NATIONAL MU ALPHA THETA
OPEN DIVISION
DISCRETE MATH TOPIC TEST

Choose letter "e" for NOTA –"None of the Above"

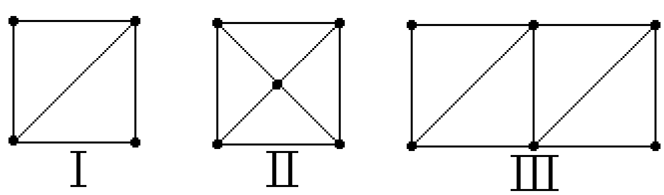
1. The notation K_n denotes the complete graph on n vertices. K_n is the simple graph that contains exactly one edge between each pair of distinct vertices. How many edges comprise a K_{10}?
    a. 100
    b. 50
    c. 45
    d. 55

2. $(p \wedge q) \rightarrow (p \vee q)$ is logically equivalent to
    a. T
    b. $p \vee q$
    c. F
    d. $p \wedge q$

3. How many relations are there on a set with n elements?
    a. $n!$
    b. n^2
    c. 2^n
    d. 2^{n^2}

4. Which of the following complete graphs is planar?
    a. K_5
    b. $K_{3,3}$
    c. K_6
    d. K_4

5. Which of the graphs shown below contain Euler paths?



    a. I
    b. I,II
    c. I,III
    d. II,III

6. Let R be a relation on the positive integers where $x\ R\ y$ if x is a factor of y. Which of the following lists of properties best describes the relation R?
    a. symmetric, transitive
    b. antisymmetric, transitive, reflexive
    c. antisymmetric, symmetric, reflexive
    d. symmetric, transitive, reflexive

7. Let Q(x,y) denote "x-y = y+x." Which of the quantifications below are true?
   I. $\exists x \exists y Q(x, y)$ II. $\forall x Q(x, 0)$ III. $\exists x \forall y Q(x, y)$
    a. I
    b. I,II
    c. II,III
    d. I,II,III

8. A crossing is defined as two edges passing through one another. The crossing number of a simple graph is the minimum number of crossings that can exist in a planar representation of the graph. What is the crossing number of K_6?
    a. 0
    b. 1
    c. 2
    d. 3

9. The modular inverse of x in modulo n, denoted $\overline{x}$, is a positive integer satisfying $x\overline{x} \equiv 1 \pmod{n}$. How many nonnegative integers < 25 have a modular inverse in modulo 25?
    a. 10
    b. 15
    c. 20
    d. 25

10. The solution of the recurrence relation $H_n = 2H_{n-1} + 1$ where $H_1 = 2$ is
    a. $H_n = 2^n - 1$
    b. $H_n = 2^n$
    c. $H_n = 2^n + 2^{n-1} - 1$
    d. $H_n = 2^n + 2^{n-1} + 1$

11. A string that contains only 0s, 1s, and 2s is called a ternary string. The recurrence relation A_n and initial conditions A_2,A_1 for the number of ternary strings that do not contain two consecutive 0s is?
    a. $A_n = A_{n-1} + A_{n-2}, A_2 = 4, A_1 = 3$
    b. $A_n = 2A_{n-1} + A_{n-2}, A_2 = 7, A_1 = 3$
    c. $A_n = 2A_{n-1} + 2A_{n-2}, A_2 = 8, A_1 = 3$
    d. $A_n = A_{n-1} + 2A_{n-2}, A_2 = 16, A_1 = 6$

12. What is the composite of the relations R and S where R is the relation from {1,2,3} to {1,2,3,4} with R = {(1,1),(1,4),(2,3),(3,1),(3,4)} and S is the relation from {1,2,3,4} to {0,1,2} with S={(1,0),(2,0),(3,1),(3,2),(4,1)}?
    a. {(1,0),(1,1),(2,1),(2,2),(3,0),(3,1)}
    b. {(1,2),(2,2),(3,1),(3,2)}
    c. {(1,0),(1,2),(3,1),(3,2)}
    d. {(1,0),(1,1),(2,0),(3,1)}

13. Assuming that no new n-tuples are added which fields are primary keys for the relation represented in the table below:

Professor	Department	Course #	Room	Time
Cruz	Zoology	335	A100	9:00 am
Cruz	Zoology	412	A101	8:00 am
Farber	Psychology	501	A100	3:00 pm
Grammer	Physics	617	A110	11:00 am
Rosen	CS	518	N521	2:00 pm
Rosen	Mathematics	575	N502	3:00 pm

    a. Professor
    b. Course #
    c. Course #, Time
    d. Time

14. The symmetric relation S containing R such that S is a subset of every symmetric relation containing R is called a symmetric closure. What is the symmetric closure of the relation $R = \{(a,b) \mid a > b\}$ on the set of positive integers?
    a. $S = \{(a,b) \mid a < b\}$
    b. $S = \{(a,b) \mid a \geq b\}$
    c. $S = \{(a,b) \mid a = b\}$
    d. $S = \{(a,b) \mid a \neq b\}$

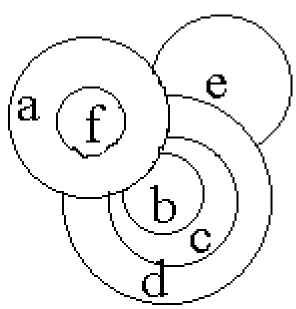
15. Find the number of paths of length 5 between two different vertices in K_4.
    a. 60
    b. 61
    c. 62
    d. 59

16. An edge whose removal changes the connectedness of a graph is called a cut edge or bridge. What is the set of cut edges for the graph represented by the following adjacency matrix?

	a	b	c	d	e	f
a	0	1	1	0	0	0
b	1	0	1	0	0	0
c	1	1	0	1	0	0
d	0	0	1	0	1	1
e	0	0	0	1	0	1
f	0	0	0	1	1	0

    a. {ab, cd}
    b. {cd}
    c. {cd, ef}
    d. {ef}

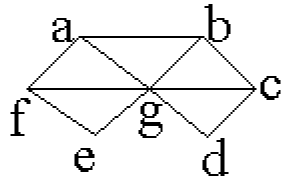
17. Find the minimum number of colors needed to color the following map such that no two adjacent regions have the same color.



    a. 1
    b. 2
    c. 3
    d. 4

18. Order the following functions in growth order from slowest to fastest.
    I. $f(x)=e^{x}$ II. $f(x)=x$ III. $f(x)=e^{.5\ln x}$ IV. $f(x)=x!$
    a. II, III, I, IV
    b. II, III, IV, I
    c. III, II, IV, I
    d. III, II, I, IV

19. What is the chromatic number of the following graph?



a. 1
b. 2
c. 3
d. 4

20. Suppose you are given three pitchers of water, of sizes 10 quarts, 7 quarts, and 4 quarts. Initially the 10 quart pitcher is full and the other two empty. If a move is defined as pouring water from one pitcher to another until the receiving pitcher is full or the pouring pitcher is empty. What is the smallest number of moves necessary to get 2 quarts in the 7 quart pitcher?
a. 2
b. 3
c. 4
d. 5

21. A relation R is called a partial order if it is reflexive, antisymmetric, and transitive. Which of the following relations represented by zero-one matrices are partial orders.

$$\text{I.}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{II.}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad \text{III.}\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

a. I,II,III
b. II,III
c. II
d. I,III

22. A small child goes to a store with one dollar to buy some candy. The store sells gumballs for 5 cents apiece, suckers for 5 cents apiece, candy canes for 5 cents apiece, and jawbreakers for 20 cents apiece. How many different purchases (subsets) of candy will this dollar buy?

a. 563
b. 536
c. 554
d. 600

23. A playoff between two teams consists of at most five games. The first team that wins three games wins the playoffs. In how many different ways can the playoff occur?
    a. 18
    b. 15
    c. 20
    d. 21

24. How many ways are there to select five bills from a money bag containing $1 bills, $2 bills, $5 bills, $10 bills, $20 bills, $50 bills, and $100 bills? Assume that the order in which the bills are chosen doesn't matter, that the bills of each denomination are indistinguishable, and that there are at least five bills of each type.
    a. 21
    b. 252
    c. 792
    d. 462

25. How many one-to-one functions are there from a set with m elements to a set with n elements ($m < n$)?
    a. nm
    b. m^n
    c. $P(n,m)$
    d. $C(n,m)$