2002 National Mu Alpha Theta Convention
Mu Division---Limits Topic Test

1. Evaluate: $\lim_{x\to\pi}\dfrac{\tan^2 x}{1+\sec(x)}$

   A. –2 B. 0 C. 1 D. 2 E. undefined

2. Determine the value of (x,y), (if any) so that $f(x)=\dfrac{x^2+5x+6}{x+2}$ is continuous at x = -2 .

   A. no value B. (-2, 3) C. (-2, 1) D. (-2, -3) E. NOTA

3. Find the radius of convergence of $\sum_{n=1}^{\infty}\dfrac{n!}{n^n}x^n$

   A. π B. 1 C. ln (2) D. e E. π/4

4. Find: $\lim_{x\to1}\dfrac{\ln x - x + 1}{x^3-3x+2}$

   A. no limit B. –2 C. $\dfrac{-1}{6}$ D. 1 E. ln(2)

5. Find: $\lim_{x\to8}\dfrac{x^3-512}{x-8}$

   A. 128 B. 192 C. 64 D. 0 E. NOTA

6. Find: $\lim_{x\to3}\dfrac{x^3-3x^2-9x+27}{-3x^2+18x-27}$

   A. $\dfrac{-7}{3}$ B. –2 C. $\dfrac{-4}{3}$ D. $\dfrac{-5}{3}$ E. NOTA

7. Find the limit as x approaches –5 of the average rate of change of the function $f(x)=3x^2-65x$ between -5 and x.

   A. –80 B. –95 C. 310 D. 400 E. NOTA

8. Determine where $f(x)=\dfrac{x+5}{x^2+5x+4}$ is continuous.

A. all x, except x = 5, 4, 1
B. all x, except x = -5, -4, -1
C. all x, except x = -4, -1
D. all x, except x = 4, 1
E. NOTA

9. Determine whether f is continuous at x = 3.

$$f(x)=\begin{cases}\dfrac{125x^3-27}{25x^2-9} & \text{if } x<3\\ \dfrac{31}{2} & \text{if } x=3\\ \dfrac{1}{2}x+14 & \text{if } x>3\end{cases}$$

A. f is continuous at x = 3.
B. f is not continuous at x =3, since $\lim\limits_{x\to 3} f(x)$ does not exist
C. f is not continuous at x = 3, since f(3) is not defined
D. f is not oontinuous at x = 3, since $\lim\limits_{x\to 3} f(x)\neq f(3)$
E. NOTA

10. Find: $\lim\limits_{x\to 0}\dfrac{x}{\tan x}$

A. 0 B. $\dfrac{\pi}{4}$ C. 1 D. Does not exist E. NOTA

11. Which of the following statements is *not* true of $f(x)=\sqrt{x^2-25}$

A. f is continuous at x = 10
B. f is continuous on the interval $(-\infty,-5)$
C. f is continuous on the interval $(5,\infty)$
D. f is continuous on the interval (-5, 5)
E. NOTA

12. Find the limit: $\lim_{x\to 1^-} \frac{-2}{x-1}$ **(limit as x approaches 1 from the left)**

A. ∞ B. -∞ C. 0 D. Limit does not exist E. NOTA

13. Given: $f(x) = \frac{4}{(x-3)(5-x)}$

f(x) decreases without bound as *x* approaches what value from the right?

A. 5 B. –3 C. –5 D. 3 E. NOTA

14. Find the limit: $\lim_{x\to\infty} \frac{a-bx^4}{cx^4+x^2}$

A. 0 B. ∞ C. $\frac{-b}{c}$ D. $\frac{a}{c}$ E. NOTA

15. Find $\lim_{x\to\infty} \frac{\sqrt{4x^2-1}}{x^2}$

A. ∞ B. 2 C. 4 D. 1 E. NOTA

16. Let $s(n) = \sum_{i=1}^{n} \left(1+\frac{i}{n}\right)^2 \left(\frac{2}{n}\right)$. Find the limit of $s(n)$ as $n \to \infty$.

A. $\frac{17}{12}$ B. $\frac{10}{3}$ C. $\frac{14}{3}$ D. $\frac{20}{3}$ E. NOTA

17. Given: $\lim_{x\to 2}(2x-1) = L$. Find δ such that

$|(2x-1)-L| < 0.01$ whenever $0 < |x-2| < \delta$

A. 3 B. 0.05 C. 0.03 D. 0.005 E. NOTA

18. Use the graph to evaluate the limit: $\lim_{x\to 2^+} \left[\ln \frac{1}{x^2+x-6}\right]$

A. ∞ B. $-\infty$ C. 0 D. 1 E. NOTA

19. If the trigonometric substitution in the variable θ is used to solve $\int_{1.25}^{2.5} \sqrt{25-4x^2}\ dx$, determine the lower and upper limits of integration for θ.

A. arcsin 1.25, arcsin 2.5

B. $\frac{\pi}{4}, \quad \pi$

C. $\frac{\pi}{6}, \quad \frac{\pi}{2}$

D. $\frac{\pi}{4}, \quad \arctan\frac{1}{2}$

E. NOTA

20. Find $\lim_{x\to 0}\left(\frac{1}{x}\right)^x$

A. e B. 0 C. 1 D. ∞ E. NOTA

21. Find $\lim_{x\to\infty} e^{-x}\ln x$

A. 0 B. ∞ C. 1 D. Limit does not exist E. NOTA

22. Determine if the following sequence converges or diverges. If the sequence converges, find its limit.

$\left\{\frac{n!}{(n-2)!}\right\}, \quad n = 2,3,4,...$

A. Converges to 1 B. Converges to 0 C. Converges to –2
D. Diverges E. NOTA

23. Find $\lim_{\Delta x\to 0}\frac{f(x+\Delta x, y)-f(x,y)}{\Delta x}$ for $f(x,y) = x^2 + y^2 - 2x$

A. $2x+\Delta x-2$ B. $2x-2$ C. $2x+2y-2$ D. 1 E. NOTA

24. Evaluate the limit: $\lim_{t\to 3}\left(\frac{t-3}{t^2+9}i+\frac{t+3}{t^2+9}j+\frac{t-3}{t^2-9}k\right)$

A. $\frac{-1}{6}i+\frac{1}{3}j+\frac{1}{6}k$ B. $\frac{1}{3}j$ C. 0 D. $\frac{1}{3}j+\frac{1}{6}k$

E.NOTA

25. For the function f(x) defined below, find all point(s) for which only the left-hand limit exists.

$$f(x)=\begin{cases}\sqrt{1-x^2} & \text{if } 0\le x<1\\ 1 & \text{if } 1\le x<2\\ 2 & \text{if } x=2\end{cases}$$

A. x = 1 only B. x = 2 only C. x = 0, 1, 2 D. x = 0, 2 only
E. NOTA

26. Determine if the following integral converges or diverges: $\int_0^3 \frac{dx}{(x-1)^{\frac{2}{3}}}$

A. converges to $3\sqrt[3]{2}$ B. diverges C. converges to $0.6\left(2^{\frac{5}{3}}\right)-1$

D. converges to $3\sqrt[3]{2}+3$ E. NOTA

27. Let $f(x)=\begin{cases}\frac{x^2+2x-15}{x-3}, & x\neq 3\\ k^3-1, & x=3\end{cases}$

Find *k* so that *f* is continuous at x = 3.

A. 3 B. 8 C. 2 D. $\sqrt[3]{9}$ E. NOTA

28. $f(x)=\left(2^x+3^x\right)^{\frac{1}{x}}$. Determine $\lim_{x\to 0^-} f(x)$, $\lim_{x\to 0^+} f(x)$, and $\lim_{x\to 0} f(x)$

A. 0, ∞, does not exist B. $-\infty, \infty$, does not exist C. 0, 0, 0

D. ∞, ∞, ∞ E. NOTA

29. Evaluate $\lim_{x\to 0} \frac{\tan 3x}{\sin 5x}$

A. does not exist B. 3/5 C. 0 D. 1/5 E. NOTA

30. Find an interval in $\left[-\frac{\pi}{2}, 0\right]$ for which $y = \cos x$ lies within 0.2 units of $y_0 = 0.4$

Round interval values to nearest hundredths.

A. (.93, 1.37) B. (.83, .98) C. (4.91, 5.35) D. (.2, .6) E. NOTA

Mu Division—LIMITS TOPIC TEST

Answer Key

1. **B**
2. **C**
3. **D**
4. **C**
5. **B**
6. **B**
7. **B**
8. **C**
9. **A**
10. **C**
11. **D**
12. **A**
13. **A**
14. **C**
15. **E**
16. **C**
17. **D**
18. **A**
19. **C**
20. **C**
21. **A**
22. **D**
23. **B**
24. **D**
25. **B**
26. **D**
27. **D**
28. **A**
29. **B**
30. **E**