**Mu Alpha Theta National Convention**
**Mississippi State University 2002**
**Mu Level—Integration Topic Test**

1. $\int (x^3 + 3)dx =$

   a. $x^4 + 3x + C$
   b. $\frac{x^4}{4} + x^3 + C$
   c. $\frac{3x^4}{4} + 3x^3 + C$
   d. $\frac{x^4}{4} + 3x + C$
   e. NOTA

2. Solve $\frac{dy}{dt} = 3t^2$ given $y = -8$ when $t = 2$.

   a. $y = t^3 + 16$
   b. $y = t^3 - 16$
   c. $y = t^2 - 24$
   d. $y = 3t^2 - 8$
   e. NOTA

3. $\int_0^2 \sqrt[3]{x^2}\,dx =$

   a. $\frac{3}{5}x^{\frac{5}{3}} + C$
   b. $\frac{2}{3}x^{\frac{5}{3}} + C$
   c. $\frac{6\sqrt[3]{4}}{5}$
   d. $\frac{2\sqrt[3]{4}}{5}$
   e. NOTA

4. $\int_0^{\frac{\pi}{2}} (2\sin x + 3\cos x)dx =$

a. 5
b. 1
c. - 1
d. - 5
e. NOTA

5. Given $f''(x) = 2$, $f'(2) = 5$, $f(2) = 10$. Find $f(x)$.

a. $x^2 + x + 4$
b. $x^2$
c. $x^2 + 3x + 3$
d. $x^2 + 5x - 2$
e. NOTA

6. $\int_0^{\frac{\pi}{4}} \frac{\sin x}{1 - \sin^2 x} dx =$

a. $\sqrt{2} - 1$
b. $\sec x + C$
c. $2 - \sqrt{2}$
d. $\sqrt{2}$
e. NOTA

7. On the moon, the acceleration due to gravity is - 1.6 meters per second. A stone dropped from a cliff on the moon hits the surface of the moon in 20 seconds. How far did it fall? What was its velocity on impact?

a. -320 meters, 32 meters per second
b. 320 meters, - 32 meters per second
c. 320 meters, - 16 meters per second
d. 640 meters, 32 meters per second
e. NOTA

8. If $\int_0^4 x\,dx$ were to be evaluated using Riemann sums, how many partitions should be used to insure an accuracy of $\pm 0.001$?

   a. 8000
   b. 4000
   c. 2000
   d. 100
   e. NOTA

9. Given $\int_1^4 f(x)dx = \frac{26}{3}$, $\int_1^4 g(x)dx = 4$, $\int_1^4 h(x)dx = 2$, $E(x) = -f(x) + 4g(x) - 3h(x)$. Evaluate $\int_1^4 E(x)dx$.

   a. $-\frac{8}{3}$
   b. $-\frac{4}{3}$
   c. $\frac{4}{3}$
   d. $\frac{14}{3}$
   e. NOTA

10. Which of the following are false?

   I. $\int_a^b f(x)g(x)dx = \left[\int_a^b f(x)dx\right]\left[\int_a^b g(x)dx\right]$
   II. If the norm of a partition approaches zero, then the number of subintervals approaches infinity.
   III. If *f* is increasing on $[a,b]$, then the minimum value of *f(x)* on $[a,b]$ is *f(a)*.
   IV. The value of $\int_a^b f(x)dx$ must be positive.
   V. If $\int_a^b f(x)dx > 0$, then *f* is nonnegative for all *x* in $[a,b]$.

   a. II, III
   b. I, IV, V
   c. I, II, III
   d. I, II, IV, V
   e. NOTA

11. Find the average value of $f(x) = 3x^2 - x$ on the interval $[1,5]$.

a. 18.5
b. 28
c. 27.25
d. 112
e. NOTA

12. If $F(x) = \int_{\frac{\pi}{2}}^{x^4} (-\cos t)dt$, find $F'(x)$.

a. $-4\cos x^4$
b. $-4\cos x^4 - \frac{\pi}{2}$
c. $-4x^3 \cos x^4 - \frac{\pi}{2}$
d. $-4x^3 \cos x^4$
e. NOTA

13. Find the area of the region bounded by the graphs of the equations $y = x^3 + x$, $x = 2$, $y = 0$.

a. 2
b. 4
c. 6
d. 8
e. NOTA

14. $\int_0^{\frac{\sqrt{3}}{2}} 5x\sqrt{1-x^2}\,dx =$

a. $\frac{5\sqrt{3}}{8}$
b. $\frac{35}{24}$
c. $\frac{5}{6}$
d. $\frac{5}{3}$
e. NOTA

15. $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^2 x dx =$

a. $\csc x \cot x + C$
b. $\frac{\pi}{4} - 1$
c. $1 + \frac{\pi}{4}$
d. $1 + \frac{3\pi}{4}$
e. NOTA

16. $\int_0^{\frac{\pi}{3}} \tan x dx =$

a. $-\frac{1}{2}$
b. $-\ln\left(\frac{1}{2}\right)$
c. $-\ln 2$
d. $\ln\left(\frac{\sqrt{3}}{2}\right)$
e. NOTA

17. $\int \frac{2x}{(x-1)^2} dx =$

a. $2\ln|x-1| - \frac{2}{x-1} + C$
b. $\ln(x-1)^2 + \frac{2}{x-1} + C$
c. $\ln|x-1|^2 + \frac{1}{x-1} + C$
d. $2\ln|x-1| + C$
e. NOTA

18. $\int_0^1 \frac{x}{1+x^4}\,dx$

a. $\frac{\pi}{8}$
b. $\frac{1}{8}$
c. $\frac{\pi}{4}$
d. $\frac{\pi}{2}$
e. NOTA

19. Solve $\frac{dy}{dx} = \frac{2x}{x^2-9}$ given $y = 4$ when $x = 0$.

a. no solution
b. $y = \tan x - \ln|x^2 - 9| + 4$
c. $y = \ln|x^2 - 9| + 4 - \ln 9$
d. $y = \ln|x^2 - 9| + 4 + \ln(-9)$
e. NOTA

20. $\int_0^1 5xe^{-x^2}\,dx =$

a. $\frac{5}{2}(e+1)$
b. $\frac{5}{2}e(e-1)$
c. $\frac{5}{2}e(e+1)$
d. $5e^2$
e. NOTA

21. $\int_0^1 \frac{4}{1+x^2} dx =$

a. $\frac{\sqrt{2}}{2}$
b. $\frac{1}{2}$
c. $\frac{\pi}{2}$
d. $\pi$
e. NOTA

22. Find the area of the region bounded by the graphs of $y = x^2 + 4$, $y = -x$, $x = 0$, and $x = 2$.

a. $\frac{38}{3}$
b. $\frac{14}{3}$
c. $\frac{7}{3}$
d. $\frac{-10}{3}$
e. NOTA

23. Find the volume of the solid formed by revolving the region bounded by $f(x) = 4 - x^2$ and $g(x) = 3$ about the line $y = 3$.

a. $\frac{8\pi}{3}$
b. $2\pi$
c. $\frac{16\pi}{15}$
d. $\frac{4\pi}{3}$
e. NOTA

24. A metal sleeve is manufactured by drilling a hole of radius 4 inches through a sphere of radius 5 inches. What is the volume of the sleeve? That is, what is the amount of material remaining after the hole is drilled?

a. $\frac{256\pi}{3}$ cubic inches
b. $36\pi$ cubic inches
c. $72\pi$ cubic inches
d. $200\pi$ cubic inches
e. NOTA

25. Find the surface area generated by revolving $f(x) = x^2$ on the interval $[0,1]$ about the $y$ - axis.

a. $\frac{\pi}{6}\left(5^{\frac{3}{2}} - 1\right)$
b. $\frac{4\pi}{3}$
c. $\frac{\pi}{8}\left(5^{\frac{1}{2}} + 1\right)$
d. $\frac{\pi}{8}\left(5^{\frac{1}{2}} - 1\right)$
e. NOTA

26. Find the center of mass of the lamina of uniform density $\rho$ bounded by the graph of $y = 16 - x^2$ and the $x$ - axis.

a. $\frac{30}{7}$
b. 6
c. $\frac{32}{7}$
d. $\frac{44}{5}$
e. NOTA

27. $\int_0^1 \frac{x+2}{\sqrt{4-x^2}}dx =$

a. This integral must be evaluated using an iterative numerical technique.

b. $2-\sqrt{3}+\frac{\pi}{3}$

c. $2+\sqrt{3}-\frac{\pi}{3}$

d. $-2-\sqrt{3}+\frac{\pi}{3}$

e. NOTA

28. $\int_0^1 \frac{x^3 e^{x^2}}{\left(x^2+1\right)^2}dx =$

a. $\frac{e}{4}-\frac{1}{2}$

b. $\frac{e}{2}+\frac{1}{2}$

c. $\frac{e^2}{2}+\frac{1}{2}$

d. $e^2-\frac{1}{2}$

e. NOTA

29. $\int_0^{\frac{\pi}{3}} \sin^5 x \cos^3 x dx =$

a. $\frac{3}{256}$

b. $\frac{\pi}{2}$

c. $\frac{257}{928}$

d. $\frac{189}{6144}$

e. NOTA

30. $\int_1^2 \frac{\sqrt{4x^2+9}}{x^4}dx =$

a. $\frac{125}{216} - \frac{13^{\frac{3}{2}}}{27}$
b. $\frac{125}{216}$
c. $\frac{250}{27} + \frac{13^{\frac{3}{2}}}{27}$
d. 13
e. NOTA

31. The $\lim_{x\to\infty} \frac{\sin x}{x}$ and $\lim_{x\to 0} \frac{\sin x}{x}$ would be _____ and _____ respectively.

a. 1, 0
b. 1, undefined
c. 0, 1
d. 0, undefined
e. NOTA

32. $\int_2^4 \frac{3}{x^2+x-2}dx =$

a. 1
b. 2
c. ln 2
d. ln 12
e. NOTA

33. Find the area under the graph of $f(x) = e^{-x}\sin(\pi x)$ on the interval $[0,1]$.

a. $\frac{1}{1+\pi}(e+1)$
b. $\frac{\pi}{1+\pi^2}\left(\frac{1}{e}+1\right)$
c. $\frac{1}{1+\pi}(e-1)$
d. $\frac{1}{1+\pi}\left(e^2-1\right)$
e. NOTA

**Mu Division—Integration Topic Test**
**Answer Key**

1. d
2. b
3. c
4. a
5. a
6. a
7. b
8. a
9. c
10. b
11. b
12. d
13. c
14. b
15. e
16. c
17. a
18. a
19. c
20. b
21. d
22. a
23. c
24. b
25. a
26. e
27. b
28. a
29. d
30. a
31. c
32. c
33. b