2002 National Mu Alpha Theta Convention
Sequences and Series Topic Test – Mu Division

1. Evaluate the sum $\sum_{k=1}^{n} k(k+1)$ for $n \ge 2$.

   (A) $\frac{n(n^2+1)}{3}$ (B) $\frac{n(n+1)(n+2)}{3}$ (C) $\frac{2n(n^2-1)}{3}$ (D) $\frac{n^2(n+1)}{2}$
   (E) NOTA

2. Evaluate the infinite series $\sum_{k=1}^{\infty}(\frac{k}{k+1}-\frac{k+1}{k+2})$.

   (A) divergent (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) NOTA

3. Evaluate the product $\prod_{k=2}^{n}(1-\frac{1}{k^2})$.

   (A) $\frac{2n-1}{2n}$ (B) $\frac{n+1}{2n}$ (C) $\frac{2n-1}{n^2}$ (D) $\frac{n^2-1}{4}$ (E) NOTA

4. Find the limit of $s_n = \frac{1}{n^2}+\frac{2}{n^2}+\cdots+\frac{n}{n^2}$ as $n \to \infty$. $n = 1,2,3,\cdots$.

   (A) 2 (B) 0 (C) diverges (D) $\frac{1}{2}$ (E) NOTA

5. Find the limit of $s_n = 1-\frac{1}{2}+\frac{1}{4}-\cdots+(-\frac{1}{2})^n$ as $n \to \infty$. $n = 1,2,3,\cdots$.

   (A) $\frac{2}{3}$ (B) 2 (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) NOTA

6. Evaluate the limit, $\lim_{n\to\infty}\sqrt{n}\left(\sqrt{n+1}-\sqrt{n}\right)$

   (A) diverges (B) 2 (C) 0 (D) $\frac{1}{2}$ (E) NOTA

7. Evaluate : $\lim_{n\to\infty} \dfrac{1-\left(1-\frac{1}{n}\right)^3}{1-\left(1-\frac{1}{n}\right)}$.

(A) 0 (B) undefined (C) $\frac{1}{3}$ (D) 3 (E) NOTA

8. The series $\sum_{n=0}^{\infty}\left(k^2-3\right)^n$ converges for which values of k?

(A) -1< k < 1 (B) $-2<k<-\sqrt{2}$ or $\sqrt{2}<k<2$ (C) $-\sqrt{2}<k<\sqrt{2}$

(D) $k<-2$ or $k>2$ (E) NOTA

9. The Fibonacci sequence satisfies the recurrence relation $F_k = F_{k-1} + F_{k-2}$, for all integers $k \geq 2$, with $F_0 = 1$ and $F_1 = 1$. Evaluate the $\lim_{k\to\infty}\dfrac{F_{k+1}}{F_k}$, assuming this limit exits.

(A) $\frac{1-\sqrt{5}}{2}$ (B) 1 (C) $\frac{\sqrt{5}}{2}$ (D) $\frac{1+\sqrt{5}}{2}$ (E) NOTA

10. Which fraction represents the repeating decimal $0.321321\ldots$?

(A) $\frac{999}{321}$ (B) $\frac{321}{99}$ (C) $\frac{321}{999}$ (D) $\frac{1000}{321}$ (E) NOTA

11. Evaluate: $\sum_{k=1}^{\infty}\dfrac{(-x)^k}{k!}$..

(A) e^x (B) e^{-x} (C) $1+e^x$ (D) e^x+1 (E) NOTA

12. Which of the following is true of the series $\sum_{n=1}^{\infty}\dfrac{(-1)^n}{1+5n}$?

(A) absolutely convergent (B) divergent (C) conditionally convergent

(D) almost convergent (E) NOTA

13. Evaluate: $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} 2^{2n-2}}{(2n-2)!}$.

(A) $\cos(\sqrt{2})$ (B) $\sin(\sqrt{2})$ (C) $\cos(2)$ (D) $\ln(2)$ (E) NOTA

14. If $\sum_{j=0}^{\infty} b_j$ is a convergent series of nonnegative terms and there are constants M and J such that $|a_j| \le Mb_j$ for $j \ge J$, then which of the following statements describes the convergence of the series $\sum_{j=0}^{\infty} a_j$:

(A) conditionally convergent (B) uniformly convergent (C) absolutely convergent (D) divergent (E) NOTA

15. Evaluate: $\sum_{k=3}^{\infty} \left[\sin\left(\frac{4}{k}\right) - \sin\left(\frac{4}{k+2}\right) \right]$.

(A) $\sin\left(\frac{4}{3}\right)$ (B) 0 (C) $\sin\left(\frac{4}{3}\right) + \sin(1)$ (D) $\sin(1)$ (E) NOTA

16. Evaluate the limit $\lim_{n \to \infty} \left\{ \frac{\arctan(n^2)}{n^2 + 1} \right\}^n$.

(A) π (B) 0 (C) $\frac{\pi}{2}$ (D) e (E) NOTA

17. Evaluate: $\sum_{j=1}^{\infty} \frac{1}{j(j+1)}$.

(A) 2 (B) 1 (C) $\frac{1}{2}$ (D) $\frac{2}{3}$ (E) NOTA

18. The 10^{th} term of an arithmetic sequence is 52 and the 15^{th} is 77. Find the 50^{th} term of this sequence.

(A) 252 (B) 250 (C) 302 (D) –48 (E) NOTA

19. The Maclaurin series for $e^x + e^{3x}$ is

(A) $\sum_{j=0}^{\infty} \frac{3^j}{j!} x^j$ (B) $\sum_{j=0}^{\infty} \frac{(1+3^j)}{j!} x^j$ (C) $\sum_{j=0}^{\infty} \frac{j!}{(1+3^j)} x^j$ (D) $\sum_{j=1}^{\infty} \frac{(1+3^j)}{j!} x^{j-1}$

(E) NOTA

20. Evaluating the improper integral $\lim_{b\to\infty} \int_2^b \frac{1}{x\ln(x)} dx$ shows that $\sum_{j=2}^{\infty} \frac{(-1)^j}{j\ln(j)}$

(A) sums to $\ln(\ln(2))$. (B) converges absolutely. (C) diverges

(D) converges conditionally. (E) NOTA

21. Applying the alternating series test to $\sum_{j=2}^{\infty} \frac{(-1)^j}{j\ln j}$ shows this series

(A) converges absolutely. (B) converges. (C) diverges.
(D) does not converge absolutely. (E) NOTA.

22. Evaluate $\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$.

(A) e (B) e^2 (C) $2e$ (D) e^{-2} (E) NOTA

23. Evaluate $\sum_{n=2}^{\infty} \frac{(-1)^{2n+1}}{n2^n}$.

(A) $\ln(2)$ (B) $-\ln(2)$ (C) 0 (D) $\ln(3)$ (E) NOTA

24. Which of the following describes the convergence of $\sum_{k=0}^{\infty} \frac{(-2)^k}{k^3}$.

(B) converges (B) converges absolutely (C) converges conditionally

(D) diverges (E) NOTA

25. Evaluate $\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!}$.

(A) 0 (B) $\sin(1)$ (C) $\cos(1)$ (D) diverges (E) NOTA

26. The interval of convergence of $\sum_{k=1}^{\infty} \frac{1}{k}\left(\frac{x}{3}\right)^k$ is

(A) $[-3,3)$ (B) $\left[-\frac{1}{3},\frac{1}{3}\right)$ (C) $(-3,3]$ (D) $[-1,1)$ (E) NOTA

27. What are the terms up to degree 4 in the Maclaurin series of $\frac{\sin(x)}{1-x}$?

(A) $x - x^2 - \frac{7}{6}x^3 - \frac{5}{6}x^4$ (B) $x - x^2 - \frac{x^3}{6} + \frac{x^4}{6}$ (C) $x + x^2 + \frac{5}{6}x^3 + \frac{5}{6}x^4$

(D) $x + x^2 + \frac{x^3}{6} + \frac{x^4}{6}$ (E) NOTA

28. At the first of each month, for ten years, \$1,000 is deposited into a saving account earning 6% a year compounded monthly. How much money is in this account when the last deposit is made? Round to nearest dollar.

(A) \$6,958,240 (B) \$165,699 (C) \$48,000 (D) \$163,879 (E) NOTA

29. Which is true of the series $\sum_{k=1}^{\infty} \frac{1}{k^{3/2}}$?

(A) it converges. (B) its sum is $\frac{\pi}{3}$. (C) it is conditionally convergent

(D) it is divergent. (E) NOTA

30. Which is true for the sequence $\left\{\frac{5}{3+(-1)^n}\right\}$?

(A) it converges to $\frac{5}{3}$. (B) it is unbounded. (C) it is divergent by oscillation.

(D) it converges to $\frac{15}{4}$ (E) NOTA.

WORK THE TIEBREAKER IN THE WHITE PORTION ON THE BACK OF THE SCANTRON SHEET.

TIEBREAKER: Find all values of a for which the series $\sum_{v=1}^{\infty} a^{\ln(v)}$ converges.