2002 National Mu Alpha Theta Convention
Mu Division----Analytic Geometry Test

1. Which nondegenerate conic is represented by the following equation?
   $3x^2 - 2\sqrt{3}\ xy + y^2 + 2x + 2\sqrt{3}\ y = 0$
   A. circle   B. parabola   C. hyperbola   D. ellipse   E. none of the above

2. An ellipse whose center is at (-2, 3) has a major axis of length 11 inches and a minor axis of length 4 inches. What is the area (in square inches) enclosed by the ellipse?
   A. 44π B. $22\ \pi$ C. 11π D. 4π E. NOTA

3. Find the area of a triangle whose vertices are (-2, 7), (6,8), and (1, -1).

   A. 18.9   B. 25.8   C. 40.2   D. 33.5   E. NOTA

4. Find the perimeter (in inches) of a regular hexagon circumscribed about a circle of radius 10 inches.
   A. $40\sqrt{3}$ B. 60   C. $60\sqrt{3}$ D. $150\sqrt{3}$ E. NOTA

5. Find the distance between the point (4,1) and the line whose equation is $y = 2x + 1$.
   A. $\dfrac{6\sqrt{5}}{5}$ B. $\dfrac{7\sqrt{5}}{5}$ C. $\dfrac{8}{3}$ D. 2   E. NOTA

6. Find all vertical asymptotes for the following:
   $$y = \frac{4x^3 + 4x^2 - 3x - 3}{x^2 + 3x + 2}$$

   A. x = -1   B. x = -2   C. x = -1, x = -2
   D. There are no vertical asymptotes   E. NOTA

7. The graph of which of the following functions does not cross the x-axis.
   A. $y = -x^3 + x^2 + 1$ B. $f(x) = x^6 + x^4 + 2$
   C. $h(x) = x^4 - x^3 - 1$ D. $g(x) = x^3 + 3x^2 - 2x - 1$
   E. NOTA

8. Points A, B, C, and D lie on the graph of $y^2 + 2xy + x^2 + 3x + 4y + 2 = 0$, and each has abscissa (x-coordinate) of 1 or −1. Of these points, let A and C be the farthest apart. If $AC = \sqrt{p + q\sqrt{3}}$, where p and q are integers, find p-q.

   A. 22   B. 10   C. 2   D. 7   E. NOTA

9. If the sides of a right triangle are *a, a-d, a+d,* where *a* and *d* are positive real numbers, then $\left(\dfrac{a}{d}\right)^2 = $ _____.

   A. 9   B. 16   C. $\dfrac{1}{9}$ D. $\dfrac{1}{16}$ E. NOTA

10. Triangle ABC has sides of lengths 20, 21, and 29 units. The diameter of the circle inscribed in the triangle is _____.

   A. 12   B. 7   C. 11   D. 6   E. NOTA

11. The length of a rectangle is increased by 15% and the width is decreased by 20%. The percentage change in the area of the rectangle is ___.
   A. 3   B. 2   C. 5   D. 35   E. NOTA

12. Find the area enclosed by the system of inequalities:
$$\begin{cases} 2x + y \le 4 \\ x - y \ge 5 \\ x \ge 0 \\ y \ge -10 \end{cases}$$

   A. 35.5   B. $33\dfrac{5}{8}$ C. $34\dfrac{3}{8}$ D. 31.75   E. NOTA

13. A sphere has an area in square units equal to the number of cubic units in its volume. Find the ratio of the circumference of a great circle of the sphere to the area of a great circle of the sphere.

   A. $\dfrac{3}{4}$ B. $\dfrac{2}{3}$ C. $\dfrac{3}{5}$ D. $\dfrac{1}{\pi}$ E. NOTA

14. Find the length of the latus rectum of the parabola $x^2 - 4x - 12y - 32 = 0$.
   A. 3   B. 6   C. 9   D. 12   E. NOTA

15. W hat are the coordinates of the focus of the parabola whose equation is $y^2 - 6y + 8x + 25 = 0$?

   A. (-2, 3)   B. (-2, 5)   C. (-2, 0)   D. (-2, 3)   E. NOTA

16. An arch is in the form of a semi-ellipse. The arch is 52 meters wide at the base and has a height of 20 meters at its highest point. How wide, in meters, is the arch at a height of 10 meters above the base?

A. 40 B. $26\sqrt{3}$ C. 26 D. $13\sqrt{3}$ E. NOTA

17. What is the volume, in cubic units, of a tetrahedron whose vertices are:
(3, 1, 0) (1, 4, 0) (5, 5, 0) (3, 1, 6)

A. 14 B. 24 C. 42 D. 84 E. NOTA

18. In triangle ABC, angle C has measure 120 degrees, side BC has length 10 units, side AC has length 12. Find the length of side AB.

A. $2\sqrt{91}$ B. $\sqrt{244-120\sqrt{3}}$ C. $2\sqrt{41}$ D. $\sqrt{244+120\sqrt{3}}$
E. NOTA

19. The graph of $r = \sin\theta$ is a:
A. circle B. parabola C. limacon D. cardioid E. NOTA

20. What is the tangent of the acute angles formed when the two lines whose equations are 3x - 4y = 7   and 2x + 3y = 8 intersect?
A. $-17/6$ B. 17/6 C. 1/6 D. 17/18 E. NOTA

21. W hat is the area, in square units, of the region enclosed by the graph of the parametric equations $x = 8\cos\theta$ and $y = 6\sin\theta$?
A. 96π B. 48π C. 12π D. 48. E. NOTA

22. What is the equation of the line tangent to the circle $x^2 + y^2 + 2x - 4y - 5 = 0$ at the point P(2,1)?

A. 3x + y = 7 B. x − 3y = -1 C. 3x − y = 5 D. x + 3y = 5
E. NOTA

23. Suppose the equation for an ellipse is given by $\dfrac{(x-3)^2}{16} + \dfrac{(y+2)^2}{b^2} = 1$.

Find b so that the eccentricity is 0.75.
A. 3 B. 6 C. $\sqrt{7}$ D. 5 E. NOTA

24. Which of the following is an asymptote to the hyperbola $\dfrac{(y+2)^2}{9}-\dfrac{(x-3)^2}{16}=1$

A. $y=\dfrac{3}{4}x-3$ B. $y=\dfrac{-3}{4}x-\dfrac{17}{4}$ C. $y=\dfrac{3}{4}x-\dfrac{17}{4}$ D. $y=\dfrac{-3}{4}x-\dfrac{1}{4}$

E. NOTA

25. Determine an angle of rotation necessary to eliminate the xy term:
$x^2+xy+2y^2+5x-3y-56=0$

A. $\dfrac{3}{8}\pi$ B. $\dfrac{3}{4}\pi$ C. $\dfrac{\pi}{8}$ D. $\dfrac{\pi}{4}$ E. NOTA

26. Calculate the area inside the cardioid $r=1+\cos\theta$

A. 3π B. $\dfrac{3}{4}\pi$ C. $\dfrac{3}{2}\pi$ D. $\dfrac{\pi}{2}$ E. NOTA

27. Find an equation for the hyperbola with eccentricity $\dfrac{3}{2}$ and directrix $x=2$.

A. $r=\dfrac{6}{2+3\cos\theta}$ B. $r=\dfrac{2}{2+3\cos\theta}$ C. $r=\dfrac{6}{1+2\cos\theta}$

D. $r=\dfrac{3}{1+3\cos\theta}$ E. NOTA

28. Find the length of the arc of the curve $x=\dfrac{8}{3}t^{\frac{3}{2}}$, $y=2t-t^2$ between $t=1$ and $t=3$.

A. 4   B. 16   C. 12   D. 8   E. NOTA

29. Find the area of a parallelogram having $v_1=-i+2j+2k$ and $v_2=3i-2j+k$ as adjacent sides.

A. 3   B. 10   C. $\sqrt{101}$ D. $\sqrt{69}$ E. NOTA

30. The region in the first quadrant bounded by $y=\sin x^2$, the coordinate axes, and the line $x=b$, $b>0$, is revolved about the y-axis. Find b such that the volume of the solid generated is $\dfrac{\pi}{2}$ cubic units.

A. $\dfrac{\pi}{2}$ B. $\sqrt{\dfrac{1}{3}}$ C. $-\sqrt{\dfrac{\pi}{3}}$ D. $\sqrt{\dfrac{\pi}{3}}$ E. NOTA

Mu Division—ANALYTIC GEOMETRY TOPIC TEST
Answer Key

1. B
2. C
3. D
4. A
5. E
6. B
7. B
8. B
9. B
10. A
11. E
12. A
13. B
14. D
15. E
16. B
17. A
18. A
19. A
20. B
21. B
22. C
23. C
24. C
25. A
26. C
27. A
28. C
29. C
30. D