Theta Equations and Inequalities

1. A $4y(2 + b) - b(3y - 1) = 5b$
   $8y + 4by - 3by + b = 5b$
   $y = \frac{4b}{8 + b}$

2. C $x^2 + 3x - 10 = 0$; $b^2 - 4ac = 9 - 4(-10) = 49$

3. C $|2 - 3x| < 4$; $-4 < 2 - 3x < 4$; $-\frac{2}{3} < x < 2$

4. E

5. B Solving the system $3x + y = 10$ and $x - 3y - 10 = 0$ you get $(4, -2)$ $x + y = 4 - 2 = 2$

6. A Percent of decrease $= \frac{90 - 75}{90} = \frac{15}{90} = \frac{1}{6} = 16\frac{2}{3}\%$

7. B $\frac{4(10i\pi)}{6} = \frac{x}{24i\pi}$; $x = 1$

8. B $6(4) = 5h$, $h = 4.8$

9. C $a^3 = 7$, $4a^6 = 4(a^3)^2 = 4(7)^2 = 196$

10. B $2x + 3 + \sqrt{29 - 4x} = 0$
    $\sqrt{29 - 4x} = -2x - 3$; $29 - 4x = 4x^2 + 12x + 9$
    $4x^2 + 16x - 20 = 0$; $4(x^2 + 4x - 5) = 0$; $x = -5, 1$ but must reject 1; only solution is $-5$.

11. D

12. D $2x^4 - x^3 - 4x^2 + 10x - 4 = 0$ factors into $(x - 1 - i)(x - 1 + i)(2x - 1)(x + 2) = 0$ therefore there are 2 rational and 2 imaginary roots.

13. B

14. D $f(x) = \frac{2x^3 + 15x^2 + 34x + 18}{x^2 + 5x + 4}$ has vertical asymptotes where $x^2 + 5x + 4 = 0$; $x = -1, x = -4$. The slant asymptote $y = 2x + 5$ is found by dividing the numerator by the denominator.

15. E If $px^2 + px + q$ is divided by $x - 1$, the remainder is 3; if $px^3 + px + q$ is divided by $x + 1$, the remainder is -7. This implies $p(1)^3 + p(1) + q = 3$ and $p(-1)^3 + p(-1) + q = -7$ which means $p + p + q = 3$ and $-p - p + q = -7$. Solving the system, $2q = -4$; $q = -2$ and $p = 2.5$

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16. D Let $r_1$ and $r_2$ are the roots of the equation $ax^2 + bx + c = 0$, then
   $r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$
   then $(r_1 - r_2)^2 = \left(\frac{-b + \sqrt{b^2 - 4ac}}{2a} - \frac{-b - \sqrt{b^2 - 4ac}}{2a}\right)^2 = \left(\frac{b^2 - 4ac}{a^2}\right)^2$
   $= \frac{b^2 - 4ac}{a^2}$

17. B $\frac{x^2 - 2}{9} - \frac{x + 1}{3} = 0$; $\left(\frac{x - 1}{3}\right)^2 = 0$; $x = 1$

18. A $y = \sqrt{(3x - 5)(4x^2 + 12x + 9)}$
   $y = \frac{\sqrt{(3x - 5)(2x - 3)^2}}{(3x - 5)(2x - 3)} = \frac{1}{(3x - 5)(2x - 3)}$
   $y = (3x - 5)^{-1}$

19. E $\log_8 (\sqrt{a + x} + \sqrt{a - x}) + \log_8 (\sqrt{a + x} - \sqrt{a - x}) = \frac{1}{3}$
   $\log_8 (a + x - a + x) = \frac{1}{3}$; $\log_8 2x = \frac{1}{3}$
   $2x = 8^{\frac{1}{3}}$; $x = 1$

20. D $x^3 - 3x^3 = 4$, $x^3 - 3x^3 - 4 = 0$
    $\frac{1}{x^3} = 4$; $x = 64$, $x^3 = -1$; $x = -1$
    sum $= 64 - 1 = 63$

21. B $a^x - a^{-x} = 3$, $a^x - \frac{1}{a^x} = 6$.
    $a^{2x} - 6a^x - 1 = 0$ Solve using the quadratic formula:
    $a^x = 3 \pm \sqrt{10}$; $x = \log_8(3 + \sqrt{10})$
    remember that you can’t take the log of a negative number.
22. B Since \( x + y + z = 100 \) and \( x, y, \) and \( z \) are proportional to 2, 3, and 5 \( \Rightarrow x = 20, \) \( y = 30, \) and \( z = 50. \) \( y = ax - 10 \) \( \Rightarrow 30 = 20a - 10, \) \( a = 2 \)

23. B \( m = \frac{4}{n} \) and \( r = \frac{9}{t} \) then \( \frac{3mr-nt}{4nt-7mr} \)
\[
m = \frac{4n}{3} \quad t = \frac{14r}{9}
\]
\[
\frac{4nr - 14nr}{9} = \frac{22nr}{9} - \frac{28nr}{14}
\]
\[
\frac{36nr - 14nr}{56nr - 84nr} = \frac{9}{9} - \frac{11}{14}
\]

24. C The base of the triangle is \( h + 2 \) and since the triangle is a 30-60-90 triangle \( h + 2 = \sqrt{3} h \)
\[
(h + 2)^2 = \left(\sqrt{3} \right)^2 \Rightarrow h^2 + 4h + 4 = 3h^2
\]
\[
h^2 - 2h - 2 = 0
\]
solve using the quadratic formula \( h = 1 \pm \sqrt{3} \) You can only use the positive one.

25. D \( y = \frac{10\log x}{x^3}; \) \( y = \frac{x}{x^3}; \) \( y = \frac{1}{x^2}; \) \( x^2 y = 1 \)
this is an inverse function

26. C Given \( x^2 + y^2 - 8x + 2y - 3 = 0 \) the center is \((x-4)^2 + (y+1)^2 = 20\) radius = \( 2\sqrt{5} \)
\[
C = 2\pi(2\sqrt{5}) = 4\pi\sqrt{5}
\]

27. A Given the points \((3,5)\) and \((-2,1)\) the slope is\( m = \frac{4}{5}, \) the \( m \perp = -\frac{5}{4}. \) The midpoint between the given points is \((.5, 3). \) The perpendicular bisector then is \(-10x - 8y + 29 = 0 \)

28. C Changing \( x^2 + 4y^2 - 2x - 24y - 19 = 0 \) into graphing form you get \( \frac{(x-1)^2}{56} + \frac{(y-3)^2}{14} = 1 \) the length of the semi-major axis is \( \sqrt{56} = 2\sqrt{14} \)
the longest chord = \( 4\sqrt{14} \)

29. C The \( y \)-intercept is when \( x = 0 \)
\[
\begin{bmatrix} 1 & 0 & -1 \end{bmatrix} = 10 \quad \begin{bmatrix} y & 3 \end{bmatrix} = 10 \quad \begin{bmatrix} y - 4 + 3y - 6 \end{bmatrix} = 10
\]
\[
\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} = 5 \quad 0, 5 \)

30. A Starting with \( xu = 400 \) substitute for \( u \)
\[
x \left( v - 20 \right) = 400, \) substituting for \( y \)
\[
x \left( \frac{400}{y} - 20 \right) = 400; \quad x \left( \frac{600}{x} - 20 \right) = 400
\]
\[
600 - 20x = 400; \quad x = 10
\]

31. A \( \left( x^2 - 9 \right)^2 = 1 \left( x^2 \right)^{\frac{3}{2}} + \frac{3}{2} \left( x^2 \right)^{\frac{1}{2}} \left( -9 \right) + \frac{3}{8} \left( x^2 \right)^{-\frac{1}{2}} \left( -9 \right)^2 + ... \)
The third term is \( \frac{243}{8x} \)

32. A Let \( b \) = rate of the boat in still water; \( s \) = rate of the current. From the first trip we know that \( 5(b-s) = 2(b+s) \) which leads to \( 3b - 7s = 0. \) From the second trip we know \( 3(b+s) - 2 = 7(b-s) \) which leads to \( 2b - 5s + 1 = 0. \) Solving the system for \( s \) you will get \( s = 3. \)

33. C Given \( \triangle ABC, BC = 1, AC = p, \) \( AB = \sqrt{p^2 + 1} \)
\[
\cos \angle A = \frac{AC}{AB} = \frac{p}{\sqrt{p^2 + 1}}
\]

34. C If \( x = \sqrt{yz}, \) then \( x^2 = yz \) and \( y = \frac{x^2}{z}. \)
Therefore, \( \log y = \log \frac{x^2}{z} = 2\log x - \log z \)

35. B the teller’s initial total could be represented by \( .25q + .1d + .05n + .01p \) What he should have had could be represented by \( .25(q-x) + .1(d+x) + .05(n+x) + .01(p-x) = .25q + .1d + .05n + .01x - .25x + .1x + .05x \) \(-.01x = \) initial total \-11x \)
36. D

\[
\begin{bmatrix}
-7/2 & 3 & b/2 & 18 \\
-21/2 & -35 & b/2 & 21/2 & = 5 \\
3/2 & 5 & r/2 & \\
\end{bmatrix}
\]

and \(18 - \frac{35}{2} = \frac{r}{2}\) then \(b = 31\) and \(r = 1\)

\(r + b = 32\)

37. D

\[6^a + b = 6^2 \quad a + b = 2; \quad 6^{a + 5} = 6^3\]

\[a + 5b = 3; \quad 4b = 1; \quad b = .25 \quad a = \frac{7}{4}\]

38. A

A matrix is considered singular if its determinant is 0. Solving for the determinant \(x(1 + 15) - 2(-30 - 2) = 0;\)

\[x = -4 \quad B^2 = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}^2 = \begin{bmatrix} 8 & -7 \\ 7 & 15 \end{bmatrix}\]

\[Q = -4 \quad R = 23; \quad Q + R = 19\]

39. C

1 is false because the domain is all real numbers. 2 is false because if \(0 < a < 1\) \(f(4) < f(-1).\) 3, 4, and 5 are true.

40. D

\[2a + 2c = 32; \quad a + c = 16; \quad a^2 + 8^2 = c^2;\]

\[c^2 - a^2 = 64; \quad (c + a)(c - a) = 64; \quad 16(c - a) = 64; \quad c - a = 4 \quad \text{solving the system} \quad -a + c = 4 \quad \text{and} \quad a + c = 16 ; \quad c = 10 \quad \text{and} \quad a = 6. \quad \text{The area then is} \quad .5(12)(8) = 48\]