

Theta Algebra Solutions

- C The range of the relation $\{(-1, 2), (0,0), (1,2)\}$ is $\{0,2\}$
- B Given $g(x) = \frac{1}{3-|x|}$, then $3-|x| \neq 0$
therefore, $|x| \neq 3$, $x \neq 3$, $x \neq -3$
- D Graphing the system of inequalities, the vertex points of the graph are $(2,2)$, $(-5, 2)$, $(-2.5, -2)$, and $(0, -2)$
- B When simplified
$$\frac{x+y}{x^{-1}+y^{-1}} = \frac{x+y}{\frac{1}{x} + \frac{1}{y}} = \frac{x+y}{\frac{x+y}{xy}} = xy$$
- D Factoring $2x^3 - 7x^2 + 2x + 3$, the factors are $(x-3)(2x+1)(x-1)$
- C Factoring $x^4 - 8x^2 + 16 = 0$ $(x+2)^2(x-2)^2 = 0$,
therefore -2 has multiplicity of 2.
- D Given $f(t) = \frac{1+t}{t}$ then
$$\frac{1}{t} f\left(\frac{1}{t}\right) = \left(\frac{1}{t}\right) \left(\frac{1+\frac{1}{t}}{\frac{1}{t}}\right) = \left(\frac{1}{t}\right)(t+1) = \frac{t+1}{t} = f(t)$$
- A $\log_b 250 = \log_b 25 + \log_b 10 = 2\log_b 5 + \log_b 10 =$
 $2\log_b \frac{10}{2} + \log_b 10 = 2(\log_b 10 - \log_b 2) + \log_b 10 =$
 $3\log_b 10 - 2\log_b 2 = 3y - 2x$
- C Using Descartes Rule of Signs,
 $f(x) = 2x^5 - x^3 - x - 1$ has at most 2 negative zeros. There are 2 sign changes when $f(-x)$ is evaluated.
 $f(-x) = 2(-x)^5 - (-x)^3 - (-x) - 1 = -2x^5 + x^3 + x - 1$
- E $N = \begin{vmatrix} 4 & 3 \\ -2 & 2 \end{vmatrix} = 14$
- D Given matrices $P = \begin{bmatrix} 4 & -6 \\ 1 & 3 \end{bmatrix}$ and
 $Q = \begin{bmatrix} -1 & 7 \\ 2 & -2 \end{bmatrix}$ then $2P + 3Q = \begin{bmatrix} 5 & 9 \\ 8 & 0 \end{bmatrix}$
- A If P is a 2 by 2 matrix, which of the following is $P^{-1} \cdot P =$ identity matrix.
- D Solving for the n th term, $a_n = 2 + (n-1)4$,
 $a_n = 4n-2$. Solving for the number of terms

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- $$1250 = \frac{n}{2}(2 + 4n - 2) \quad 2500 = 4n^2 \quad n=25$$
- C Solving for the common ratio $\frac{-9}{4} = \frac{1}{12}r^3$
 $r^3 = -27$, $r = -3$
 - B Find the distance between a point on one of the lines and the other line using $\frac{|Ax+By+C|}{\sqrt{A^2+B^2}}$
$$\frac{|2(0) - 1(4) + 1|}{\sqrt{2^2 + (-1)^2}} = \frac{3\sqrt{5}}{5}$$
 - D $\sum_{b=1}^{\infty} 4b = 4 + 8 + 12 + \dots$
 - C $g(f(x)) = 2(3x-4)^3 + 1 = 54x^3 - 216x^2 + 288x - 127$
 - B Given $f(x) = \frac{x+2}{x+1}$, then the inverse is found as follows:
 $x = \frac{y+2}{y+1}$; $xy+x=y+2$; $y = \frac{x-2}{1-x}$; $f^{-1}(x) = \frac{x-2}{1-x}$
 - B If you let $a = 1$ you easily see that you have an equilateral triangle.
 - C Given the parabola $(x-6)^2 = 8(y+1)$ the vertex is $(6,-1)$ and $p = 2$ therefore the focus is $(6,1)$.
 - A Rewriting the equation of the ellipse
 $4x^2 + 9y^2 + 16x + 18y - 11 = 0$
 $4(x^2 + 4x + 4) + 9(y^2 + 2y + 1) = 36$
 $\frac{(x+2)^2}{9} + \frac{(y+1)^2}{4} = 36$ then $a = 3$, $b = 2$, and
 $c = \sqrt{5}$.
The sum $= 2(3) + 2(2) + 2(\sqrt{5}) = 10 + 2(\sqrt{5})$
 - C For an ellipse, $a^2 - b^2 = c^2$ and eccentricity is
 $e = \frac{c}{a}$ Since the minor axis has length $2\sqrt{21}$,
the value of $b = \sqrt{21}$. From the focus the value of $c = 2$. Then
 $a^2 - (2\sqrt{21})^2 = 2^2 \quad a^2 = 25$, $a = 5$
This makes the eccentricity $e = \frac{2}{5}$
 - B For the equation $\frac{x^2}{3} - \frac{y^2}{3} = 1$, $a^2 = 3$ and
 $b^2 = 3$ then solving for $a^2 + b^2 = c^2 \quad c = \sqrt{6}$
The focus points are $(\sqrt{6}, 0)$ and $(-\sqrt{6}, 0)$

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24. B First fix a person and then everyone else is relative to them. $5! = 120$
25. A The number of possible arrangements of the letters in the word *banana* is $\frac{6!}{3! 2!} = 60$
26. A If the probability of an event is $\frac{a}{b}$, then the odds of the event occurring a: b-a
27. D The largest possible median will occur when the three numbers not given are larger than those given. Let a, b, and c denote the three missing numbers, where $9 \leq a \leq b \leq c$. Listed from smallest to largest, the list is 3, 5, 5, 7, 8, 9, a, b, c so the median is 8
28. A Solving $H = \frac{A}{1 + Be^{-rt}}$ for t you get
- $$H + HBe^{-rt} = A$$
- $$HBe^{-rt} = A - H$$
- $$\ln e^{-rt} = \ln\left(\frac{A - H}{HB}\right)$$
- $$-rt = \ln(A - H) - \ln H - \ln B$$
- $$t = \left(\frac{1}{r}\right)(\ln H + \ln B - \ln(A - H))$$
29. C let L = length and W = width; $LW = 6$;
 $2W + 2L = 6\sqrt{3}$; $W = \frac{6}{L}$; $\frac{6}{L} + L = 3\sqrt{3}$;
 $L^2 - 3\sqrt{3}L + 6 = 0$ $L = \sqrt{3}$ or $L = 2\sqrt{3}$.
 Shorter side therefore is $\sqrt{3}$.
30. A For $f(x) = x^3 + a_1x^2 + a_2x + a_3$, the maximum number of zeros would be 3. Since the equation is an odd degree, the must be at least 1 zero. Therefore $1 \leq n \leq 3$.
31. D If you graph each of the given functions, they all pass the horizontal line test. Therefore each relation has an inverse that is also a function.
32. D let x = amount of solution C, let 2x = amount of solution B, and let y = amount of solution A
 Solving the system
 $3x + y = 50$; $1.1x + .1y = 17$ $x = 15$, $y = 5$

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33. C Average speed = $\frac{\text{total distance}}{\text{total time}}$
 Ave. Speed = $\frac{2}{\frac{1}{x} + \frac{1}{y}} = \frac{2}{\frac{x+y}{xy}} = \frac{2xy}{x+y}$ mph
34. B The sum of the coefficients is $(2+1-1)^8 = 256$
35. E Since $0^z = 0$ for any $z > 0$, $f(0) = f(-2) = 0$
 $f(0)+f(-1)+f(-2)+f(-3)=(-1)^0(1)^2+(-3)^2(-1)^0=1+\frac{1}{9}=\frac{10}{9}$
36. D $r \# s = \frac{r+s}{rs}$ is commutative because
 $\frac{r+s}{rs} = \frac{s+r}{sr}$
37. A $144_b = 1 \cdot b^2 + 4b + 4 = (b+2)^2$
 The number is a perfect square for any integral value of b. However, since the digits up to 4 are used to write the number, $b > 4$.
38. C Let the original radius be 1. The original volume = $\frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi$. The new radius is 2.
 The new volume is $\frac{4}{3}\pi(2)^3 = \frac{4}{3}\pi(8)$
 The increase in volume is $\frac{4}{3}\pi(7)$. The
 percent increase = $\frac{\frac{4}{3}\pi(7)}{\frac{4}{3}\pi} \cdot 100 = 700$
39. B $\log x \geq \log 2 + \log x^{\frac{1}{2}} \rightarrow \log x - \log x^{\frac{1}{2}} \geq \log 2$
 $\log \frac{x}{x^{\frac{1}{2}}} \geq \log 2 \rightarrow \log x^{\frac{1}{2}} \geq \log 2$
 $x^{\frac{1}{2}} \geq 2 \rightarrow x \geq 4$
40. C By the Pythagorean theorem, $85^2 = 75^2 + y^2$
 $y = 40$ and $85^2 = 68^2 + (x+y)^2$
 $2601 = (x+40)^2$ $x + 40 = 51$ $x = 11$

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