Differential Equations (Mu Division) Topic Test

Note: For each problem, where there is no choice (e), assume (e) none of the above.

1. State the order of the differential equation: $(y')^3 = \sin x$

   a) 1st b) 2nd c) 3rd d) 4th

2. The solution to the differential equation $x\,dy - y\,dx = 0$ is

   a) $y = e^x + C$ b) $y = Cx$ c) $y = x$ d) cannot be solved

3. Solve $x\cos x\,dx + (1 - 6y^5)\,dy = 0$; The graph passes through $(\pi, 0)$.

   a) $y^6 - y = \cos x - x\sin x + C$ b) $x\sin x - \cos x + \pi = y^6 - y$

   c) $y^6 - y = x\sin x + \cos x + 1$ d) no real valued solution exists

4. Solve the differential equation: $(x^2 - xy + y^2)\,dx - xy\,dy = 0$

   a) $xy = Ce^{\frac{1}{x-y}}$ b) $(y - x)e^{\frac{y}{x}} = C$ c) $x = Ce^{(2x-y)}$ d) $C = \dfrac{2x}{2y-1}$

5. Solve the differential equation : $(x + y)y' + (y + 3x) = 0$

   a) $xy + \frac{3}{2}x^2 + \frac{1}{2}y^2 = C$ b) $3x^2 + y^2 = C$ c) $1 + \frac{3}{2}x^2 = \frac{y^2}{2}$ d) $C = x^3 - y^2 - xy$

6. Solve the differential equation: $(1 + 3x\sin y)\,dx - x^2\cos y\,dy = 0$

   a) $\dfrac{4}{x}\cos y = Cx^3 - 1$

   b) $3x\sin y = Cx^2 - \ln|x| - 2$

   c) $4x\sin y = Cx^4 - 1$

   d) not solvable

7. What is the velocity of a projectile at an altitude of 8000 feet after it was fired directly upward from the ground with a muzzle velocity of 1000 feet per second? (g = 32ft/sec)

   a) 698.570 b) 770.366 c) 1229.634 d) 1698.570

8. A certain type of glass is such that a slab 1 inch thick absorbs one-quarter of the light which starts to pass through it. How thin must a pane be made to absorb only 1% of the light? (all answers are in inches).

   a) 0.007 b) 0.015 c) 0.028 d) 0.035

9. A certain radioactive material loses mass at a rate proportional to the mass present. If the material has a half-life of 30 minutes, what percent of the original mass is expected to remain after 0.9 hours?

   a) 8% b) 29% c) 47% d) 98%

10. If the marginal cost, y, of producing a certain item (x) is $\frac{dy}{dx} = 3 + x + \frac{e^{-x}}{4}$, what is the cost of producing one item if there is a fixed cost of \$4.00?
    a) \$6.76 b) \$7.66 c) \$8.16 d) \$9.26

11. Solve $y'' - y' - 2y = 0$.
    a) $y = C_1e^{2x} + C_2e^{-x}$ b) $y = C_1e^{x} + C_2e^{-2x}$ c) $y = C_1e^{-x} + C_2e^{-2x}$ d) not solvable

12. What is the time required for one dollar to double when invested at the rate of 5% per annum compounded continuously?
    a) 0.139 yrs b) 1.386 yrs c) 13.863 yrs d) 138.629 yrs

13. Solve the differential equation: $\frac{dy}{dx} = 3x^2$.
    a) $y = x^3 + c$ b) y= 6x +c c) $y = 3x^3 + c$ d) $y = 3x^2y + c$
    E) NOTA

14. The temperature inside a house is 70° F. A thermometer is taken from the house and placed outside. The outside air is 10° F. After 3 minutes, the thermometer reads 25° F. What is the thermometer temperature after 7 minutes?
    a) 19° F b) 12° F c) 9° F d) 7° F

15. A pipe 10 cm in diameter contains steam at 100° C. It is covered with asbestos 5 cm thick. The thermal conductivity, k, is 0.00060 cal/cm deg sec. The outside surface is at 30° C. Find the heat loss per hour from a meter length of pipe. (answers are in cal/hr)
    a) 380 b) 38,500 c) 138,000 d) 1,380,000

16. What integrating factor would make the differential equation $2\left(y - 4x^2\right)dx + xdy = 0$ exact?
    a) xy^2 b) $\frac{y}{x}$ c) x^2y d) x^2

17. Water flows down a river at the rate $9 + t^{\frac{3}{2}}$ million ft^3/day, *t* days after a rain. How much water will flow past a given point during the first 4 days after a rain? (answer in million ft^3)
    a) 39.4 b) 48.8 c) 61 d) 116

18. A solution of the differential equation 2y dy = x dx is
    a) $x^2 - 2y^2 = 8$
    b) $x^2 + 2y^2 = 8$
    c) $2y^2 = -x^2$
    d) $x^2 - 8y^2 = 0$
    e) $x^2 = 16 - 2y^2$

19. If a car accelerates from 0 to 70 mph in 10 sec, what distance does it travel in those 10 sec? (assume acceleration is constant and 60 mph=88ft/sec)
    a) 51 ft. b) 513 ft. c) 616 ft. d) 1027 ft.

20. The growth size of an animal population at time *t* is denoted by $\frac{dp}{dt}$ = 0.002P (1000 – P). The population is growing fastest
    a) initially
    b) at the carrying capacity
    c) when P=500
    d) when $\frac{d^2P}{dt^2} > 0$

21. Functions *g* and *h* are twice differentiable such that h(x)= ln(g(x)) and $h''(x) = f(x)/(g(x))^2$ Find f(x).
    a) $g(x)g''(x) - 2g'(x)$
    b) $g(x)g''(x) - g'(x)$
    c) $g(x)[g''(x)]^2 - g'(x)$
    d) $g(x)g''(x) - [g'(x)]^2$

22. Given $\frac{ds}{dt} = t^2 - t - 1$. If $s = 0$ when $t = 1$, then what is the value of *s* when $t = 0$?
    a) $\frac{7}{6}$ b) $\frac{8}{7}$ c) $-\frac{4}{5}$ d) $\frac{1}{2}$

23. Find the general solution for $y' + 2y = x^2$
    a) cannot be done
    b) $y = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4} + Ce^{-2x}$
    c) $y = 2x^2 - 8x + 19 + Ce^{-2x}$
    d) $y = \frac{1}{4}x^2 - \frac{1}{2}x - \frac{1}{4} + Ce^{-2x}$

24. The motion of a particle on the x-axis has acceleration $\frac{d^2x}{dt^2} = t^2 - 2t$. It is stationary at 1 when t = 1. Find $12x(t)$.
    a) $t^4 + 4t^3$
    b) $t^4 - 4t^3 + 8t + 7$
    c) $4t^4 + 8t^3$
    d) $t^4 - 4t^3 + 15t^2$

25. The general solution of $x\,dy = y\,dx$ is a family of a) circles b) parabolas
c) hyperbolas d) lines passing through the origin

26. If radium decomposes at a rate proportional to the amount present, then the amount R left after t years, if R_0 is present initially and k is a negative constant of proportionality, is given by

a) $R = R_0 kt$ b) $R = R_0 e^{kt}$ c) $R = R_0 + \frac{1}{2}kt^2$ d) $R = e^{R_0 kt}$

27. Given $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$ when t = 0, and s = 1. Find t when $s = \frac{3}{2}$.

a) $\frac{1}{2}$ b) $\frac{\pi}{2}$ c) 1 d) $\frac{2}{\pi}$

28. In 1970, the earth's population was 3.5 billion. If the rate of increase is 2% per year, then the year in which the population reaches 50 billion is closest to which of the following years?

a) 2100 b) 2150 c) 2200 d) 2300

29. Use Euler's method and 4 steps with $\Delta x = 0.1$ for the differential equation $y' = 2y$ to find an approximation for y, when $y(0) = 1$ and $x = 0.4$.

a) 1.452 b) 1.597 c) 1.872 d) 2.074

30. Which of the following differential equations is NOT logistic?

a) $P' = P - P^2$

b) $\frac{dy}{dt} = 0.01y(100 - y)$

c) $\frac{dx}{dt} = 0.8x - 0.004x^2$

d) $\frac{dR}{dt} = 0.16(350 - R)$

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ANSWER KEY

1. a
2. b
3. c
4. b
5. a
6. c
7. a
8. d
9. b
10. b
11. a
12. c
13. a
14. b
15. d
16. d
17. b
18. a
19. b
20. c
21. d
22. a
23. b
24. b
25. d
26. b
27. d
28. a
29. d
30. d