Mu Alpha Theta National Convention
Mississippi State University 2002
Mu Division--Vector Calculus Topic Test

1. The vector **V** has a length of 4 and makes an angle of $\frac{\pi}{6}$ with the positive x - axis. Write **V** as a linear combination of the unit vectors **i** and **j**.

   a. $\mathbf{V} = \frac{4\pi}{3}\mathbf{i} + \frac{\pi}{2}\mathbf{j}$
   b. $\mathbf{V} = -2\sqrt{3}\mathbf{i} - \mathbf{j}$
   c. $\mathbf{V} = 2\mathbf{i} - 2\sqrt{3}\mathbf{j}$
   d. $\mathbf{V} = 2\sqrt{3}\mathbf{i} + 2\mathbf{j}$
   e. NOTA

2. Find the magnitude of $\mathbf{V} = 2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$.

   a. 5
   b. 7
   c. 9
   d. $\sqrt{31}$
   e. NOTA

3. Find a unit vector normal to $f(x) = x^3 + 3$ at $(1,1)$.

   a. $\pm\frac{1}{\sqrt{10}}(3\mathbf{i} - \mathbf{j})$
   b. $\pm\frac{1}{\sqrt{10}}(3\mathbf{i} + \mathbf{j})$
   c. $\pm\frac{1}{\sqrt{10}}(\mathbf{i} - 3\mathbf{j})$
   d. $\pm\frac{1}{\sqrt{10}}(\mathbf{i} + 3\mathbf{j})$
   e. NOTA

4. Find the direction cosines of the vector $\mathbf{V} = 3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$.

   a. $\frac{3}{5}, \frac{4}{5}, 1$
   b. $\frac{9}{5\sqrt{2}}, \frac{16}{5\sqrt{2}}, \frac{25}{\sqrt{2}}$
   c. $\frac{3}{5\sqrt{2}}, \frac{4}{5\sqrt{2}}, \frac{1}{\sqrt{2}}$
   d. $\frac{1}{\sqrt{5}}, \frac{3}{\sqrt{5}}, \frac{4}{\sqrt{5}}$
   e. NOTA

5. A particle traveled from $A = (6,2,9)$ to $B = (126,26,153)$ in 24 seconds with a constant velocity. Find its position after 3 seconds from the start.

   a. $[21,5,27]$
   b. $\left[7,\frac{5}{3},9\right]$
   c. $[18,4,25]$
   d. $[28,12,34]$
   e. NOTA

6. The length of **u** is 4 and the length of **v** is 9. The angle between **u** and **v** is 60°. Find the length of **u** + **v**.

   a. $\sqrt{87}$
   b. $\sqrt{113}$
   c. $\sqrt{133}$
   d. $\sqrt{155}$
   e. NOTA

7. Vector $\mathbf{V} = [-72,54]$ is rotated counter clockwise 30°. Find the rotated vector.

   a. $[36 - 27\sqrt{3}, -36\sqrt{3} - 27]$
   b. $[36\sqrt{3} - 27, 36 - 27\sqrt{3}]$
   c. $[-36\sqrt{3} + 27, -36 - 27\sqrt{3}]$
   d. $[-36\sqrt{3} - 27, -36 + 27\sqrt{3}]$
   e. NOTA

8. Vector $\mathbf{u} = [4,1]$ is rotated counter clockwise until it points in the direction of $\mathbf{v} = [-3,8]$. Find the angle of rotation θ.

a. $\theta = \cos^{-1}\left(\frac{-4}{\sqrt{225}}\right)$
b. $\theta = 92^{\circ}$
c. $\theta = \cos^{-1}\left(\frac{-4}{\sqrt{17}+\sqrt{73}}\right)$
d. $\theta = \cos^{-1}\left(\frac{-4}{\sqrt{1241}}\right)$
e. NOTA

9. Find the projection of $\mathbf{A} = 3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ onto $\mathbf{B} = 6\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$ and the component of **A** orthogonal to **B**.

a. $\left\langle \frac{129}{35}, \frac{24}{7}, \frac{-58}{12} \right\rangle, \left\langle \frac{-24}{35}, \frac{4}{7}, \frac{-12}{35} \right\rangle$
b. $\left\langle \frac{3}{5}, \frac{4}{5}, \frac{-2}{5} \right\rangle, \left\langle \frac{6}{5}, -1, \frac{3}{5} \right\rangle$
c. $\left\langle \frac{129}{35}, \frac{24}{7}, \frac{-58}{12} \right\rangle, \langle 9, -1, 1 \rangle$
d. $\left\langle \frac{-24}{35}, \frac{4}{7}, \frac{-12}{35} \right\rangle, \left\langle \frac{129}{35}, \frac{24}{7}, \frac{-58}{35} \right\rangle$
e. NOTA

10. If $(\mathbf{u} \times \mathbf{v}) = [3,-8,3]$, find $[-1\mathbf{u} + 3\mathbf{v}] \times [\mathbf{u} + 3\mathbf{v}]$.

a. $[6,-16,6]$
b. $[-18,48,-18]$
c. This problem cannot be work with the information provided.
d. $[-36,96,-36]$
e. NOTA

11. Find the area of a triangle with vertices at $P(-9,7,2)$, $Q(6,-5,7)$, $R(3,1,6)$.

a. $\sqrt{3240}$
b. $\dfrac{\sqrt{3240}}{2}$
c. $\dfrac{\sqrt{3240}}{4}$
d. $\dfrac{\sqrt{2916}}{2}$
e. NOTA

12. Find the volume of a parallelpiped having $\mathbf{A} = 3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$, $\mathbf{B} = 2\mathbf{j} - 2\mathbf{k}$, $\mathbf{C} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$ as adjacent edges.

a. 20
b. 24
c. 30
d. 36
e. NOTA

13. Given $\mathbf{s}(t) = \frac{1}{t}\mathbf{i} - \mathbf{j} + (\ln t)\mathbf{k}$ and $\mathbf{u}(t) = t^2\mathbf{i} - 2t\mathbf{j} + \mathbf{k}$. Find $D_t[\mathbf{s}(t) \cdot \mathbf{u}(t)]$.

a. $t^2\mathbf{i} - 2t\mathbf{j} + 3\mathbf{k}$
b. $3 + \dfrac{1}{t}$
c. $t^2 + 2t + 2$
d. $1 + \dfrac{1}{2t^2}$
e. NOTA

14. Given $\mathbf{V}(t) = t^2\mathbf{i} - 2t\mathbf{j} + \mathbf{k}$. Find $D_t[\mathbf{V}(t) \times \mathbf{V}'(t)]$

a. $2\mathbf{j} + 4t\mathbf{k}$
b. $2\mathbf{i} - 2\mathbf{j} - 4t\mathbf{k}$
c. $2t\mathbf{i} - 2t\mathbf{j} + 4t\mathbf{k}$
d. $8t^2\mathbf{i} - 4t\mathbf{j} + 5t\mathbf{k}$
e. NOTA

15. Given $\mathbf{r}'(t) = 4\cos 2t\mathbf{i} - 4\cos t\mathbf{j} + \dfrac{1}{1+t^2}\mathbf{k}$ and $\mathbf{r}(0) = 2\mathbf{i} - 4\mathbf{j} + \mathbf{k}$. Find **r***(t)*.

a. $\mathbf{r}(t) = \left(\dfrac{\cos 4t}{2} + 2\right)\mathbf{i} - (\sin 4t + 4)\mathbf{j} + (\arccos t + 1)\mathbf{k}$
b. $\mathbf{r}(t) = \left(\dfrac{\sin 8t}{2} + 2\right)\mathbf{i} - (\sin 4t + 4)\mathbf{j} + (arc\sec t + 1)\mathbf{k}$
c. $\mathbf{r}(t) = (2\sin 2t + 2)\mathbf{i} - (4\sin t + 4)\mathbf{j} + (\arctan t + 1)\mathbf{k}$
d. $\mathbf{r}(t) = (2\sin 2t + 2)\mathbf{i} - 4(\sin t - 1)\mathbf{j} + (arc\cot + 1)\mathbf{k}$
e. NOTA

16. A particle moves along a plane curve described by $\mathbf{P}(t) = 4\sin\dfrac{t}{4}\mathbf{i} + 4\cos\dfrac{t}{4}\mathbf{j}$. Find the speed of the particle at any time *t*.

a. $\cos\dfrac{t}{4}\mathbf{i} - \sin\dfrac{t}{4}\mathbf{j}$
b. $\dfrac{1}{4}\sin\dfrac{t}{4}\mathbf{i} - \dfrac{1}{4}\cos\dfrac{t}{4}\mathbf{j}$
c. $\dfrac{1}{4}\cos\dfrac{t}{4}\mathbf{i} - \dfrac{1}{4}\sin\dfrac{t}{4}\mathbf{j}$
d. 1
e. NOTA

17. Find the unit tangent vector **T** to the curve $y^3x^2 + 2x + 3y + xy = -3$ at the point $P(1,-1)$ pointing up. That is, the *y* component of **T** is positive.

a. $\dfrac{1}{\sqrt{50}}[-7,1]$
b. $\dfrac{1}{\sqrt{50}}[7,1]$
c. $\dfrac{1}{\sqrt{50}}[-1,7]$
d. $\dfrac{1}{\sqrt{50}}[1,7]$
e. NOTA

18. Find the line integral of $(2x+2y+1)dx+(2x+4)dy$ along the curve C given by

$$\begin{cases} x(t)=t \\ y(t)=t^2 \end{cases} \text{ for } 0 \le t \le 2.$$

a. 38
b. 36
c. 21
d. 10
e. NOTA

19. Let $\Phi(x,y)$ be the potential of the vector field $\mathbf{F}(x,y)=[3y-4,3x+2]$ so that $\Phi(0,0)=2$. Find $\Phi(1,2)$.

a. 2
b. 4
c. 6
d. 8
e. NOTA

20. Which of the following vector fields is/are conservative?

a. $\mathbf{F}(x,y)=2xy\mathbf{i}+x^2\mathbf{j}$
b. $\mathbf{F}(x,y)=xe^{x^2y}(2y\mathbf{i}+x\mathbf{j})$
c. $\mathbf{F}(x,y)=\dfrac{x\mathbf{i}+y\mathbf{j}}{x^2+y^2}$
d. All are conservative.
e. NOTA

21. Find the curl of the vector field $\mathbf{F}(x,y,z)=e^x \sin y\mathbf{i}-e^x \cos y\mathbf{j}$ at the point $P(0,0,3)$.

a. $2\mathbf{i}-2\mathbf{j}$
b. $-2\mathbf{k}$
c. $2\mathbf{i}-2\mathbf{j}-2\mathbf{k}$
d. $2\mathbf{j}-2\mathbf{k}$
e. NOTA

22. Find the conservative vector field for the potential function $f(x,y,z)=\frac{y}{z}+\frac{z}{x}-\frac{xz}{y}$.

a. $\mathbf{F}(x,y,z)=\left(\frac{-z}{x^2}-\frac{z}{y}\right)\mathbf{i}+\left(\frac{1}{z}+\frac{xz}{y^2}\right)\mathbf{j}+\left(\frac{-y}{z^2}+\frac{1}{x}-\frac{x}{y}\right)\mathbf{k}$

b. $\mathbf{F}(x,y,z)=\left(\frac{-z}{x^2}\right)\mathbf{i}+\left(\frac{xz}{y^2}\right)\mathbf{j}+\left(\frac{-y}{z^2}\right)\mathbf{k}$

c. $\mathbf{F}(x,y,z)=\left(\frac{-z}{x^2}-\frac{z}{y}+\frac{y}{z}\right)\mathbf{i}+\left(\frac{1}{z^2}+\frac{xz}{y^2}-xz\right)\mathbf{j}+\left(\frac{-y}{z^2}+\frac{1}{x^2}-\frac{x}{y}\right)\mathbf{k}$

d. $\mathbf{F}(x,y,z)=\left(\frac{-z}{x^2}+\frac{z}{y}\right)\mathbf{i}+\left(-\frac{1}{z}-\frac{xz}{y^2}\right)\mathbf{j}+\left(\frac{y}{z^2}+\frac{1}{x}\right)\mathbf{k}$

e. NOTA

23. Find the work done by the vector field $\mathbf{F}(x,y)=-y\mathbf{i}+x\mathbf{j}$ on the particle moving counterclockwise around the ellipse $5x^2+4y^2=2$.

a. 0

b. $\frac{2\sqrt{5}\pi}{5}$

c. $\frac{5\sqrt{2}\pi}{2}$

d. $\frac{2\pi}{5}$

e. NOTA

24. Find the flux of $\mathbf{F}(x,y,z)=(x+y)\mathbf{i}+y\mathbf{j}+z\mathbf{k}$ over the closed surface S: $z=1-x^2-y^2, z=0$. Let N be the outward unit normal vector of the surface.

a. $\frac{3\pi}{4}$

b. $\frac{\pi}{2}$

c. $\frac{3\pi}{2}$

d. $\frac{3\pi}{5}$

e. NOTA

25. Let $\mathbf{F}(x,y) = \left[7x^3 + 9xy, 8x + 8y^3\right]$. Find $\int_C F \cdot n ds$, where C is the square with vertices $(1,-1)$, $(1,1)$, $(-1,1)$, $(-1,-1)$.

a. 60
b. 50
c. 30
d. 20
e. NOTA

26. Let $\mathbf{F}(x,y) = \left[4 + 9(x-4) + 5(y-8), 6 + 3(x-4) + (y-8)\right]$. Find $\int_C F \cdot n ds$, where C is the circle of radius 7 and center $(4,8)$.

a. 50π
b. 226π
c. 490π
d. 500π
e. NOTA

27. Find the outward flux of $\mathbf{F}(x,y,z) = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ through the surface of the solid bounded by the graphs of the equations. The surface is S: $x^2 + y^2 + z^2 = 4$.

a. 4π
b. 8π
c. 16π
d. 32π
e. NOTA

28. Given the field $\mathbf{F}(x,y,z) = z^2\mathbf{i} + x^2\mathbf{j} + y^2\mathbf{k}$ and surface, S: $z = 4 - x^2 - y^2, z \geq 0$. Evaluate $\int_C F \cdot dr$.

a. 8π
b. 6π
c. 0
d. -2π
e. NOTA

29. Given the position function $\mathbf{r}(t) = a\cos wt\mathbf{i} + b\sin wt\mathbf{j}$. Find the normal component of acceleration at $t = 0$.

a. abw^2
b. abw^3
c. bw^2
d. aw^2
e. NOTA

30. Find the equation of a tangent vector to the helix $\mathbf{r}(t) = 2\cos t\mathbf{i} + 2\sin t\mathbf{j} + t\mathbf{k}$.

a. $-2\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$
b. $2\sin t\mathbf{i} + 2\cos t\mathbf{j}$
c. $2\sin t\mathbf{i} + 2\cos t\mathbf{j} + \mathbf{k}$
d. $-2\sin t\mathbf{i} - 2\cos t\mathbf{j} - \mathbf{k}$
e. NOTA

31. Evaluate the definite integral $\int_0^{\frac{\pi}{2}} \left[a\cos t\mathbf{i} + a\sin t\mathbf{j} + \mathbf{k}\right] dt$.

a. $a\mathbf{i} + a\mathbf{j} + \frac{\pi}{2}\mathbf{k}$
b. $-a\mathbf{i} - a\mathbf{j} + \frac{\pi}{2}\mathbf{k}$
c. $-a\mathbf{i} + a\mathbf{j} + \mathbf{k}$
d. $a\mathbf{i} - a\mathbf{j} + \frac{\pi}{2}\mathbf{k}$
e. NOTA

32. Find the directional derivative of $f(x,y) = 3x - 4xy + 5y$ at $(1,2)$ in the direction of $\mathbf{V} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$.

a. $\left(-5 + \sqrt{3}\right)$
b. $\frac{1}{2}\left(-5 + \sqrt{3}\right)$
c. $-\frac{5}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$
d. $\frac{\sqrt{3}}{2}\mathbf{i} - \frac{5}{2}\mathbf{j}$
e. NOTA

33. Find the equation of the tangent plane to the hyperboloid given by $x^2 + y^2 - z^2 = 0$ at the point $(5,12,13)$.

a. $2x + 2y - 2z = 8$
b. $-2x - 2y - 2z = 8$
c. $5x + 12y + 13z = 0$
d. $-5x + 12y - 13z = 0$
e. NOTA

Mu Division---Vector Calculus
Answer Key

1. d
2. b
3. a
4. c
5. a
6. c
7. d
8. d
9. d
10. b
11. b
12. d
13. b
14. a
15. c
16. d
17. b
18. a
19. d
20. d
21. b
22. a
23. b
24. c
25. a
26. c
27. d
28. c
29. d
30. a
31. a
32. b
33. e