DERIVATIVES AND APPLICATIONS

PROBLEM #1

Which of the following is not true concerning a function f?

A. If f is differentiable at x = c, then f is continuous at x = c.

B. If f is continuous at c, then f is differentiable at c.

C. If f is constant, then the derivative of f is zero.

d. $f'(x)$, dy/dx, y', $\frac{d}{dx}[f(x)]$, $D_x[Y]$ are all acceptable notations for the derivative.

e. NOTA

Answer B

A function with a sharp corner at point c is not ~~continuous~~ differentiable at c.

Derivatives and Applications

2. Evaluate the derivative of $\tan x \cot x$ at $(1,1)$.

A. π

B. $\pi/2$

C. L

D. 0

E. NOTA

Solution ANSWED D

$f(x) = \tan x \cot x = 1$

$f'(x) = 0$

$f'(1) = 0$

derivatives
and applications

#3 Which of the following statements is/are true?

I. If $f'(x) = g'(x)$ then $f(x) = g(x)$
II. If $f(x) = g(x) + C$ then $f'(x) = g'(x)$
III. If $Z = \pi^2$, then $dZ/dx = 2\pi$
IV. If $y = x/\pi$ then $dy/dx = 1/\pi^2$

A. I, II

B. II

C. I, II, III, IV

D. III, IV

E. NOTA

Solution: B

I. False $f'(x)$ AND $g'(x)$ CAN be = every where but $f(x)$ AND $g(x)$ NOT be equal $\Rightarrow$ They could vary by a CONSTANT.

II. TRUE In this case $f'(x) = g'(x)$ Since the derivative of a CONSTANT IS Zero

III. π^2 IS a CONSTANT, its derivative is Zero

IV. If $y = \frac{x}{\pi} = \frac{1}{\pi} x$ $\quad$ $\frac{1}{\pi}$ is a Constant

hence the derivative is $\frac{1}{\pi}$

$$\frac{dy}{dx} = \frac{d}{dx} x = 1$$

derivatives
and applications

4 given $f(x) = 3x^3 - x^2 + 2$, which of the following should be evaluated to find the slope of a tangent line to this function at any x.

A. $\lim_{\Delta x \to 0} \dfrac{3(x+\Delta x)^3 - (x+\Delta x)^2 + 2 - 3x^3 - x^2 + 2}{x + \Delta x - x}$

B. $\lim_{\Delta x \to 0} \dfrac{3(x+\Delta x)^3 - (x+\Delta x)^2 + 2 - (3x^3 - x^2 + 2)}{x + \Delta x}$

C. $\lim_{\Delta x \to 0} \dfrac{3(x+\Delta x)^3 - (x+\Delta x)^2 + 2 - (3x^2 - x^2 + 2)}{\Delta x}$

D. $\lim_{\Delta x \to 0} \dfrac{3x^3 - x^2 + 2 - [3(x+\Delta x)^3 - (x+\Delta x)^2 + 2]}{\Delta x}$

E. NOTA

Solution: C

The definition of the derivative $\Rightarrow$ Slope of the tangent line at any point in the domain of a function is

$$m = f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Hence

$$f'(x) = \lim_{\Delta x \to 0} \frac{3(x+\Delta x)^3 - (x+\Delta x)^2 + 2 - (3x^2 - x^2 + 2)}{\Delta x}$$

DERIVATIVES AND APPLICATIONS

#5

$f(x) = \sin x \cos x$, find $f'(x)$ at $x = \pi/6$ $\nearrow \pi/6$

A $1 + 2\sin^2 x$

B $3/2$

C $1/2$

D $1 - 2\sin^2 x$

E NOTA

Solution C

$\sin x(-\sin x) + \cos x \cos x$

$f'(x) = -\sin^2 x + \cos^2 x$

$f'(x) = -\sin^2 x + 1 - \sin^2 x = 1 - 2\sin^2 x$

$f'(\pi/6) = 1 - 2(1/2)^2 = 1/2$

Derivatives and Applications

#6 $f(x) = \dfrac{x}{\sqrt{x^2+1}}$

find $f'(x)$.

A. $1/(x^2+1)^{3/2}$

B. $1/(x^2+1)$

C. $\left[(x^2+1)^{1/2} - x/2\,(x^2+1)^{-1/2}\right] / (x^2+1)$

D. $(1-2x^2)/(x^2+1)^{3/2}$

E. NOTA

Solution: A

$$f'(x) = \frac{(x^2+1)^{1/2} - x\ \frac{1}{2}(x^2+1)^{-1/2}\,2x}{x^2+1}$$

$$= \frac{(x^2+1)^{1/2} - \frac{x^2}{(x^2+1)^{1/2}}}{x^2+1}$$

$$\frac{x^2+1-x^2}{(x^2+1)^{3/2}} = \frac{1}{(x^2+1)^{3/2}}$$

Derivatives
and Applications

#7 Given: $f(x) = [g(x)]^3 h(x)$, $g(5) = -2$, $g'(5) = 8$
$h(5) = 1/2$, $h'(5) = 4$

find $f'(5)$

A. 16

B. 4

C. −16

D. 0

E. NOTA

Solution: A

$$f(x) = [g(x)]^3 \, h(x)$$

$$f'(x) = [g(x)]^3 \, h'(x) + h(x) \; 3[g(x)]^2 \, g'(x)$$

$$f'(5) = (-2)^3 (4) + 1/2 \; 3[-2]^2 \, 8$$

$$= -32 + 48 = 16$$

Derivatives
AND
APPLICATIONS

#8. Find d^2y/dx^2 in terms of x and y for

$x^2 - y^2 = 16$

A x/y

B x^2/y^2

C $(y^2 - x^2)/y$

D $16/y^3$

E NOTA

Solution: (E) NOTA

$x^2 - y^2 = 16$

differentiate implicitly:

$$2x - 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = x/y$$

$$\frac{d^2y}{dx^2} = \frac{y - x(1)\,dy/dx}{y^2} = \frac{y - x\frac{x}{y}}{y^2} = \frac{y^2 - x^2}{y^3} = \frac{-16}{y^3}$$

Derivatives
AND
Applications

#9

A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is PULLED AWAY from the house at 3 feet ~~/s~~ per second. How fast is the top of the ladder moving down the wall when the ~~base~~ bottom is 24 ft from the base of the wall?

A. 7 feet Per Second

B. -144 feet Per Second

C. $-72/7$ feet per second

D. -3 feet per second

E. NOTA

Solution (C)

$$x^2 + y^2 = 625$$

$$2x \frac{dx}{dt} + 2y \frac{dy}{dt} = 0$$

$$2(24\,ft)(3)\ ft/sec = -2(7)\,ft \frac{dy}{dt}$$

$$-72/7\ ft/sec = dy/dt$$

Derivatives and Applications

\# 10. $f(t) = \sin^3 4t$ find $\frac{d}{dt} f(t)$

A. $12t^2 \sin^3(4t^2) \cos(4t^2)$

B. $3 \sin^2(4t^2) \cos(4t^2)$

C. $3 \sin^2(4t^2)$

D. $24t \sin^2(4t^2) \cos(4t^2)$

E. NOTA

Solution: D

$$f'(t) = 3 \sin^2 4t^2 \cos 4t^2 \, (8t) =$$
$$24t \sin^2 4t^2 \cos 4t^2$$

DERIVATIVES AND APPLICATIONS

#11.

The temperature T of food placed in a freezer is given by $T = \frac{700}{t^2+4t+10}$ where t is time in hours. At how many degrees per hour is T changing when t = 2 hours?

A. $-1400/121$ degrees/hour

B. $1400/121$ degrees/hour

C. $2800/11$ degrees/hour

D. 1800 degrees/hour

E. NOTA

SOLUTION (A)

$$\frac{d}{dt}T = \frac{-700(2t+4)}{(t^2+4t+10)^2} \quad @ \ t=2$$

$$\longrightarrow \frac{-700((2\cdot2)+4)}{(2^2+8+10)^2}$$

$$\frac{-\cancel{700}^{350}\cdot\cancel{8}^{4}}{\cancel{22}_{11}\cdot\cancel{22}_{11}} = \frac{-1400}{121} \text{ degrees/hour}$$

Derivatives and Applications

#12

Find y'' if $y = 3x^4 - 2x^2 + 10x - 1$

A. $12x^3 - 4x + 10$

B. $36x^2 - 4x + 10$

C. $36x^2 - 4$

D. $12x^2 - 4$

E. NOTA

Solution: (C)

$y = 3x^4 - 2x^2 + 10x - 1$

$y' = 12x^3 - 4x + 10$

$y'' = 36x^2 - 4$

Derivatives and APPLICATIONS

#13

Find any critical numbers of the function

$f(x) = x^2(x-3)$

A. $x=0$, $x=2$

B. $x=0$, $x=-2$

C. $x=2$, $x=-2$

D. $x=-3$ $x=2$

E. NOTA

Solution (A)

$f(x) = x^3 - 3x^2$

$f'(x) = 3x^2 - 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$\left.\begin{array}{l} x=0 \\ x=2 \end{array}\right\}$ critical Numbers

#14

Derivatives and applications

Given the function $f(x) = \dfrac{x^2 - 2x - 3}{x+2}$, FIND that value on the interval $[-1, 3]$ for which Rolle's theorem is satisfied.

A. $-2 \pm \sqrt{5}$

B. $-2 - \sqrt{5}$

C. $-2 + \sqrt{5}$

D. $2, \frac{3}{2}$

E. NOTA

Solution: (C)

$$f(-1) = \frac{(-1)^2 - 2(-1) - 3}{-1+2} = \frac{0}{2} = 0$$

$$f(3) = \frac{9 - 6 - 3}{3+2} = \frac{0}{5} = 0$$

$$f' = \frac{(2x-2)(x+2) - (x^2-2x-3)\,1}{(x+2)^2} = 0$$

$\cancel{(-2+\sqrt{5})^2 - 2(-2+\sqrt{5}) - 3}$

$\cancel{4 - 4\sqrt{5} + 5 + 4 - 2\sqrt{5} - 3}$

$$2x^2 + 2x - 4 - x^2 + 2x + 3 = 0$$

$$x^2 + 4x - 1 = 0$$

$\cancel{((-2 - \sqrt{5})^2 - 2(-2-\sqrt{5}) - 3)}$

$$x = \frac{-4 \pm \sqrt{16 - 4(-1)}}{2}$$

$\cancel{4 + 4\sqrt{5} + 5 + 4 + 2\sqrt{5} - 3}$

$$x = -2 \pm \sqrt{5}$$

$-2-\sqrt{5}$ NOT on the interval

$(-2-\sqrt{5})^2 + 4(-2-\sqrt{5}) - 1$

$\cancel{4 + 4\sqrt{5} + 5 - 8 - 4\sqrt{5} - 1}$

Derivative and applications

#15

Given $f(x) = \sin x$ find that value on the interval $[0, \pi]$ of this function for which the mean value theorem is satisfied.

A. $\pi/6$

B. $\pi/3$

C. $\pi/2$

D. $2\pi/3$

E. NOTA

Solution: (C)

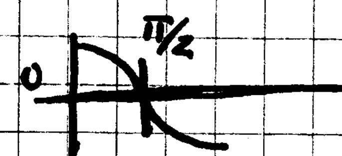
The mean value says

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{\sin \pi - \sin(0)}{\pi - 0} = \frac{0}{\pi} = 0$$

$f'(c) = 0$

$f' = \cos(c) = 0$ $\quad c = \cos^{-1} 0$

$c = \pi/2$



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Derivative and applications

Find the open intervals on which the function is increasing or decreasing and critical numbers if any.

$$f(x) = x^{1/3} + 1$$

A. Decreasing $(-\infty, \infty)$, critical number 0

B. Decreasing $(-\infty, 0)$, increasing $(0, \infty)$, critical number 1

C. Decreasing $(0, \infty)$, increasing $(-\infty, 0)$, critical number 1

D. Increasing $(-\infty, \infty)$ critical number 0

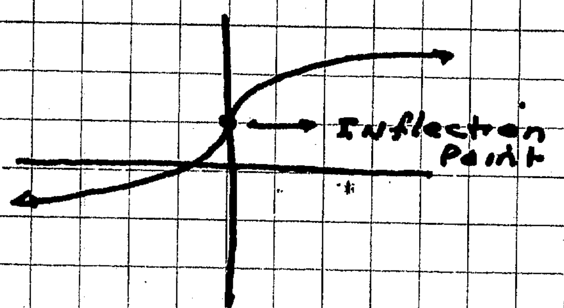
E. NOTA

Solution: (D)

$$f'(x) = \tfrac{1}{3}x^{-2/3} = 0$$

$$\tfrac{1}{3}x = 0$$

$$x = 0$$



$$f(0) = 1$$

$$f'(-1) = \tfrac{1}{3}(-1)^{-2/3}$$

~~-1/2 Decreasing~~

$$= \tfrac{1}{3}(-1)^{2/3} = \tfrac{1}{3} +$$

increasing

#17

Derivatives and applications

Locate all relative extrema of the function.

$$f(x) = \frac{x^2 - 2x + 1}{x + 1}$$

A. Maximum at $(1, 0)$
   Minimum at $(-3, -8)$

B. Maximum at $(-3, 8)$
   Minimum at $(-1, 0)$

C. Maximum at $(-3, -8)$
   Minimum at $(1, 0)$

D. no relative extrema exists for this function

E. NOTA

Solution: C

$$f'(x) = \frac{(x+1)(2x-2) - (x^2-2x+1)1}{(x+1)^2} = 0$$

$$2x^2 - 2 - x^2 + 2x - 1 = 0$$

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x = 1$$

$$x = -3$$

$$f''(x) = \frac{(x+1)^2(2x+2) - (x^2+2x-3)\,2(x+1)}{(x+1)^4}$$

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$$f''(1) = \frac{4(4) - 4}{(2)^4} = + \quad \therefore \text{ at } x = 1 \text{ we have minimum}$$

$$f''(-3) = \frac{4(-4) - (9-6-3)(-4)}{(-3+1)^4} = - \quad \text{at } x = -3 \text{ maximum}$$

Derivatives and applications

#18

Which of the following Statements is true If f is differentiable on an open interval I.

A. the graph of f is concave upward on I if f' is positive on the interval and concave donward on I if f' is negative on the interval.

B. the graph of f is concave upward on I if f'' is increasing on the interval and concave downward on I if f'' is increasing on the interval.

C. the graph of f is concave upward if f' is is zero at some point on the interval.

d. the graph of f is concave downward if f' is zero at some point on the interval.

E. NOTA

Solution E.

Let f be differentiable on an open interval I. The graph of f is concave upward on I if f' is increasing on the interval and concave downward if f' is decreasing on the interval.

#19

Determine the open intervals on which the graph of $f(x) = 4(x^2+3)^{-1}$ is concave upward or downward.

A concave up on $(-\infty, -1)$ and $(1, \infty)$
concave down on $(-1, 1)$

B concave up on $(-1, 1)$
concave down on $(-\infty, -1)$ and $(1, \infty)$

C concave up on $(-\infty, -4)$ $(4, \infty)$
concave down on $(-4, 4)$

D concave up on the entire real number line

E NOTA

Solution: (A)

f is continuous on the entire real number line.

$$f(x) = \frac{4}{(x^2+3)^{+1}}$$

$$f'(x) = -4(x^2+3)^{-2}\, 2x = \frac{-8x}{(x^2+3)^2}$$

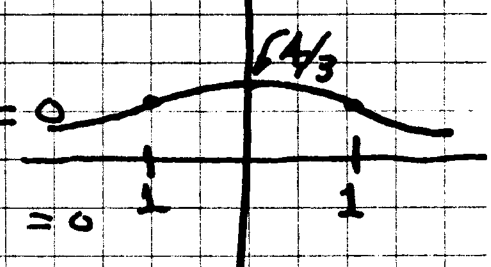
$$f''(x) = \frac{(x^2+3)^{\cancel{2}\,1}(-8) - (-8x)\,2\cancel{(x^2+3)}\,2x}{(x^2+3)^{\cancel{4}\,3}}$$

$$= \frac{(x^2+3)(-8) + 32x^2}{\cancel{(x^2+3)^3}} = 0$$

$$-8x^2 - 24 + 32x^2 = 0$$

$$24x^2 = 24$$

$x = \pm 1$ (Inflection points)



TEST $(-\infty, -1)$
TRY (-2)

$$\frac{24(-2)^2 - 24}{((-2)^2+3)^3} = +$$

slope increasing concave up

$(-1, 1)$
TRY (0)

$$\frac{24(0) - 24}{3} = -$$

slope decreasing concave down

Derivatives and applications

#20

Find a, b, c, d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the following conditions:

I. a relative maximum at (3,3)
II. a relative minimum at (5,1)
III. an inflection point at (4,2)

A. $f(x) = 6x^3 - \frac{1}{2}x^2 + \frac{45}{2}x - 24$

B. $f(x) = 4x^3 - 6x^2 + \frac{45}{2}x + 24$

C. $f(x) = -\frac{1}{2}x^3 + 6x^2 - \frac{45}{2}x + 24$

D. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$

E. NOTA

Solution: (D)

$f(x) = ax^3 + bx^2 + cx + d \Rightarrow$ generates

condition I: $3 = 27a + 9b + 3c + d$

condition II: $1 = 125a + 25b + 5c + d$

condition III: $2 = 64a + 16b + 4c + d$

$f'(x) = 3ax^2 + 2bx + c \Rightarrow$

condition I: $27a + 6b + c = 0$

condition II: $75a + 10b + c = 0$

$f''(x) = 6a + 2b$

$$\begin{aligned}
1 &= 125a + 25b + 5c + d\\
-(3 &= 27a + 9b + 3c + d)\\
\hline
-2 &= 98a + 16b + 2c\\
+\ -2\cdot(0 &= 75a + 10b + c)\\
\hline
-2 &= -52a - 4b
\end{aligned}$$

$$\begin{aligned}
1 &= 125a + 25b + 5c + d\\
-(2 &= 64a + 16b + 4c + d)\\
\hline
-1 &= 61a + 9b + c\\
0 &= 27a + 6b + c\\
\hline
-1 &= 34a + 3b
\end{aligned}$$

$\times 3 \Rightarrow -6 = -156a - 12b$

$\times 4 \Rightarrow -4 = 136a + 12b$

$-10 = -20$

$a = \frac{1}{2}$

$-1 = 34(\frac{1}{2}) + 3b$

$-18 = 3b$

$-6 = b$

$c = -27a - 6b$
$= -13\frac{1}{2} + 36 = 22\frac{1}{2}$

$d = 2 - 64a - 16b - 4c$
$= 2 - 32 + 96 - 90$

$d = -24$

#21

Derivatives and applications

A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

A 3 × 3 × 6 inches

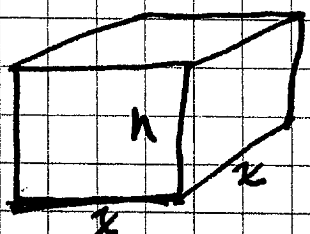
B 36 × 36 × 1/12 inches

C 6 × 6 × 3 inches

D 4 × 4 × 6 3/4 inches

E. NOTA inches

Solution: (C)



$V = x^2 h \qquad S = x^2 + 4xh = 108$

$h = \frac{108 - x^2}{4x}$

$V = x^2\left(\frac{108 - x^2}{4x}\right)$

$\frac{dV}{dx} = \frac{d}{dx}\left(27x - \frac{1}{4}x^3\right)$

$= 27 - \frac{3}{4}x^2 = 0$

$x^2 = 27 \cdot \frac{4}{3} = 36$

$x = \pm 6$

$h = \frac{108 - 36}{24} = \frac{72}{24} = 3$

Derivatives and Applications

#22

Which points on the graph of $y = 9 - x^2$ are closest to the point $(0, 3)$?

A. $\pm\sqrt{11}/2,\ 6\frac{1}{4}$

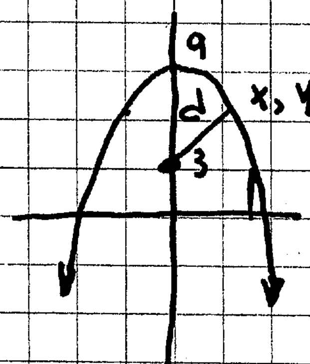
B. $\pm\sqrt{22}/2,\ 7/2$

C. $\pm\sqrt{22}/4,\ 7\frac{5}{8}$

D. $\pm 3,\ 0$

E. NOTA

Solution: E



$$d = \sqrt{(x-0)^2 + (y-3)^2}$$

$$d = \sqrt{x^2 + y^2 - 6y + 9}$$

$$= \sqrt{x^2 + (9-x^2)^2 - 6(9-x^2) + 9}$$

$$= \sqrt{x^2 + 81 - 18x^2 + x^4 - 54 + 6x^2 + 9}$$

$$y = 9 - \frac{11}{4} = 25/4$$

$$d = \sqrt{x^4 - 11x^2 + 36}$$

$$\frac{dd}{dx} = \frac{1}{2}\left(x^4 - 11x^2 + 36\right)^{-1/2}\left(4x^3 - 22x\right) = 0$$

$$x(4x^2 - 22) = 0$$

$\cancel{x=0}$ $\quad x = \pm\sqrt{22/4} = \sqrt{\frac{11}{2}}$

$$y = 9 - \left(\sqrt{\tfrac{11}{2}}\right)^2 = 9 - 11/2 = 3\frac{1}{2}$$

#23

Differentiate $f(x) = x^2 \ln x$

A. $2x + \ln x$

B. $x(1 + \ln x^2)$

C. 2

D. $x^2 \ln 1/x + 2 \ln x$

E. NOTA

Solution: (B)

$$\cancel{x}^2 \cdot \frac{1}{\cancel{x}} + 2x \ln x$$

$$x + 2x \ln x$$

$$x(1 + \ln x^2)$$

derivatives
and applications

#24. $\frac{d}{dx} 2^{6x} =$

A. $(6 \ln 2)\, 2^{6x}$

B. $\ln 6 \;\; 2^{3x}$

C. $\ln 8^{x}$

D. $2^{3x} \ln x$

E. NOTA

Solution A

$$\frac{d}{dx} 2^{6x} = (\ln 2)(2^{6x})(6)$$

$$6 \ln 2 \;\; 2^{3x}$$

$$\ln 64\,(2^{3x})$$

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*25. $\frac{d}{dx}\left[\arcsin(8x)\right] =$

A. $8/\sqrt{1+64x^2}$

B. $8/(1+64x^2)$

C. $8/(1+8x)^2$

D. $8/\sqrt{1-64x^2}$

E. NOTA

Solution D

$$\frac{d}{dx}\arcsin u = \frac{u'}{\sqrt{1-u^2}}$$

$u = 8x$

$$\frac{d}{dx}\arcsin 8x = \frac{8}{\sqrt{1-64x^2}}$$

#26

derivatives and applications

given $g(t) = \sin(\arccos t)$ find the derivative of the function.

A. $\cos(t)$

B. $-\arcsin(t)$

C. $\cos(\arccos t)$

D. $(-t)(1-t^2)^{1/2}/(1-t^2)$

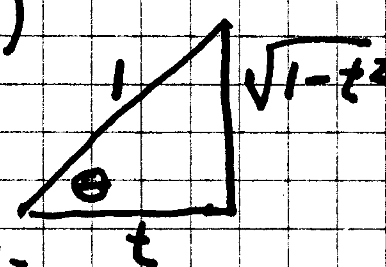
E. NOTA

Solution: D

(1) $$g'(t) = \cos(\arccos t)\left(\frac{-1}{\sqrt{1-t^2}}\right)$$

$$t \quad \frac{-1}{\sqrt{1-t^2}} = \frac{-t}{\sqrt{1-t^2}}$$

(2) $$g(t) = \sin(\arccos t)$$



$$g(t) = \sin\theta = \frac{\sqrt{1-t^2}}{1}$$

$$g'(t) = \tfrac{1}{2}\sqrt{(1-t^2)}^{-1/2}(-2t) = \frac{-t}{(1-t^2)^{1/2}}$$

#27

derivatives and applications

given $g(x) = \ln(\sinh x)$

A. $(e^{x}-e^{-x})/2$

B. $\tanh x$

C. $(e^{x}+e^{-x})/(e^{x}-e^{-x})$

D. $1/(\sinh x)$

E. NOTA

Solution: C

$$g'(x) = \frac{1}{\sinh x} \cdot \cosh x = \coth x = \frac{\frac{e^{x}+e^{-x}}{2}}{\frac{e^{x}-e^{-x}}{2}} \Rightarrow$$

derivatives
and applications

28 The displacement from the equilibrium of an object in harmonic motion on the end of a spring is

$$y = \tfrac{1}{2}\cos(6t) - \tfrac{1}{3}\sin(6t)$$

Where y is measured in feet and t is the time in seconds. Determine the position and velocity of the object when $t = \pi/4$

(POSITION) , (VELOCITY)

A. $5\sqrt{2}/12$ feet , $-\sqrt{2}/2$ feet/second

B. $-5\sqrt{2}/12$ feet , $\sqrt{2}/2$ feet/second

C. $\sqrt{2}/12$ feet , $-\sqrt{2}$ feet/second

D. $-\sqrt{2}/12$ feet , $-\sqrt{2}/2$ feet/second

E. NOTA

Solution : E

$$y = \tfrac{1}{2}\cos 6(\pi/4) - \tfrac{1}{3}\sin 6(\pi/4)$$

$$y = \tfrac{1}{2}\left(-\tfrac{\sqrt{2}}{2}\right) - \tfrac{1}{3}\tfrac{\sqrt{2}}{2} =$$

$$-\frac{3\sqrt{2}}{12} - \frac{2\sqrt{2}}{12} = -\frac{5\sqrt{2}}{12} \text{ Position}$$

Velocity

$$y' = -\tfrac{1}{2}(\sin 6t)(6) - \tfrac{1}{3}(\cos 6t)\,6$$

$$y' = -3\frac{\sqrt{2}}{2} - 2\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{2} \text{ Velocity}$$

29. Evaluate

$$\lim_{x \to \infty} \frac{3x - 2}{\sqrt{2x^2 + 2}}$$

A. This limit does not exist.

B. 0

C. 1

D. $3/\sqrt{2}$

E. NOTA.

Solution: D

$$\lim_{x \to \infty} \frac{\frac{3x}{x} - \frac{2}{x}}{\sqrt{2 + 2/x^2}} = \frac{3}{\sqrt{2}}$$

30. A wooden beam has a rectangular cross section of height h and width w. The strength S of the beam is directly proportional to the width and the square of the height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 15 inches?

A. $5\sqrt{3}$ inches by $5\sqrt{6}$ inches

B. $5\sqrt{3}$ inches by $5\sqrt{3}$ inches

C. $5\sqrt{6}$ inches by $5\sqrt{6}$ inches

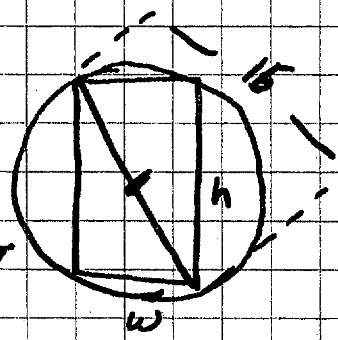
D. 5 inches by $10\sqrt{2}$ inches

E. NOTA

Solution A

$S = kwh^2$

constant of proportionality



$h^2 + w^2 = 15^2$

$h^2 = 15^2 - w^2$

$S = kw(15^2 - w^2)$

$\frac{dS}{dw} = k(15^2 - 3w^2) = 0$

$15^2 = 3w^2$

$\frac{15 \cdot \cancel{15}^{5}}{\cancel{3}} = w^2$

$\underline{5\sqrt{3} = w}$

$h^2 = 15^2 - 75$

$h^2 = 150$

$\Rightarrow h = 5\sqrt{6}$

$25(6) + 25 \cdot 3$

$150 \cdot 75 = 225$

derivatives
and applications

31. Find any relative extrema of the function

$$y = \ln(x^2+2x+4)$$

A. MAXIMUM at $x = 1$

B. MINIMUM at $x = -1$

C. MAXIMUM at $x = -1$

D. MINIMUM at $x = 1$

E. NOTA

Solution: B

$$y' = \frac{1}{x^2+2x+4}\cdot(2x+2) = 0$$

$$2x+2 = 0$$
$$2x = -2$$
$$x = -1$$

$$y'' = \frac{(x^2+2x+4)2 - (2x+2)(2x+2)}{(x^2+2x+4)^2} \quad @\ x = -1 \quad = \frac{2(1-2+4) - \cancel{(-2+2)}^{\,0}}{(1-2+4)^2}$$

Slope is increasing
concave up at $x = -1$ $\quad \frac{6}{9} > 0$
hence relative minimum

32. ON what portion of the domain of $y = x^4 - 4x^3$ is the graph of the function concave down?

A. $-\infty < x < 0$

B. $0 < x < 2$

C. $2 < x < \infty$

D. $-\infty < x < \infty$

E. NOTA

Solution: B

$y' = 4x^3 - 12x^2$

$y'' = 12x^2 - 24x = 0$

$x(12x - 24) = 0$

@ $\left.\begin{array}{l} x = 0 \\ x = 2 \end{array}\right\}$ slope is NOT changing

@ $x = 1$

$y'' = 12 - 24 = -12 < 0$ Slope is decreasing meaning concave down

$\left.\begin{array}{l} x > 2 \\ x < 0 \end{array}\right\}$ $y'' > 0$ slope increasing concave up and applications

33 A rectangle is inscribed in a semi circle such that one side lies on the diameter. If the diameter of the semi circle is 10 inches What is the area of the largest rectangle that can be inscribed?

A. 5 in^2

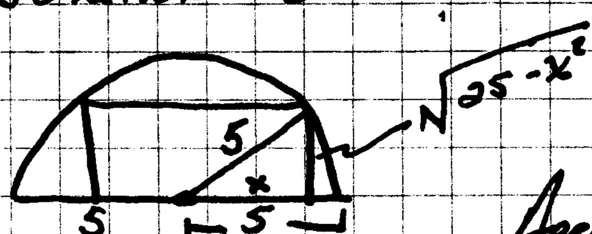
B. $5\sqrt{2}$ in^2

C. 15 in^2

D. 25 in^2

E. NOTA

Solution D



Area of $\square$ = $2x\sqrt{25-x^2}$

$$\frac{dA}{dx} = 2x \tfrac{1}{2}(25-x^2)^{-1/2}(-2x) + 2\sqrt{25-x^2} = 0$$

$$\frac{x(-2x)}{(25-x^2)^{-1/2}} + 2\sqrt{25-x^2} = 0$$

$5\sqrt{2} \cdot 5/\sqrt{2}$ (25)

$-2x^2 + 2(25-x^2) = 0$

$-4x^2 = 50$

$x^2 = 50/4 = \frac{5\sqrt{2}}{2}$