Theta Algebra Solutions

1. C  The range of the relation \{(-1, 2), (0,0), (1,2)\} is \{0,2\}

2. B  Given \( g(x) = \frac{1}{3-|x|} \), then \( 3-|x| \neq 0 \)
therefore, \(|x| \neq 3, x \neq 3, x \neq -3\)

3. D  Graphing the system of inequalities, the vertex points of the graph are (2,2), (-.5, 2), (-2.5, -2), and (0, -2)

4. B  When simplified
\[
\frac{x + y}{x - y} = \frac{x + y}{\frac{1}{1} - \frac{1}{y}} = \frac{x + y}{x y} = xy
\]

5. D  Factoring \( 2x^3 - 7x^2 + 2x + 3 \), the factors are \((x-3)(2x+1)(x-1)\)

6. C  Factoring \( x^4 - 8x^2 + 16 = 0 \) \((x+2)^2 (x-2)^2 = 0\), therefore \(-2\) has multiplicity of 2.

7. D  Given \( f(t) = \frac{1+t}{t} \) then
\[
\frac{1}{f(t)} = \left(\frac{1}{t}\right) \left(\frac{1}{1}\right) = \left(\frac{1}{t}\right) = \frac{t+1}{t} = f(t)
\]

8. A  \( \log_b 250 = \log_b 25 + \log_b 10 = 2\log_b 5 + \log_b 10 = \log_b 10 + \log_b 10 = 2(\log_b 10 - \log_b 2) + \log_b 10 = 3\log_b 10 - 2\log_b 2 = 3y - 2x \)

9. C  Using Descartes Rule of Signs, \( f(x) = 2x^5 - x^3 - x - 1 \) has at most 2 negative zeros. There are 2 sign changes when \( f(-x) \) is evaluated.
\( f(-x) = 2(-x)^5 - (-x)^3 - (-x) - 1 = -2x^5 + x^3 + x - 1 \)

10. E  \( N = \begin{vmatrix} 4 & 3 \\ 2 & -2 \end{vmatrix} = 14 \)

11. D  Given matrices \( P = \begin{bmatrix} 4 & -6 \\ 1 & 3 \end{bmatrix} \) and
\( Q = \begin{bmatrix} -1 & 7 \\ 2 & -2 \end{bmatrix} \) then \( 2P + 3Q = \begin{bmatrix} 5 & 9 \\ 8 & 0 \end{bmatrix} \)

12. A  If \( P \) is a 2 by 2 matrix, which of the following is \( P^{-1} \). \( P = \) identity matrix.

13. D  Solving for the nth term, \( a_n = 2 + (n - 1)4, \) \( a_n = 4n - 2 \). Solving for the number of terms

14. C  Solving for the common ratio \( \frac{-9}{4} = \frac{1}{12} r^3 \)
\( r^3 = -27, r = -3 \)

15. B  Find the distance between a point on one of the lines and the other line using \( \frac{|Ax+By+C|}{\sqrt{A^2 + B^2}} \)
\( \frac{2(0) - 1(4) + 1}{\sqrt{2^2 + (-1)^2}} = \frac{3\sqrt{5}}{5} \)

16. D  \( \sum_{b=1}^{\infty} 4b = 4 + 8 + 12 + \ldots \)

17. C  \( g(f(x)) = 2(3x - 4)^3 + 1 = 54x^3 - 216x^2 + 288x - 127 \)

18. B  Given \( f(x) = \frac{x + 2}{x + 1} \), then the inverse is found as follows:
\( x = \frac{y + 2}{y + 1}; xy + x = y + 2; y = \frac{x - 2}{1 - x}; f^{-1}(x) = \frac{x - 2}{1 - x} \)

19. B  If you let \( a = 1 \) you easily see that you have an equilateral triangle.

20. C  Given the parabola \((x - 6)^2 = 8(y + 1)\) the vertex is (6,-1) and \( p = 2 \) therefore the focus is (6,1).

21. A  Rewriting the equation of the ellipse
\( 4x^2 + 9y^2 + 16x + 18y - 11 = 0 \)
\( 4(x^2 + 4x + 4) + 9(y^2 + 2y + 1) = 36 \)
\( (x + 2)^2 + (y + 1)^2 = 36 \) then \( a = 3, b = 2, \) and \( c = \sqrt{5} \).

The sum = \( 2+2+2(\sqrt{5}) = 2(3) + 2(\sqrt{5}) = 20+2(\sqrt{5}) \)

22. C  For an ellipse, \( a^2 - b^2 = c^2 \) and eccentricity is \( e = \frac{c}{a} \). Since the minor axis has length \( 2\sqrt{21} \), the value of \( b = \sqrt{21} \). From the focus the value of \( c = 2 \). Then
\( a^2 - (2\sqrt{21})^2 = 2^2 \) \( a^2 = 25, a = 5 \)

This makes the eccentricity \( e = \frac{2}{5} \)

23. B  For the equation \( \frac{x^2}{3} - \frac{y^2}{3} = 1 \), \( a^2 = 3 \) and \( b^2 = 3 \) then solving for \( a^2 + b^2 = c^2 \) \( c = \sqrt{6} \)
The focus points are \((\sqrt{6}, 0)\) and \(( -\sqrt{6}, 0)\)
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24. B First fix a person and then everyone else is relative to them. 5! = 120

25. A The number of possible arrangements of the letters in the word banana is \( \frac{6!}{3!2!} = 60 \)

26. A If the probability of an event is \( \frac{a}{b} \), then the odds of the event occurring is \( a:b-a \)

27. D The largest possible median will occur when the three numbers not given are larger than those given. Let a, b, and c denote the three missing numbers, where 9 < a < b < c. Listed from smallest to largest, the list is 3, 5, 5, 7, 8,

9, a, b, c so the median is 8

28. A Solving \( H = \frac{A}{1 + Be^{-rt}} \) for \( t \) you get

\[
H + HBe^{-rt} = A \\
HBe^{-rt} = A - H \\
\ln e^{-rt} = \ln \left( \frac{A - H}{HB} \right) \\
-rt = \ln(A - H) - \ln H - \ln B \\
t = \left(\frac{1}{r}\right) \left( \ln H + \ln B - \ln(A - H) \right)
\]

29. C Let \( L = \) length and \( W = \) width; \( LW = 6; \)

\[
2W + 2L = 6\sqrt{3}; \ W = \frac{6}{L}; \ \frac{6}{L} + L = 3\sqrt{3}; \L^2 - 3\sqrt{3}L + 6 = 0 \ \ L = \sqrt{3} \ or \ L = 2\sqrt{3}.
\] Shorter side therefore is \( \sqrt{3} \).

30. A For \( f(x) = x^3 + ax^2 + ax + a_3 \), the maximum number of zeros would be 3. Since the equation is an odd degree, the must be at least 1 zero. Therefore \( 1 \leq n \leq 3 \).

31. D If you graph each of the given functions, they all pass the horizontal line test. Therefore each relation has an inverse that is also a function.

32. D Let \( x = \) amount of solution C, let \( 2x = \) amount of solution B, and let \( y = \) amount of solution A. Solving the system

\[
3x + y = 50; \ 1.1x + .1y = 17 \ \ x = 15, y = 5
\]

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33. C Average speed = \( \frac{\text{total distance}}{\text{total time}} \)

\[
\text{Ave. Speed} = \frac{2}{1 + \frac{1}{\frac{x+y}{x+y}}} = \frac{2xy}{x+y} \text{ mph}
\]

34. B The sum of the coefficients is \( (2+1-1)^8 = 256 \)

35. E Since \( 0^0 = 0 \) for any \( z > 0 \), \( f(0) = f(-2) = 0 \)

\[
f(0)+f(-1)+f(-2)+f(-3) = (-1)^0(1)^2 + (-3)^2(-1)^3 = 1 + \frac{1}{9} = \frac{10}{9}
\]

36. D \( r \# s = \frac{r+s}{rs} \) is commutative because

\[
\frac{r+s}{rs} = \frac{s+r}{sr}
\]

37. A \( 144_b = 1 \cdot b^2 + 4b + 4 = (b+2)^2 \)

The number is a perfect square for any integral value of \( b \). However, since the digits up to 4 are used to write the number, \( b > 4 \).

38. C Let the original radius be 1. The original volume = \( \frac{4}{3} \pi(1)^3 = \frac{4}{3} \pi \). The new radius is 2.

The new volume is \( \frac{4}{3} \pi(2)^3 = \frac{4}{3} \pi(8) \)

The increase in volume is \( \frac{4}{3} \pi(7) \). The percent increase = \( \frac{4}{3} \pi \cdot 100 = 700 \)

39. B \( \log x \geq \log 2 + \log x^{1/2} \rightarrow \log x \cdot \log x^{-1/2} \geq \log 2 \)

\[
\log \frac{x}{1} \geq \log 2 \rightarrow \log x^2 \geq \log 2 \rightarrow \frac{1}{x^2} \geq 2 \rightarrow x \leq 4
\]

40. C By the Pythagorean theorem, \( 85^2 = 75^2 + y^2 \)

\[
y = 40 \ \ \text{and} \ 85^2 = 68^2 + (x+y)^2 \\ 2601 = (x+40)^2 \ x + 40 = 51 \ x = 11
\]