Mu Alpha Theta National Convention
Mississippi State University 2002
Mu Division--Derivatives and Applications

1. Which of the following is not true concerning a function *f*?

   a. If *f* is differentiable at $x = c$, then *f* is continuous at $x = c$.
   b. If *f* is continuous at *c*, then *f* is differentiable at *c*.
   c. If *f* is a constant, then the derivative of *f* is zero.
   d. $f'(x), \frac{dy}{dx}, y', \frac{d}{dx}\left[f(x)\right], D_x\left[y\right]$ are all acceptable notation for the derivative
   e. NOTA

2. Evaluate the derivative of $\tan x \cot x$ at $(1,1)$.

   a. π
   b. $\frac{\pi}{2}$
   c. 1
   d. 0
   e. NOTA

3. Which of the following statements is/are true?

   I. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
   II. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.
   III. If $z = \pi$, then $\frac{dz}{dx} = 2\pi$.
   IV. If $y = \frac{x}{\pi}$, then $\frac{dy}{dx} = \frac{1}{\pi^2}$.

   a. I, II
   b. II
   c. I, II, III, IV
   d. III, IV
   e. NOTA

4. Given $f(x)=3x^3-x^2+2$, which of the following should be evaluated to find the slope of a tangent line to this function at any x:

a. $\lim\limits_{\Delta x\to 0}\dfrac{3(x+\Delta x)^3-(x+\Delta x)^2+2-3x^3-x^2+2}{x+\Delta x-x}$

b. $\lim\limits_{\Delta x\to 0}\dfrac{3(x+\Delta x)^3-(x+\Delta x)^2+2-\left(3x^3-x^2+2\right)}{x+\Delta x}$

c. $\lim\limits_{\Delta x\to 0}\dfrac{3(x+\Delta x)^3-(x+\Delta x)^2+2-\left(3x^3-x^2+2\right)}{\Delta x}$

d. $\lim\limits_{\Delta x\to 0}\dfrac{3x^3-x^2+2-\left[3(x+\Delta x)^3-(x+\Delta x)^2+2\right]}{\Delta x}$

e. NOTA

5. If $f(x)=\sin x\cos x$, find $f'(x)$ at $x=\dfrac{\pi}{6}$.

a. $1+2\sin^2 x$

b. $\dfrac{3}{2}$

c. $\dfrac{1}{2}$

d. $1-2\sin^2 x$

e. NOTA

6. If $f(x)=\dfrac{x}{\sqrt{x^2+1}}$, find $f'(x)$.

a. $\dfrac{1}{\left(x^2+1\right)^{\frac{3}{2}}}$

b. $\dfrac{1}{\left(x^2+1\right)}$

c. $\dfrac{\left(x^2+1\right)^{\frac{1}{2}}-\frac{x}{2}\left(x^2+1\right)^{-\frac{1}{2}}}{\left(x^2+1\right)}$

d. $\dfrac{1-2x^2}{\left(x^2+1\right)^{\frac{3}{2}}}$

e. NOTA

7. Given $f(x)=[g(x)]^3 h(x), g(5)=-2, g'(5)=8, h(5)=\frac{1}{2}, h'(5)=4$, find $f'(5)$.

a. 16
b. 4
c. - 16
d. 0
e. NOTA

8. Find $\frac{d^2y}{dx^2}$ in terms of x and y for $x^2-y^2=16$.

a. $\frac{x}{y}$
b. $\frac{x^2}{y^2}$
c. $\frac{y^2-x^2}{y}$
d. $\frac{16}{y^3}$
e. NOTA

9. A ladder 25 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the house at 3 feet per second. How fast is the top of the ladder moving down the wall when the bottom is 24 feet from the base of the wall?

a. 7 feet per second
b. - 144 feet per second
c. $\frac{-72}{7}$ feet per second
d. - 3 feet per second
e. NOTA

10. If $f(t)=\sin^3(4t)$, find $\frac{d}{dt}[f(t)]$.

a. $12t^2\sin^2(4t^2)\cos(4t^2)$
b. $3\sin^2(4t^2)\cos(4t^2)$
c. $3\sin^2(4t^2)$
d. $24t\sin^2(4t^2)\cos(4t^2)$
e. NOTA

11. The temperature T of food placed in a freezer is given by $T = \frac{700}{t^2 + 4t + 10}$ where t is time in hours. At how many degrees per hour is T changing when $t = 2$ hours?

a. $-\frac{1400}{121}$ degrees per hour
b. $\frac{1400}{121}$ degrees per hour
c. $\frac{2800}{11}$ degrees per hour
d. 1800 degrees per hour
e. NOTA

12. Find y'' if $y = 3x^4 - 2x^2 + 10x - 1$.

a. $12x^3 - 4x + 10$
b. $36x^2 - 4x + 10$
c. $36x^2 - 4$
d. $12x^2 - 4$
e. NOTA

13. Find any critical numbers of the function $f(x) = x^2(x-3)$.

a. $x = 0, x = 2$
b. $x = 0, x = -2$
c. $x = 2, x = -2$
d. $x = -3, x = 2$
e. NOTA

14. Given the function $f(x) = \frac{x^2 - 2x - 3}{x+2}$, find the value on the interval $[-1,3]$ for which Rolle's Theorem is satisfied.

a. $-2 \pm \sqrt{5}$
b. $-2 - \sqrt{5}$
c. $-2 + \sqrt{5}$
d. $2, \frac{3}{2}$
e. NOTA

15. Given $f(x) = \sin x$, find the value on the interval $[0, \pi]$ of this function for which the mean value theorem is satisfied.

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$
c. $\frac{\pi}{2}$
d. $\frac{2\pi}{3}$
e. NOTA

16. For $f(x) = x^{\frac{1}{3}} + 1$, find the critical numbers (if any) and the open intervals on which the function is increasing or decreasing.

a. decreasing $(-\infty, \infty)$, critical number 0
b. decreasing $(-\infty, 0)$, increasing $(0, \infty)$, critical number 1
c. decreasing $(0, \infty)$, increasing $(-\infty, 0)$, critical number 1
d. increasing $(-\infty, \infty)$, critical number 0
e. NOTA

17. Find all relative extrema of the function $f(x) = \dfrac{x^2 - 2x + 1}{x + 1}$.

a. maximum at $(1,0)$, minimum at $(-3,-8)$
b. maximum at $(-3,8)$, minimum at $(-1,0)$
c. maximum at $(-3,8)$, minimum at $(1,0)$
d. no relative extrema exists for this function
e. NOTA

18. If f is differentiable on an open interval I, then which of the following statements is/are true?

a. The graph of f is concave upward on I if f' is positive on the interval and concave downward on I if f' is negative on the interval.
b. The graph of f is concave upward on I if f'' is increasing on the interval and concave downward on I if f'' is increasing on the interval.
c. The graph of f is concave upward if f' is zero at some point on the interval.
d. The graph of f is concave downward if f' is zero at some point on the interval.
e. NOTA

19. Determine the open intervals on which the graph of $f(x) = 4(x^2 + 3)^{-1}$ is concave upward or downward.

a. concave up on $(-\infty,-1)$ and $(1,\infty)$, concave down on $(-1,1)$
b. concave up on $(-1,1)$, concave down on $(-\infty,-1)$ and $(1,\infty)$
c. concave up on $(-\infty,-4)$ and $(4,\infty)$, concave down on $(-4,4)$
d. concave up on the entire real number line
e. NOTA

20. Find *a, b, c, d* such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ satisfies the following conditions:

I. *f* has a relative maximum at $(3,3)$
II. *f* has a relative minimum at $(5,1)$
III. *f* has and inflection point at $(4,2)$

a. $f(x) = 6x^3 - \frac{1}{2}x^2 + \frac{45}{2}x - 24$
b. $f(x) = 4x^3 - 6x^2 + \frac{45}{2}x + 24$
c. $f(x) = -\frac{1}{2}x^3 + 6x^2 - \frac{45}{2}x + 24$
d. $f(x) = \frac{1}{2}x^3 - 6x^2 + \frac{45}{2}x - 24$
e. NOTA

21. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. What dimensions will produce a box with maximum volume?

a. 3 inches by 3 inches by 6 inches
b. 36 inches by 36 inches by $\frac{1}{12}$ inches
c. 6 inches by 6 inches by 3 inches
d. 4 inches by 4 inches by $6\frac{3}{4}$ inches
e. NOTA

22. Which points on the graph of $y = 9 - x^2$ are closest to the point $(0,3)$?

a. $\left(\pm\frac{\sqrt{11}}{2}, 6\frac{1}{4}\right)$
b. $\left(\pm\frac{\sqrt{22}}{2}, \frac{7}{2}\right)$
c. $\left(\pm\frac{\sqrt{22}}{4}, 7\frac{5}{8}\right)$
d. $(\pm 3,0)$
e. NOTA

23. Differentiate $f(x) = x^2 \ln x$.

a. $2x + \ln x$
b. $x(1 + \ln x^2)$
c. 2
d. $x^2 + \ln\left(\frac{1}{x}\right) + 2\ln x$
e. NOTA

24. $\frac{d}{dx}\left[2^{6x}\right] =$

a. $(6\ln 2)2^{6x}$
b. $(\ln 6)2^{3x}$
c. $\ln 8^x$
d. $2^{3x} \ln x$
e. NOTA

25. $\frac{d}{dx}\left[\arcsin(8x)\right] =$

a. $\frac{8}{\sqrt{1+64x^2}}$
b. $\frac{8}{1+64x^2}$
c. $\frac{8}{(1+8x)^2}$
d. $\frac{8}{\sqrt{1-64x^2}}$
e. NOTA

26. Given $g(t) = \sin(\arccos t)$, find $g'(t)$.

a. $\cos t$
b. $-\arcsin t$
c. $\cos(\arccos t)$
d. $\frac{-t\left(1-t^2\right)^{\frac{1}{2}}}{1-t^2}$
e. NOTA

27. Given $q(x) = \ln(\sinh x)$, find $q'(x)$.

a. $\frac{e^x - e^{-x}}{2}$
b. $\tanh x$
c. $\frac{e^x + e^{-x}}{e^x - e^{-x}}$
d. $\frac{1}{\sinh x}$
e. NOTA

28. The displacement for the equilibrium of an object in harmonic motion on the end of a spring is $y = \frac{1}{2}\cos(6t) - \frac{1}{3}\sin(6t)$. Where y is measured in feet and t is the time in seconds. Determine the position and velocity of the object when $t = \frac{\pi}{4}$.

a. position $\frac{5\sqrt{2}}{12}$, velocity $-\frac{\sqrt{2}}{2}$ feet per second
b. position $-\frac{5\sqrt{2}}{12}$, velocity $\frac{\sqrt{2}}{2}$ feet per second
c. position $\frac{\sqrt{2}}{12}$, velocity $-\sqrt{2}$ feet per second
d. position $-\frac{\sqrt{2}}{12}$, velocity $-\frac{\sqrt{2}}{2}$ feet per second
e. NOTA

29. Evaluate $\lim_{x\to\infty} \frac{3x-2}{\sqrt{2x^2+2}}$.

a. This limit does not exist.
b. 0
c. 1
d. $\frac{3}{\sqrt{2}}$
e. NOTA

30. A wooden beam has a rectangular cross section of height h and width w. The strength S of the beam is directly proportional to the width and the square of the height. What are the dimensions of the strongest beam that can be cut from a round log of diameter 15 inches?

a. $5\sqrt{3}$ inches by $5\sqrt{6}$ inches
b. $5\sqrt{3}$ inches by $5\sqrt{3}$ inches
c. $5\sqrt{6}$ inches by $5\sqrt{6}$ inches
d. 5 inches by $10\sqrt{2}$ inches
e. NOTA

31. Find any relative extrema of the function $y = \ln\left(x^2 + 2x + 4\right)$.

a. maximum at $x = 1$
b. minimum at $x = -1$
c. maximum at $x = -1$
d. minimum at $x = 1$
e. NOTA

32. On what portion of the domain of $y = x^4 - 4x^3$ is the graph of the function concave down?

a. $-\infty < x < 0$
b. $0 < x < 2$
c. $2 < x < \infty$
d. $-\infty < x < \infty$
e. NOTA

33. A rectangle is inscribed in a semi circle such that one side lies on the diameter. If the diameter of the semi circle is 10 inches, what is the area of the largest rectangle what can be inscribed?

a. 5 square inches
b. $5\sqrt{2}$ square inches
c. 15 square inches
d. 25 square inches
e. NOTA

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Answer Key

1. b
2. d
3. b
4. c
5. c
6. a
7. a
8. e
9. c
10. d
11. a
12. c
13. a
14. c
15. c
16. d
17. c
18. e
19. a
20. d
21. c
22. e
23. b
24. a
25. d
26. d
27. c
28. e
29. d
30. a
31. b
32. b
33. d