos(\arccos\frac{1}{2})\cdot\sin(\arcsin(-1))$

$= \left(\frac{\sqrt{3}}{2}\right)(0) + \left(\frac{1}{2}\right)(-1)$

$= 0 + \left(-\frac{1}{2}\right)$

$= -\frac{1}{2}$

21



$\cos\theta = \frac{3.75}{4.25}$

$\cos\theta = 0.8824$

$\theta = \cos^{-1}(0.8824)$

TRIGONOMETRY
TOPIC TEST

C 20. Find the EXACT value for $\sin\left[\arccos\frac{1}{2}+\arcsin(-1)\right]$

(a) -2 (b) -1 (c) -0.5 (d) 0 (e) NOTA

B 21. A person is standing at an observation deck at the Atlanta Hartsfield airport. He sees an airplane begin the final approach for landing. If the plane's horizontal distance from the observation deck at the time of the sighting is 3.75 km and the line-of-sight distance is 4.25 km, then determine the *angle of elevation* from the observation deck to the plane to the nearest degree.

(a) 41° (b) 28° (c) 62° (d) 49° (e) NOTA

E 22. Solve for the radian measure of x in $6\cos^2 x = -7\sin x - 14$ over the interval $0 \le x \le 2\pi$

(a) $0.41^R, 2.73^R$ (b) $1.16^R, 5.12^R$ (c) $2.42^R, 3.86^R$ (d) $5.43^R, 3.99^R$ (e) NOTA

$6(1-\sin^2 x) + 7\sin x + 14 = 0$

$6 - 6\sin^2 x + 7\sin x + 14 = 0$

$6\sin^2 x - 7\sin x - 20 = 0$

$(2\sin x - 5)(3\sin x + 4) = 0$

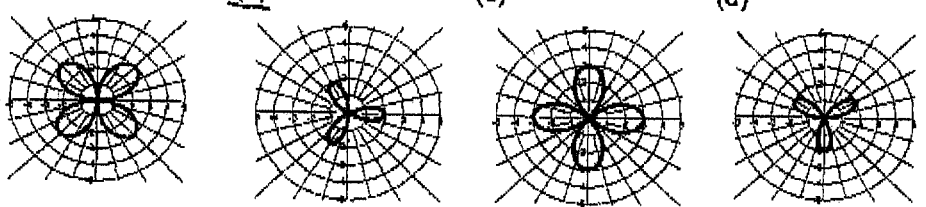
$\sin x = \frac{5}{2}$ extraneous

$\sin x = -\frac{4}{3}$

error on answer key

B 23. The sketch of $r = 2\cos 3\theta$ is:

(a) (b) (c) (d)



(e) NOTA

Use of calculator (graphing)

C 24. *Simplify* $\frac{\sin^2 x}{\cos^2 x} + \sin x \csc x$

(a) $\tan^2 x$ (b) $\cot^2 x$ (c) $\sec^2 x$ (d) $\csc^2 x$ (e) NOTA

$\tan^2 x + (\sin x)\left(\frac{1}{\sin x}\right)$

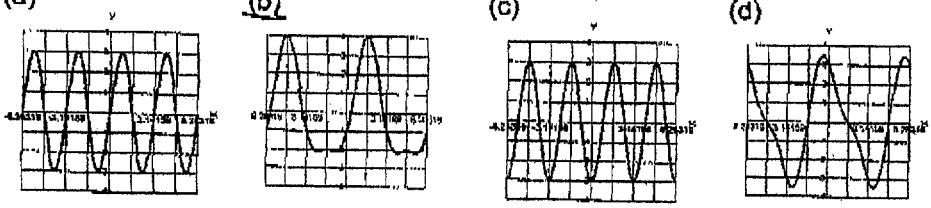
$\tan^2 x + 1$

A 25. Simplify completely: $\frac{(\sec y - \tan y)^2 + 1}{\sec y \csc y - \tan y \csc y}$

(a) $2\tan y$ (b) $2\cot y$ (c) $2\sec y$ (d) $2\csc y$ (e) NOTA

B 26. The sketch of $y = 3\sin x - \cos 2x, -2\pi \le x \le 2\pi$ is

(a) (b) (c) (d)



(e) NOTA

Use of calculator (graphing)

25 $\dfrac{(\sec y - \tan y)^2 + 1}{\sec y \cos y - \tan y \csc y}$

$\dfrac{\sec^2 y - 2\sec y \tan y + \tan^2 y + 1}{\left(\frac{1}{\cos y}\right)\left(\frac{1}{\sin y}\right) - \left(\frac{\sin y}{\cos y}\right)\left(\frac{1}{\sin y}\right)}$

$\dfrac{2\sec^2 y - 2\sec y \tan y}{\frac{1 - \sin y}{\sin y \cos y}}$

$\dfrac{2\sec y(\sec y - \tan y)}{\frac{1 - \sin y}{\sin y \cos y}}$

$2\sec y(\sec y - \tan y) \cdot \dfrac{\sin y \cos y}{1 - \sin y}$

$\dfrac{2\sin y\left(\frac{1}{\cos y} - \frac{\sin y}{\cos y}\right)}{1 - \sin y}$

$\dfrac{2\sin y\left(\frac{1}{\cos y}\right)(1 - \sin y)}{1 - \sin y}$

$2\tan y$

TRIGONOMETRY
TOPIC TEST

27. Find the EXACT value for $\cos(x+y)$ if $\sin x = \frac{13}{85}$ and $\cos y = -\frac{7}{25}$. (x lies in QII and y lies in QIII)

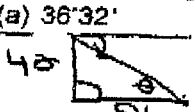
D

(a) $\frac{276}{2125}$ (b) $\frac{399}{425}$ (c) $\frac{2016}{2125}$ (d) $\frac{36}{85}$ (e) NOTA

28. A company safety committee has recommended that a floodlight be mounted in a parking lot so as to illuminate the employee exit for nighttime usage. Find the angle of depression, to the nearest degree, of the light if it is mounted 40 feet above the ground and 54 feet from the exit.

A

(a) 36°32' (b) 53°28' (c) 36°53' (d) 53°47' (e) NOTA



$\tan\theta = \frac{40}{54}$

29. A circle with its center at the origin in a rectangular coordinate system passes through the point (4,5). What is the length of the arc on the circle in the first quadrant between the positive horizontal axis and the point (4,5)? Compute your answer ~~in radian measure to two decimal places.~~ to the nearest unit.

D

(a) ~~2.07~~ units (b) ~~6.40~~ units (c) ~~2,051.~~83 units (d) ~~5.09~~ units (e) NOTA

3 8 2052 6

30. Find the EXACT value of $\tan 2x$, given $\cos x = \frac{9}{41}, \frac{3\pi}{2} \le x \le 2\pi$ $\sin x = -\frac{40}{41}$

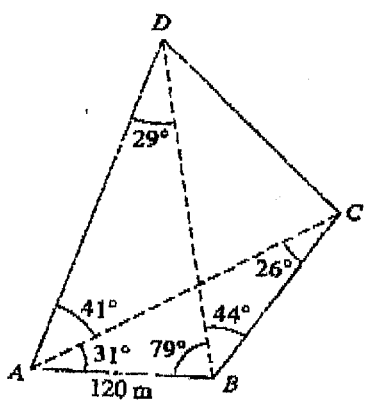
C

(a) $-\frac{720}{1681}$ (b) $\frac{3280}{81}$ (c) $\frac{720}{1519}$ (d) $\frac{369}{800}$ (e) NOTA

$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$ $\tan x = -\frac{40}{9}$

TIE BREAKER: (Show all work)

A plot of land has been surveyed with the resulting information as shown in the diagram. Find the length of $\overline{CD}$ to the nearest meter.



$\triangle ABD$, Find AD using Law of Sines

$\frac{\sin 79}{AD} = \frac{\sin 29}{120}$

$AD = \frac{120 \cdot \sin 79}{\sin 29}$ $AD = 243$ m.

$\triangle ABC$, Find AC using Law of Sines

$\frac{\sin 26}{120} = \frac{\sin 123}{AC}$

$AC = \frac{120 \cdot \sin 123}{\sin 26}$ $AC = 229$ m

$\triangle ACD$ Find CD using Law of Cosines

$CD = \sqrt{AD^2 + AC^2 - 2(AD)(AC)(\cos 41)}$

$= \sqrt{243^2 + 229^2 - 2(243)(229)(\cos 41)}$

$CD = 166$ m

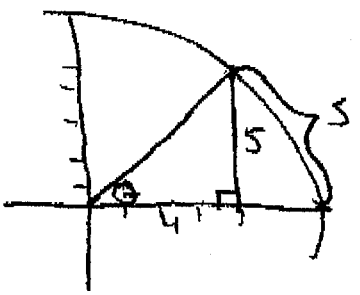
27. $\cos(x+y) = \cos x \cdot \cos y - \sin x \sin y$

$= \left(-\frac{84}{85}\right)\left(-\frac{7}{25}\right) - \left(\frac{13}{85}\right)\left(-\frac{24}{25}\right)$

$= \frac{588 - (-312)}{2125}$

$= \frac{900}{2125} \div \frac{25}{25} = \frac{36}{85}$

29.



s = arc length

$\tan\theta = \frac{5}{4}$

$\tan\theta = 1.25$

$\theta = \tan^{-1} 1.25$

$\theta = .90^R$

$r = \sqrt{4^2 + 5^2}$

$= \sqrt{41}$

$\frac{\theta}{2\pi} = \frac{s}{2\pi r}$

$\theta = \frac{s}{r}$

$\theta \cdot r = s$

$(0.90)(6.40) = 5$

$5.69 = 5$

2002 MU ALPHA THETA NATIONAL CONVENTION
ALPHA DIVISION TRIGONOMETRY TOPIC TEST
ANSWER KEY

ITEM	ANSWER
1.	D
2.	A
3.	C
4.	C
5.	A
6.	C
7.	A
8.	B
9.	D
10.	D
11.	C
12.	A
13.	B
14.	A
15.	D

ITEM	ANSWER
16.	B
17.	C
18.	C
19.	B
20.	C
21.	B
* 22.	D
23.	B
24.	C
25.	A
26.	B
27.	D
28.	A
29.	D
30.	C

TIE-BREAKER 166 meters

* Check #22 Answer choice E is correct (?)