Integrals and applications

1. $\int (x^3+3)\,dx =$

A. x^4+3x+C
B. $x^4/4+x^3+C$
C. $3x^4/4+3x^3+C$
D. $\dfrac{x^4}{4}+3x+C$
E. NOTA

Solution: D

$$\int x^3+3 = \frac{x^4}{4}+3x+C$$

Integrals and applications

2. Solve the differential equation given $y = -8$ when $t = 2$.

$$\frac{dy}{dt} = 3t^2$$

A. $y = t^3 + 16$

B. $y = t^3 - 16$

C. $y = t^2 - 24$

D. $y = 3t^2 - 8$

E. NOTA

Solution (B)

$$\frac{dy}{dt} = 3t^2$$

$$dy = 3t^2\,dt$$

$$\int dy = \int 3t^2\,dt$$

$$y + C_1 = t^3 + C_2$$

$$y = t^3 + C_3$$

$y = -8$ when $t = 2$

$$-8 = 8 + C_3$$

$$-16 = C_3$$

$$y = t^3 - 16$$

Integrals and Applications

3. $\int_0^2 \sqrt[3]{x^2}\,dx$

A. $\frac{3}{5}x^{5/3}+C$

B. $\frac{2}{3}x^{5/3}+C$

C. $\frac{6\sqrt[3]{4}}{5}$

D. $\frac{2\sqrt[3]{4}}{5}$

E. NOTA

Solution: C

$$\int_0^2 \sqrt[3]{x^2}\,dx = \int_0^2 x^{2/3}\,dx = \frac{3}{5}x^{5/3}+C\Big|_0^2$$

$$\frac{3\cdot 2\sqrt[3]{4}}{5} = \frac{6\sqrt[3]{4}}{5}$$

Integrals and applications

4. $\int_0^{\pi/2} (2\sin x + 3\cos x)\, dx$

A. 5

B. 1

C. -1

D. -5

E. NOTA

Solution: A

$$-2\cos x + 3\sin x + C \Big|_0^{\pi/2}$$

$$0 + 3 - \left[-2\right] = 5$$

Integrals and applications

5. Given

$f''(x) = 2$

$f'(2) = 5$

$f(2) = 10$

Solve for $f(x)$

A. $x^2 + x + 4$

B. x^2

C. $x^2 + 3x + 3$

D. $x^2 + 5x - 2$

E. NOTA

Solution: $f''(x) = 2$ → (A)

$$\frac{d^2y}{dx^2} = 2$$

$$\int \frac{dy}{dx}\, dy = \int 2\, dx$$

$$\frac{dy}{dx} = 2x + C \qquad f'(2) = 5$$

$$5 = 4 + C$$

$$1 = C$$

$$\frac{dy}{dx} = 2x + 1$$

$$\int dy = \int (2x+1)\, dx = x^2 + x + C$$

$$10 = 4 + 2 + C$$

$$C = 4$$

$y = x^2 + x + 4$

Integrals and applications

6. evaluate the integral.

$$\int_0^{\pi/4} \frac{\sin x}{1-\sin^2 x}\, dx$$

A. $\sqrt{2}-1$

B. $\sec x + C$

C. $2-\sqrt{2}$

D. $\sqrt{2}$

E. NOTA

$\to$ A

Solution $\int \frac{\sin x}{\cos^2 x}\, dx$

$$\int \cos^{-2} x \sin x \, dx$$

$$\cos x^{-1}$$

$$\left.\frac{1}{\cos x}\right|_0^{\pi/4} = \frac{1}{\sqrt{2}/2} - \frac{1}{1}$$

$\sqrt{2}-1$ $= \frac{2}{\sqrt{2}} - 1 = \frac{2-\sqrt{2}}{\sqrt{2}}$

Integrals and applications

7. On the moon, the acceleration due to gravity is -1.6 meters per second. A stone dropped from a cliff on the moon hits the surface of the moon in 20 seconds. How far did it fall? What was it's velocity on impact?

A. -320 meters, 32 meters/sec

B. 320 meters, -32 meters/sec

C. 320 meters, -16 meters/sec

D. 640 meters, 32 meters/sec

E. NOTA

Solution: B

$$\frac{d^2h}{dt^2} = -1.6$$

$$\frac{dh}{dt} = -1.6t + C$$

$v = -1.6t + C$ $\quad$ $v = -1.6(t)$ @ $t=20$

$v_0 = 0 \quad \therefore \quad C = 0$ $\quad$ $v = \underline{-32 \text{ m/sec}}$

$h = -.8t^2 + \cancel{Ct} + C$ $\quad$ $h = 0$ at $t = 20$

$C = 320$

@ $t = 0$ $\quad h = C$

$-.8(20)^2 = -.8 \; 400 = \underline{-320 \text{ m}}$

INTEGRALS AND APPLICATIONS

8. If $\int_0^4 x\,dx$ were to be evaluated using Riemann Sums, how many partitions should be used to insure an accuracy of $\pm .001$?

A. 8000

B. 4000

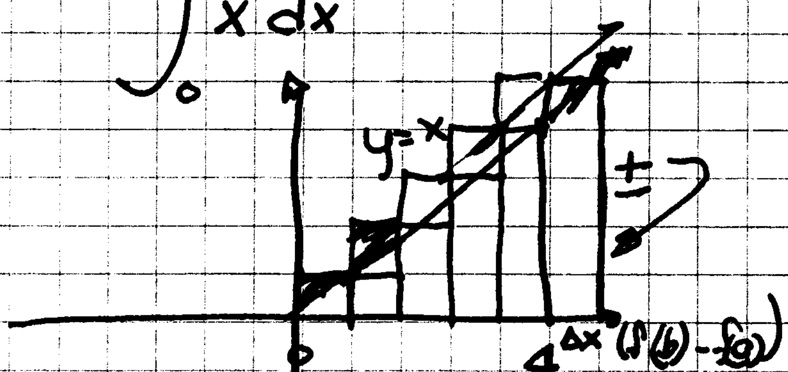
C. 2000

D. 100

E. NOTA

Solution (A)

$$\int_0^4 x\,dx$$



$$\frac{\Delta x\,(4-0)}{2} = .001$$

$$\frac{\Delta x\,(4-0)}{2} = .001 \qquad \Delta x = \frac{.001\cdot 2}{4} = \Delta x = .0005 \qquad n = \frac{4-0}{.0005} = \frac{40000}{5} = 8000 \text{ part.}$$

Integrals and applications

9. Given: $\int_1^4 f(x)\,dx = 26/3$

$\int_1^4 g(x)\,dx = 4$

$\int_1^4 h(x)\,dx = 2$

$E(x) = -f(x) + 4g(x) - 3h(x)$

Evaluate $\int_1^4 E(x)\,dx$

A. $-8/3$

B. $-4/3$

C. $4/3$

D. $14/3$

E. NOTA

Solution: C

$$\int_1^4 E(x) = -\int_1^4 f(x)\,dx + 4\int_1^4 g(x)\,dx - 3\int_1^4 h(x)\,dx$$

$$-26/3 + 16 - 6 = 4/3$$

Integrals
and
applications

10 Which of the following are false?

I. $\int_a^b f(x)\, g(x)\, dx = \left[\int_a^b f(x)\, dx\right]\left[\int_a^b g(x)\, dx\right]$

II. If the norm of a partition approaches zero, then the number of subintervals approaches infinity.

III. If f is increasing on $[a,b]$ then the minimum value of $f(x)$ on $[a,b]$ is $f(a)$.

IV. The value of $\int_a^b f(x)\, dx$ must be positive.

V. If $\int_a^b f(x)\, dx > 0$, then f is nonnegative for all x in $[a,b]$.

A. II, III

B. I, IV, V

C. I, II, III

D. I, II, IV, V

E. NOTA

False: I, IV, V

True: II, III

Solution (B)

Integrals and applications

11. Find the average value of $f(x) = 3x^2 - x$ on the interval $[1, 5]$.

A 18.5

B 28

C 27.75

D 112

E NOTA

Solution (B)

$$\frac{1}{b-a}\int_a^b f(x)\,dx = \frac{1}{5-1}\int_1^5 (3x^2 - x)\,dx$$

$$\frac{1}{4}\int_1^5 3x^2 - x\,dx$$

$$\frac{1}{4}\left[x^3 - \frac{x^2}{2}\right]_1^5$$

$$\left[\left[125 - \frac{25}{2}\right] - \left[1 - \frac{1}{2}\right]\right]$$

$$4\overline{)112}$$

$$28$$

Integrals and applications

12. If $F(x) = \int_{\pi/2}^{x^4} -\cos t \, dt$

find $F'(x)$.

A. $-4\cos x^4$

B. $-4\cos x^4 - \pi/2$

C. $-4x^3\cos x^4 - \pi/2$

D. $-4x^3\cos x^4$

E. NOTA

Solution D

$$F(x) = \int_{\pi/2}^{x^4} -\cos t \, dt$$

$$\sin \pi/2 = 1$$

$$F x = -\sin t \Big|_{\pi/2}^{x^4} = -\sin x^4 - (-\sin \pi/2)$$

$$F(x) = -\sin x^4 + 1$$

$$F'(x) = -4x^3\cos x^4$$

Integrals
and
applications

13. Find the area of the region bounded by the graphs of the equations: $y = x^3 + x$, $x = 2$, $y = 0$

A. 2

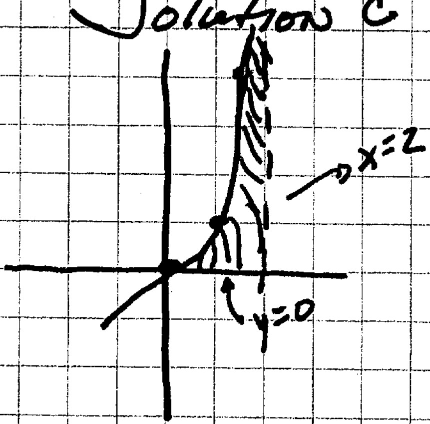
B. 4

C. 6

D. 8

E. NOTA

Solution C



$$\int_0^2 x^3 + x = \frac{x^4}{4} + \frac{x^2}{2}\Big|_0^2$$

$$4 + 2 = 6$$

Integrals and Applications

Evaluate:

14. $$\int_0^{\sqrt{3}/2} 5x\sqrt{1-x^2}\,dx =$$

A. $5\sqrt{3}/8$

B. $35/24$

C. $5/6$

D. $5/3$

E. NOTA

Solution B

$$\int_0^{\sqrt{3}/2} 5x\sqrt{1-x^2}\,dx$$

Let $u = 1-x^2$

$$du = -2x\,dx$$

$$-\frac{du}{2} = x\,d\cancel{x}$$

$$-\frac{5}{2}\int u^{1/2}\,du = \frac{-5/2}{3/2}u^{3/2} + C$$

$$-5/3\left(1-x^2\right)^{3/2}\Big|_0^{\sqrt{3}/2}$$

$$-5/24 - -5/3 =$$

$$40/24 - 5/24 = 35/24$$

integration
and
applications

Evaluate

15. $\displaystyle\int_{\pi/4}^{\pi/2} \cot^2 x \, dx$

A. $\csc x \cot x + C$

B. $\pi/4 - 1$

C. $1 + \pi/4$

D. $1 + 3\pi/4$

E. NOTA

Solution E.

$\displaystyle\int_{\pi/4}^{\pi/2} \cot^2 x \, dx$ $\cot^2 x$

$\dfrac{\sin^2 x}{s} + \dfrac{\cos^2 x}{s} = \dfrac{1}{s}$

$1 + \cot^2 = \csc^2 x$

$\displaystyle\int (\csc^2 x - 1)\, dx$

$-\cot x - x \Big|_{\pi/4}^{\pi/2}$

$0 - \pi/2 - [-1 - \pi/4]$

$= 1 - \pi/4$

16. Evaluate

$$\int_0^{\pi/3} \tan x \, dx$$

A. $-1/2$

B. $-\ln(1/2)$

C. $-\ln 2$

D. $\ln\sqrt{3}/2$

E. NOTA

Solution: C

$$\int \frac{\sin x}{\cos x} \, dx$$

Let $u = \cos x$

$$\frac{du}{dx} = \sin x$$

$$du = \sin x \, dx$$

$$\int \frac{du}{u} = \ln u + C \Rightarrow \ln \cos x \Big|_0^{\pi/3}$$

$$\ln 1/2 - \ln 1 = \ln(1/2)$$

$$\ln 1 - \ln 2 = -\ln 2$$

Integrals and applications

Find the indefinite integral:

17. $\displaystyle\int \frac{2x}{(x-1)^2}\,dx$

A. $2\ln|x-1| - \frac{2}{x-1} + C$

B. $\ln(x-1)^2 + \frac{2}{x-1} + C$

C. $\ln|x-1|^2 + \frac{1}{x-1} + C$

D. $2\ln|x-1| + C$

E. NOTA

Solution: A

$$\int \frac{2x-2}{(x-1)^2}\,dx + \int \frac{2}{(x-1)^2}\,dx$$

$u = x^2-2x+1$ Let $u = x-1$

$du = dx$

$\frac{du}{dx} = 2x-2$

$du = (2x-2)\,dx$ $+ \displaystyle\int \frac{2\,du}{u^2}$

$$\int \frac{du}{u} + \int \frac{2\,du}{u^2}$$

$$\ln|u| + 2(-u^{-1}) + C$$

$$2\ln|x-1| + 2\cdot\frac{1}{(x-1)} + C$$

negative

INTEGRALS and Applications

18. Evaluate

$$\int_0^1 \frac{x}{1+x^4}\,dx$$

A. $\pi/8$

B. $1/8$

C. $\pi/4$

D. $\pi/2$

E. NOTA

Solution: A

$$\int_0^1 \frac{x}{1+x^4}\,dx$$

Let $x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$$\frac{1}{2}\int_0^1 \frac{du}{1+u^2} = \frac{1}{2}\arctan u$$

$$\frac{1}{2}\arctan x^2 \Big|_0^1$$

$$\frac{1}{2}\cdot\frac{\pi}{4} = \frac{\pi}{8}$$

Integrals and applications

19. Solve the differential equation:

$$\frac{dy}{dx} = \frac{2x}{x^2-9} \qquad \text{given: } y = 4 \text{ when } x = 0$$

A. No Solution

B. $y = \tan x - \ln|x^2-9| + 4$

C. $y = \ln|x^2-9| + 4 - \ln 9$

D. $y = \ln|x^2-9| + 4 + \ln(-9)$

E. NOTA

Solution: C ⇒ C

$$\frac{dy}{dx} = \frac{2x}{x^2-9}$$

$$dy = \frac{2x}{x^2-9}\,dx$$

$$\int dy = \int \frac{2x}{x^2-9}\,dx \Rightarrow \int dy = \int \frac{du}{u}$$

$$u = x^2-9 \qquad \int \frac{du}{u} = \ln u$$
$$du = 2x\,dx$$

$$y = \ln|x^2-9| + C$$

$$y = \ln|x^2-9| + 4 - \ln 9$$

Integrals and applications

Evaluate

20. $\int_0^1 5xe^{-x^2}dx =$

A. $\frac{5}{2}(e+1)$

B. $\frac{5}{2e}(e-1)$

C. $\frac{5}{2e}(e+1)$

D. $5e^2$

E. NOTA

Solution: B

$\int_0^1 5xe^{-x^2}dx$

let $u = e^{-x^2}$

$\frac{du}{dx} = e^{-x^2}(-2x)$

$du = -2xe^{-x^2}dx$

$-\frac{du}{2} = xe^{-x^2}dx$

$-\frac{5}{2}\int du = -\frac{5}{2}u + C$

$-\frac{5}{2}e^{-x^2} + C \Big|_0^1$

$-\frac{5}{2e} + \frac{5}{2} = \frac{5}{2}\left(1 - \frac{1}{e}\right)$

21. Evaluate

$$\int_0^1 \frac{4}{1+x^2}\,dx$$

A. $\sqrt{2}/2$

B. $1/2$

C. $\pi/2$

D. π

E. NOTA

Solution: D

$$\int_0^1 \frac{4}{1+x^2}\,dx$$

$$4\int_0^1 \frac{dx}{1+x^2} = 4\tan^{-1}x\Big|_0^1$$

$$4\left[\pi/4 - 0\right] = \pi$$

Integrals and applications

22. Find the area of the region bounded by the graphs of $y = x^2 + 4$, $y = -x$, $x = 0$ and $x = 2$.

A. $38/3$

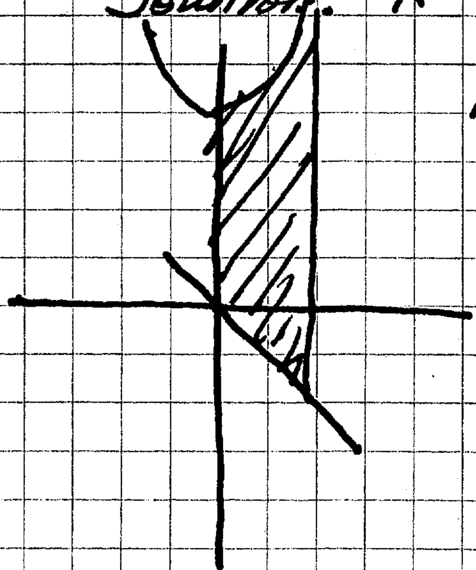
B. $14/3$

C. $7/3$

D. $-10/3$

E. NOTA

Solution: A



$$Area = \int (f(x) - g(x))\, dx$$

$$= \int_0^2 x^2 + 4 - (-x)$$

$$= \frac{x^3}{3} + \frac{x^2}{2} + 4x \Big|_0^2$$

$$\frac{8}{3} + \frac{4}{2} + 8$$

$$16/6 + 12/6 + 48/6 = 76/6 = 38/3$$

INTEGRALS
AND
APPLICATIONS

23. Find the volume of the solid formed by revolving the region bounded by $f(x) = 4 - x^2$ and $g(x) = 3$ about the line $y = 3$.

A. $8\pi/3$

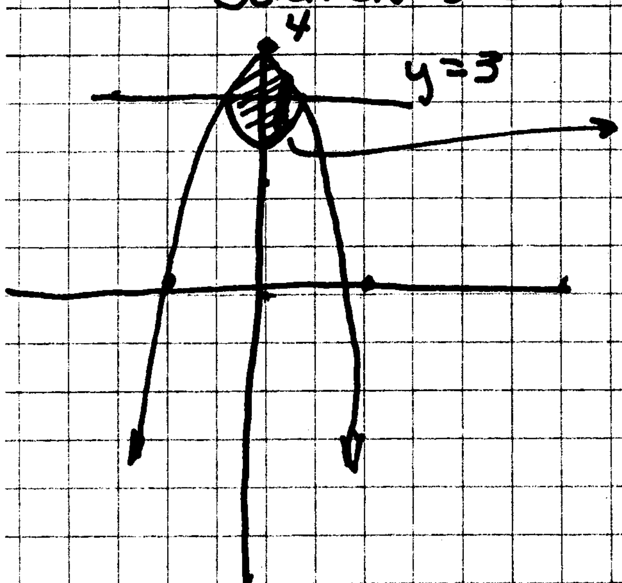
B. 2π

C. $16\pi/15$

D. $4\pi/3$

E. NOTA

Solution : C



$$R(x) = (4 - x^2 - 3)$$

$$V = \int_{-1}^{1} \pi R(x)^2\,dx = 2\int_0^1 \pi R(x)^2\,dx$$

$$V = 2\pi \int \left(1 - \frac{x^2}{2}\right) dx \Big| = \cancel{4/3}$$

$$(1 - x^2)^2$$

$$(1 - 2x^2 + x^4)$$

$$x - \tfrac{2}{3}x^3 + x^5/5 \Big|_0^1 = 2\pi\left(8/15\right)$$

$$1 - 2/3 + 1/5$$

INTEGRALS AND APPLICATIONS

24. A metal sleve is manufactured by drilling a hole radius 4 inches through a Sphere radius 5 inches. What is the Volume of the Sleve? ie the amount of material remaining after the hole is drilled?

A. $256\pi/3$ cubic inches

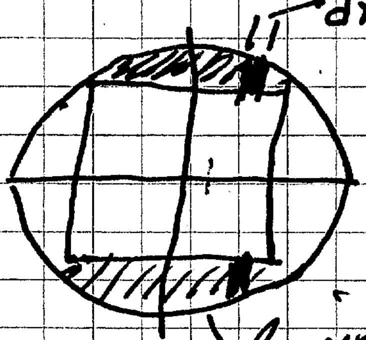
B. 36π cubic inches

C. 72π cubic inches

D. 200π cubic inches

E. NOTA

Solution: (B)



$$V = \pi \int_{-3}^{3} \left(R_{(x)}^2 - r_{(x)}^2 \right) dx$$

$$y = \sqrt{25 - x^2}$$

$$V = \pi 2 \int_0^3 25 - x^2 - 16$$

$$= 2\pi \left(9x - \frac{x^3}{3} \Big|_0^3 \right) \qquad \frac{4}{3}\pi r^3$$

$$= 2\pi (27 - 9) = 36\pi$$

Integrals and applications

25. Find the Surface area generated by revolving the line $f(x) = x^2$ on the interval $[0,1]$ about the y axis.

A $\pi/6\,[5^{3/2} - 1]$

B $4\pi/3$

C $\pi/8\,[5^{1/2} + 1]$

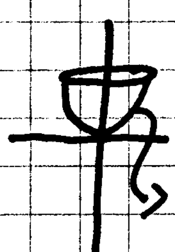
D $\pi/8\,[5^{1/2} - 1]$

E NOTA

Solution:

$$S = 2\pi \int_0^1 x \sqrt{1 + (2x)^2}\, dx$$

$$2\pi \int x (1 + 4x^2)^{1/2}\, dx = \frac{2\pi}{8} \int u^{1/2}\, du$$



Surface

$u = 1 + 4x^2$

$du = 8x\, dx$

$\frac{du}{8} = x\, dx$

$$\frac{\pi}{4} \frac{u^{3/2}}{3/2} = \frac{\pi}{6} (1 + 4x^2)^{3/2} \Big|_0^1$$

$$\frac{\pi}{6}\left[5^{3/2} - 1\right]$$

Integrals
and
applications

26. Find the center of mass of the lamina of uniform density ρ bounded by the graph of $y = 16 - x^2$ and the x axis.

A. $30/7$

B. 6

C. $32/7$

D. $44/5$

E. NOTA

Solution: E

$$\text{Mass} = \rho \int_{-4}^{4} (16 - x^2)\,dx = \rho\left[16x - \frac{x^3}{3}\right]_{-4}^{4}$$

$$\rho\, 2\left[64 - \frac{64}{3}\right] = \frac{128}{3} 2\rho$$

$$\text{Mass} = \frac{256}{3}\rho$$

$$M_x = \rho \int_{-4}^{4} \frac{16 - x^2}{2}\; 16 - x^2$$

$$= \rho \int_{0}^{4} (256 - 32x^2 + x^4)\,dx$$

$$\rho\left[256x - \frac{32x^3}{3} + \frac{x^5}{5}\right]_0^4$$

$$\left[1024 - \frac{2048}{3} + \frac{1024}{5}\right]_0^4 \rho = \frac{8192}{15}\rho$$

$$\bar{y} = \frac{\frac{8192}{15}\cancel{\rho}\;\frac{3}{256}}{\frac{256}{3}\cancel{\rho}} = \frac{32}{5}$$

INTEGRALS
and
applications

27. Evaluate

$$\int_0^1 \frac{x+2}{\sqrt{4-x^2}}\,dx.$$

A. This integral must be evaluated using an iterative numerical technique.

B. $2-\sqrt{3}+\pi/3$

C. $2+\sqrt{3}-\pi/3$

D. $-2-\sqrt{3}+\pi/3$

E. NOTA

Solution: (B)

$$\int_0^1 \frac{x}{\sqrt{4-x^2}}\,dx + \int_0^1 \frac{2}{\sqrt{4-x^2}}\,dx$$

$u = 4-x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$-\frac{du}{2} = x\,dx$

$$\int_0^1 \frac{1}{\sqrt{1-\frac{x^2}{4}}}\,dx$$

let

$u = \frac{x}{2}$

$du/dx = 1/2$

$2du = dx$

$$2\arcsin\frac{x}{2}\Big|_0^1$$

$$\int -\frac{u^{-1/2}}{2}\,du \quad + \quad \frac{\pi}{3} - 0$$

$-u^{1/2}$

$$-(4-x^2)^{1/2}\Big|_0^1 \qquad 2-\sqrt{3}+\pi/3$$

$-\sqrt{3}--2$

INTEGRALS AND APPLICATIONS

28. Evaluate

$$\int_0^1 \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$$

A. $e/4 - 1/2$

B. $e/2 + 1/2$

C. $e^2/2 + 1/2$

D. $e^2 - 1/2$

E. NOTA

Solution: A

Integrate by Parts

$$dv = \frac{x}{(x^2+1)^2} \Rightarrow v = \int (x^2+1)^{-2}\, x\,dx = -\frac{1}{2(x^2+1)}$$

$$u = x^2 e^{x^2} \Rightarrow du = 2xe^{x^2}(x^2+1)\,dx$$

$$\int \frac{x^3 e^{x^2}dx}{(x^2+1)^2} = -\frac{x^2 e^{x^2}}{2(x^2+1)} - \int -\frac{1}{\cancel{2}\cancel{(x^2+1)}}\,\cancel{2}x e^{x^2}\cancel{(x^2+1)}$$

$$+\int x e^{x^2}dx$$

$$\frac{-x^2 e^{x^2}}{2(x^2+1)} + \frac{e^{x^2}}{2}\Bigg|_0^1$$

$$\frac{-e}{4} + \frac{e}{2} - \left[\tfrac{1}{2}\right] = \frac{e}{4} - \tfrac{1}{2}$$

29. Evaluate

$$\int_0^{\pi/3} \sin^5 x \cos^3 x \, dx$$

A. $3/256$

B. $\pi/2$

C. $257/928$

D. $189/6144$

E. NOTA

Solution: D

$$\int \sin^5 x \cos^3 x \, dx =$$

$$\int \sin x (1-\cos^2 x)^2 \cos^3 x \, dx$$

$$\int \sin x (1 - 2\cos^2 x + \cos^4 x) \cos^3 x \, dx$$

$$\int [\cos^3 x \sin x - 2 \cos^5 x \sin x + \cos^7 x \sin x] \, dx$$

$$-\frac{\cos^4 x}{4} + \frac{\cos^6 x}{3} - \frac{\cos^8 x}{8} \Big|_0^{\pi/3}$$

$$-1/64 + \frac{1}{192} - \frac{1}{2048} - \left[-1/4 + 1/3 - 1/8\right]$$

$$-\frac{64}{6144} - \frac{3}{6144} + \frac{256}{6144} \quad -\frac{3}{192} + \frac{1}{192} \quad -2/192 - \frac{1}{2048} \qquad + 1/24 \qquad = 189/6144$$

Integrals and Applications

$$\int_1^2 \frac{\sqrt{4x^2+9}}{x^4}\,dx$$

A. $\frac{125}{216} - \frac{13^{3/2}}{27}$

B. $\frac{125}{216}$

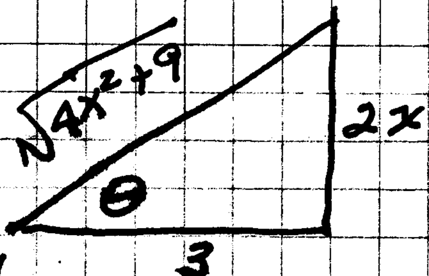
C. $\frac{250}{27} + \frac{13^{3/2}}{27}$

D. 13

E. NOTA

Solution: A

TRIG SUBSTITUTION



$$\sec\theta = \frac{\sqrt{4x^2+9}}{3}$$

$$3\sec\theta = \sqrt{4x^2+9}$$

$$\tan\theta = \frac{2x}{3}$$

$$\frac{3}{2}\tan\theta = x$$

$$dx = \frac{3}{2}\sec^2\theta\,d\theta$$

$$\int \frac{\sqrt{4x^2+9}}{x^4}\,dx = \int \frac{3\sec\theta \ \frac{3}{2}\sec^2\theta\,d\theta}{\left(\frac{3}{2}\right)^4 \tan^4\theta}$$

$$= \frac{8}{9}\int \frac{\cos\theta}{\sin^4\theta}\,d\theta = -\frac{8}{27\sin^3\theta} + C$$

$$= -\frac{(4x^2+9)^{3/2}}{27x^3}\Bigg|_1^2 = \frac{125}{216} - \frac{13^{3/2}}{27}$$

Integrals and applications

31. The $\lim_{x \to \infty} \frac{\sin x}{x}$ and $\lim_{x \to 0} \frac{\sin x}{x}$

Woud be ______ and ______ respectively.

A. 1, 0

B. 1, undefined

C. 0, 1

D. 0, undefined

E. NOTA

Solution: 0, 1

Integrals
and
applications

Evaluate

32. $$\int_2^4 \frac{3}{x^2+x-2}\,dx$$

A. 1

B. 2

C. ln 2

D. ln 12

E. NOTA

Solution:

$$\frac{3}{x^2+x-2} = \frac{1}{x-1} - \frac{1}{x+2}$$

$$\int \frac{3}{x^2+x-2}\,dx = \int \frac{dx}{x-1} - \int \frac{1}{x+2}\,dx$$

$$\ln|x-1| - \ln|x+2|$$

$$\ln\left|\frac{x-1}{x+2}\right|\Bigg|_2^4$$

$$\ln\left|\frac{3}{6}\right| - \ln\left|\frac{1}{4}\right|$$

$$= \ln\frac{3/6}{1/4} = \ln 2$$

Integrals and applications

33. $f(x) = e^{-x}\sin(\pi x)$

Find the area under the graph of $f(x)$ on the interval $[0, 1]$.

A. $\frac{1}{1+\pi}[e+1]$

B. $\frac{\pi}{1+\pi^2}\left[\frac{1}{e}+1\right]$

C. $\frac{1}{1+\pi}[e-1]$

D. $\frac{1}{1+\pi}[e^2-1]$

E. NOTA

Solution B

$u = e^{-x}$ $dv = \sin \pi x$

$du = -e^{-x}dx$ $v = -\frac{\cos \pi x}{\pi}$

$$-\frac{e^{-x}\cos \pi x}{\pi} - \int +\frac{\cos \pi x}{\pi}(+e^{-x})dx$$

$u = e^{-x}$ $dv = \frac{\cos \pi x}{\pi}$

$du = -e^{-x}dx$ $v = \frac{\sin \pi x}{\pi^2}$

$1 + 1/\pi^2$

$$-\frac{e^{-x}\cos \pi x}{\pi} - \left[e^{-x}\frac{\sin \pi x}{\pi^2} - \int -e^{-x}\frac{\sin \pi x}{\pi^2}\right]$$

$\int e^{-x}\sin \pi x$
$+\int e^{-x}\sin \pi x$

$$= \frac{-\frac{e^{-x}\cos \pi x}{\pi} - \frac{e^{-x}\sin \pi x}{\pi^2}}{1 + 1/\pi^2}\Bigg|_0^1 = \frac{\pi}{1+\pi^2}\left[\frac{1}{e}+1\right]$$