Theta Number Theory Topic Test

Note: For each problem, where there is no choice (e), assume (e) none of the above.

1. What is the arithmetic mean of the first 10 prime numbers? Express answer as a decimal.
   a) 10.1 b) 11.6 c) 12.9 d) 13.0

2. Narcissistic numbers are numbers whose values can be expressed using only the digits in the numbers and operations to include four basic operations, powers, factorials, nth roots, etc. Some narcissistic numbers follow:
   $2427 = 2^1 + 4^2 + 2^3 + 7^4$
   $81 = (8+1)^2$
   $355 = 3(5!) - 5$
   Which of these is narcissistic?
   a) 121 b) 135 c) 217 d) 273

3. A two – digit number is a perfect square and has exactly nine positive integral factors. What is the number?
   a) 25 b) 36 c) 49 d) 64

4. What value to the nearest percent of the perfect squares between 0 and 1000, exclusive are odd numbers?
   a) 45% b) 50% c) 52% d) 54%

5. If *5 @ 2 = 13, 3 @ 3 = 6, 2 @ 4 = 2,* find *2 @ 8.*
   a) - 4 b) - 2 c) 4 d) 12

6. A school hallway has a long row of lockers. Every sixth locker contains a package of chewing gum, with the 6^{th} locker being the first to have chewing gum, every eighth locker (the first one is locker #8) contains a hockey stick, and every ninth locker (locker #9 is the first) contains a mirror. Which is the first locker to contain all three items?
   a) 18 b) 24 c) 64 d) 72

7. One rectangular swimming pool has an area of 24 square yards. Another rectangular swimming pool has an area of 90 square yards. What is the largest possible integral dimension common to both pools?
   a) 6 b) 12 c) 15 d) 18

8. Which of the following is a prime number between 60 and 69?
   a) none b) 63 c) 65 d) 67

9. The number 279 is composite. Which of the following is a factor of 279?
   a) 6 b) 27 c) 31 d) 43

10. How many whole numbers are between $\sqrt{8}$ and $\sqrt{80}$?
    a) 5 b) 6 c) 7 d) 8 e) 9

11. Which of the following is closest to $\sqrt{65} - \sqrt{63}$?
    a) 0.12 b) 0.13 c) 0.14 d) 0.15 e) 0.16

12. What is the maximum number of Friday the 13ths that can occur in one year?
a) 2 b) 3 c) 5 d) 6

13. Find the units digit of 2^{2391}.
a) 2 b) 4 c) 6 d) 8

14. What is the smallest prime number that is a factor of the sum of 3^{15} and 5^{17}?
a) 2 b) 3 c) 5 d) 17

15. What is the greatest number of factors that a positive integer less than 100 can have?
a) 8 b) 10 c) 12 d) 15

16. A digital root is the value of the sum of the digits of a number until only one digit remains. For example, for the number 625, first add 6+2+5 = 13. Now since 13 has 2 digits, add those digits; that is, 1+3=4. So the digital root of 625 is 4. What is the digital root is of 6^6? (Hint: Consider the first 5 terms of the sequence of 6^{th} powers and their digital roots.)
a) 0 b) 1 c) 6 d) 9

17. How many zeros are at the end of the product of the first 50 positive integers?
a) 9 b) 10 c) 11 d) 12

18. Find the smallest possible length for a diagonal of a rectangle with a perimeter of 10 feet?
a) 3.317 b) 3.536 c) 3.606 d) 4.123

19. How many positive factors does 1000 have? a) 35 b) 27 c) 22 d) 16

20. The Fibonacci Middle School Math Club is preparing for their first competition. The entrance rules of the competition call for teams of three, where each team must include at least one girl and one boy. No student can be on more than one team. Of the 44 members that will be able to attend the competition, there are eighteen more girls than boys. What is the maximum number of teams the club can enter in the competition? a) 13 b) 18 c) 26 d) 31

21. Find two 4-digit numbers from the digits 2 through 9, using each digit exactly once, so that the absolute value of the difference of the two numbers is as small as possible.
a) 6234 & 5987 b) 5942 & 6843 c) 4576 & 3894 d) 9642 & 8735

22. Evaluate: 1 - 2 + 3 - 4 + . . . - 98 + 99
a) - 50 b) - 49 c) 0 d) 49 e) 50

23. An integer that is greater than 5 and less than 15 is chosen at random and denoted by x. What is the probability that x^2 - 29 is prime?
a) $\frac{1}{6}$ b) $\frac{5}{9}$ c) $\frac{2}{9}$ d) $\frac{1}{3}$

24. Find the smallest integer greater than 1000 that is divisible by 5 and 13 but not 4.
a) 1040 b) 1105 c) 1625 d) 4225

25. $2\left(1-\frac{1}{2}\right)+3\left(1-\frac{1}{3}\right)+4\left(1-\frac{1}{4}\right)+\ldots+10\left(1-\frac{1}{10}\right)=$

a) 45 b) 49 c) 50 d) 54 e) 55

26. For $x=7$, which of the following is the smallest?

a) $\frac{6}{x}$ b) $\frac{6}{x+1}$ c) $\frac{6}{x-1}$ d) $\frac{x}{6}$ e) $\frac{x+1}{6}$

27. $100 \cdot 19.98 \cdot 1.998 \cdot 1000 = ?$

a) $(1.998)^2$ b) $(19.98)^2$ c) $(199.8)^2$ d) $(1998)^2$ e) $(19980)^2$

28. What is the sum of the digits of the decimal form of the product $2^{1999} \cdot 5^{2001}$?

a) 1 b) 2 c) 5 d) 7 e) 10

29. Find the sum of all prime numbers between 1 and 100 that are simultaneously 1 greater than a multiple of 4 and 1 less than a multiple of 5.

a) 118 b) 137 c) 158 d) 187 e) 245

30. The point (4, 3) is reflected about the x-axis to a point P. Then P is reflected about the y-axis to a point Q. What is the sum of the coordinates of Q?

a) - 7 b) - 1 c) 1 d) 7

31. Which of the following numbers can be expressed as the sum of the squares of six odd integers? a) 1997 b) 1998 c) 1999 d) 2000

32. The sum of the positive divisors of 360 is 1170. What is the sum of the reciprocals of the positive divisors of 360? a) $\frac{1}{1170}$ b) $\frac{1}{360}$ c) $\frac{36}{117}$ d) $\frac{117}{36}$

33. The fourteen digits in a credit card number are represented by the boxes below. If the sum of any three consecutive digits is 18, what is the value of X?

			7				X				8		

a) 2 b) 3 c) 4 d) 5 e) 6

34. What is the smallest integral multiple of 30 that has exactly thirty-six divisors?

a) 750 b) 960 c) 1260 d) 1560

35. How many odd numbers are in the 100^{th} row of Pascal's triangle?

a) 7 b) 8 c) 9 d) 10