2002 National Theta Geometry Test Solutions

1. $\triangle ADE$ is equilateral: $Area = \frac{\sqrt{3}}{4} \bullet 3^2 = \boxed{A}$

2. $\boxed{A}$ The intersection can be a plane, or a line. II is only the definition of a plane; two intersecting lines can't be an intersection.

3. Taking n = 9, $\frac{n(n-3)}{2} + 180(n-2) + 360 + 9 + 1 \Rightarrow 1657 \Rightarrow \boxed{E}$

4. $(x-3)^2 + (y-7)^2 = 169$, r = 13. Twice circumference $= 52\pi \Rightarrow \boxed{B}$

5. Area of framed picture – area of picture = area of frame
   36(26) – 30(20) = 336 $\boxed{C}$

6. A = ½ (product of diagonals) = 65/2 $\boxed{D}$

7. $\pi r^2 = 2\pi r$, 2 $\boxed{B}$

8. $\frac{4}{3}\pi\left(\sqrt{e\pi}\right)^3 = \frac{\pi 5h}{3}, h = 19.96...\ Slant = \sqrt{h^2 + 5} = 20.0892... \Rightarrow \boxed{D}$

9. Circle is centered at (3, 0) with radius 3. Distance from point to center is $\sqrt{50}$. This is the hypotenuse of a triangle, with one leg of length 3, and other x. "x" is the length we want: $\sqrt{41} \Rightarrow \boxed{A}$

10. 2(CD + 2) = 9, CD = 5/2. Triangle with side lengths 3, 5, and 9/2. Heron's formula (maybe a program on calculator) yields 6.66585 $\boxed{B}$

11. Radius of circle is 2: $\frac{16 - 4\pi}{56\pi} = 0.0195 \Rightarrow \boxed{C}$

12. Law of sines: $\frac{\sin 30°}{4} = \frac{\sin 45°}{x}, x = 4\sqrt{2} \Rightarrow \boxed{C}$

13. $\boxed{C}$

14. $\frac{180(n-2)}{n} = 171;\ n = 40$ $\boxed{A}$

15. $A = 4\pi r^2,\ V = \frac{4}{3}\pi r^3;\ 288\pi = \frac{4}{3}\pi r^3;$ r=6; $A = 4\pi(6)^2 = 144\pi$ $\boxed{E}$

16. $3<x<21$ This interval contains 17 integer values $\boxed{B}$

17. $\boxed{B}$

18. (-10, -1), (-8, 2), (-6, -2) $\boxed{A}$

19. $r=\sqrt{7.5^2+3.1^2}$, Area of octagon is $\frac{1}{2}(49.6)(7.5)$, so we have $\pi r^2-186=20.90...\boxed{B}$

20. $\dfrac{\frac{4}{3}\pi(\frac{1}{2}r)^3}{\frac{4}{3}\pi r^3}=\frac{1}{8}$ $\boxed{D}$

21. We get $2\cdot3\cdot\frac{1}{2}\cdot2\cdot3\cdot\left(\frac{2\sqrt{2}}{\sqrt{2}}\right)^2\cdot\frac{\pi}{4\pi}\cdot\frac{1}{2}\cdot2\cdot3\cdot\frac{1}{2}\cdot3\cdot6=486$ $\boxed{C}$

22. Triangular numbers: $\frac{n(n+1)}{2}$, count up to the thirteenth prime number. So 210+41 $\boxed{E}$

23. CP = 2PE, AP = 2PD, so 4 + 7 + 6 + 10.5 = 27.5 $\boxed{D}$

24. lw=18, wh=15, lh=30. w=18/l , 18/l * h = 15 ; l = 6h/5; 6h/5*h = 30;
h = 5, w = 3, l = 6; V= (5)(3)(6) = 90 $\boxed{B}$

25. Equation of the line is x – 7y = 13. Midpoint is (1/4, -13/4). Distance is the square root of 40. Slope is 19. Answer: $\boxed{B}$

26. Area of square = 25*25 = 625. Area of circle =
$\pi\left(\frac{25\sqrt{2}}{2}\right)^2=\frac{625\pi}{2}$; $\dfrac{625}{\frac{625\pi}{2}}=\frac{2}{\pi}$; To change to percentage, multiply by 100. $\frac{200}{\pi}$ $\boxed{C}$

27. It works for any angle: $\sin^2 x+\cos^2 x=1$ $\boxed{D}$

28. The area of the triangle is 16.5 . The area of the pentagon is 72. $\boxed{B}$

29. Using the formula for volume of icosahedron given side length “a”:
$\frac{5a^3(3+\sqrt{5})}{12}\Rightarrow\boxed{A}$

30. Cone of radius 6 and height 4: 48π $\boxed{B}$

Tie Breakers

14-sided polygon, interior angle of $\frac{1080}{7}$. Using Area = .5ab(sinC) ,

A = $.5(4)(4)\sin(1080/7) = 3.4710$