## Question 1
## Alpha State Bowl
## Mu Alpha Theta National Convention 2003

Given $12y^2 - 4x^2 + 72y + 16x + 44 = 0$ find ABC if

A = product of the slopes of the asymptotes

B = distance from the intersection of the asymptotes to one foci

C = eccentricity of this conic section

## Question 2
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Find the absolute value of $(a - b + c)$ given

a = the coefficient of the middle term in the expansion of $(3x - y^2)^4$

b = length of the vector $(-9,12,20)$

c = the square of the reciprocal of $0.5 - 0.5i$ ($i = \sqrt{-1}$)

## Question 3
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If $f(x) = f(x-1) + x^2$ and $f(2) = -5$, find $\sum_{x=1}^{6} f(x)$.

## Question 4
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Solve $\sum_{x=0}^{2}(n^3x^3 - 3n^2x^2 + 3nx - 1) = 4n^2 - 4$ for all rational values of n.

## Question 5
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Solve for all real values of x that satisfy $8^a + 2x^2 = x(\ln e^3)$, if $a = \log_2 x$.

## Question 6
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Given:

$A = \log_{58}|3 - 7i|$

B = the vertical shift for $3y = 4\sin(3x - \frac{\pi}{4}) + 6$. Express shift in degrees.

$C = x_1 \cdot x_2 \cdot y_1 \cdot y_2$ given $(x_1,y_1)$ and $(x_2,y_2)$ are the foci of

$4x^2 - 24x + 9y^2 - 36y = -36$

Find $(A + C)/B$

## Question 7
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Given $5^x = 3^{2x+1}$, $a = \ln 3$, and $b = \ln 5$ express x in terms of a and b.

## Question 8
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Evaluate $(A / B - C)^2$, given

A =the sum of the numerator and the denominator to the simplified improper fractional equivalent of $3.7\overline{2}$,

$$B = -\frac{2}{3} + \frac{2}{9} - \frac{2}{27} + \frac{2}{81} - \ldots$$

$$C = \sum_{k=1}^{2003} i^k, \; i = \sqrt{-1}$$

## Question 9
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Let A = the 4th term of the expansion of $(x + y)^5$

B = the area of a trapezoid with a height of 6x and bases with length $2xy^3$ and $8xy^3$

C= the tenth term of the arithmetic sequence

$x^2y^3 + 4x^2y^3 + 7x^2y^3 \ldots$

Find A + B + C

## Question 10
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Let A = the remainder when $(x^4 - 7x^3 + 9x^2 + 13x - 4)$ is divided by $(x + 1)$

B = the value of $x^4 - 7x^3 + 9x^2 + 13x - 4$ when $x = 4$.

C = the sum of the irrational roots of $x^4 - 7x^3 + 9x^2 + 13x - 4 = 0$

D = the average of the roots of $x^4 - 7x^3 + 9x^2 + 13x - 4 = 0$

Find $\frac{A+B+C}{D}$

## Question 11
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Simplify completely: $\frac{1-\cos x}{\sin^2 x} - \frac{\tan x}{\tan x + \sin x}$ for $0 < x < \frac{\pi}{2}$

## Question 12
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Find the area of an equilateral triangle inscribed in a circle which is inscribed in a triangle with sides 20, 21, and 29.

## Question 13
## Alpha State Bowl
## Mu Alpha Theta National Convention 2003

If $\tan x = a$, express $\tan(x + 45^\circ) + \tan(x - 45^\circ)$ as a function of a for $0 < x < \frac{\pi}{4}$

## Question 14
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Solve for $(x, y, z)$:

$$\frac{2}{x} + \frac{1}{y} + \frac{7}{z} = 0$$

$$\frac{3}{x} + \frac{2}{y} + \frac{6}{z} = 1$$

$$\frac{5}{x} + \frac{5}{y} + \frac{1}{z} = 4$$

**Question 15**
**Alpha State Bowl**
**Mu Alpha Theta National Convention 2003**

In right triangle ABC one leg $AC = 2(\sin 15^\circ - \cos 15^\circ)$ and the other leg $BC = \cos 60^\circ$. Find the length of hypotenuse AB.

# Solutions for Alpha State Bowl Mu Alpha Theta National Convention

1. Complete the square:

$$\frac{(y+3)^2}{4}-\frac{(x-4)^2}{12}=1 \Rightarrow m=\pm\frac{1}{\sqrt{3}}$$

Product of slopes= $-\frac{1}{3}$

Distance to one foci= $\sqrt{4+12}=4$

Eccentricity= $\frac{c}{a}=\frac{4}{2}=2$

ABC = $(-\frac{1}{3})(4)(2)=\frac{-8}{3}$

2. a= ${}_4C_2(3x)^2(y^2)^2=6(9)(1)=54$

b= $\sqrt{(-9)^2+(12)^2+(20)^2}=25$

c= $\frac{1}{2}-\frac{1}{2}i=\frac{1-i}{2}\Rightarrow(\frac{2}{1-i})^2=\frac{4}{-2i}=2i$

$|a-b+c|=|29+2i|=13\sqrt{5}$

3. Let x=2, f(2)=f(1)+4⇒f(1)=-9.
f(3)=4, f(4)=20, f(5)=45, and f(6)=81.
Sum =136

4. $(-1)+(n^3-3n^2+3n-1)+(8n^3-12n^2+6n-1)=4n^2-4 \Rightarrow 9n^3-19n^2+9n+1=0$

Use synthetic division:

| 1 | 9 | -19 | 9 | 1 |
|---|---|---|---|---|
| | | 9 | -10 | -1 |
| | 9 | -10 | -1 | |

Remaining roots not rational. n=1

5. $\ln e^3=3$, $8^a=2^{3a}=x^3$
$x^3+2x^2=3x \Rightarrow x=0, -3, 1$ but only 1 is usable.

6. A= $\log_{58}|3-7i|=\log_{58}58=1$

B= vertical shift $3y=4\sin(3x-\frac{\pi}{4})+6$,

factor 3 from $3x-\frac{\pi}{4}\Rightarrow 3(x-\frac{\pi}{12})\Rightarrow\frac{\pi}{12}=15^\circ$.

C= Complete square, distance to foci= $\sqrt{5}$, foci $(3+\sqrt{5},2)$ and $(3-\sqrt{5},2)$. Product=16.
(A+C)/B=17/15

7. $5^x=3^{2x+1}\Rightarrow x\ln5=(2x+1)\ln3\Rightarrow$
$xb=(2x+1)a\Rightarrow xb-2xa=a\Rightarrow x=\frac{a}{b-2a}$.

8. $A=3.7\overline{2}=3\frac{65}{90}=3\frac{13}{18}=\frac{67}{18}\Rightarrow sum=85$

$$B=\frac{\frac{-2}{3}}{1-\frac{-1}{3}}=\frac{-1}{2}$$

C=4 Consecutive powers of i have sum of 0. Remaining powers are i-1-i = -1.
$(A/B-C)^2=(85/-.5+1)^2=29241$

9. A= ${}_5C_3(x)^2(y)^3=5x^2y^3$
B= area= $.5(6x)(2xy^3+8xy^3)=30x^2y^3$
C⇒ $a_{10}=a_1+(n-1)d=x^2y^3+9(3x^2y^3)=28x^2y^3$
A+B+C= $68x^2y^3$

10. A ⇒ let x=-1, remainder = 0.
B⇒let x=4, value = 0.
C⇒Use synthetic division and divide by -1 and 4. Remaining quadratic⇒ $x^2-4x+1$ has two irrational roots and sum=4.
D= sum/4= 7/4

$$\frac{A+B+C}{D}=\frac{16}{7}$$

11.

$$\frac{1-\cos x}{\sin^2 x}-\frac{\tan x}{\tan x+\sin x}=$$

$$\frac{1-\cos x}{1-\cos^2}-\frac{\frac{\sin x}{\cos x}}{\frac{\sin x}{\cos x}+\sin x}=$$

$$\frac{1}{1+\cos x}-\frac{\sin x}{\sin x(1+\cos x)}=0$$

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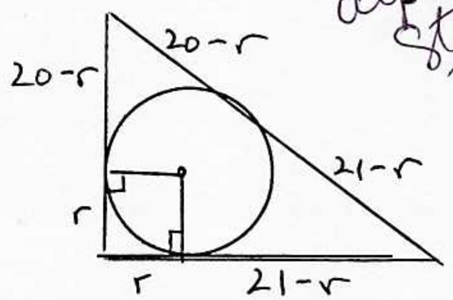
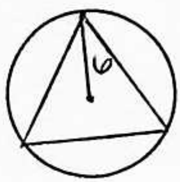
12.




29=(21-r)+(20-r)$\Rightarrow$r=6




Height triangle = 9, base=6$\sqrt{3}$
Area=27$\sqrt{3}$.

13. $\tan(a \pm b) = \dfrac{\tan a \pm \tan b}{1 \mp \tan a \tan b}$ and tan 45=1

tan(x+45)=$\dfrac{\tan x + \tan 45}{1 - \tan x \tan 45} = \dfrac{a+1}{1-a}$,

tan(x-45)=$\dfrac{\tan x - \tan 45}{1 + \tan x \tan 45} = \dfrac{a-1}{1+a}$,

$$\frac{a+1-1}{1-a} + \frac{a-1}{1+a} = \frac{a^2+2a+1-a^2+2a-1}{(1-a)(1+a)} =$$

$$\frac{2a(a+2)}{(1-a)(1+a)}$$

14.

$$\frac{2}{x} + \frac{1}{y} + \frac{7}{z} = 0$$

$$\frac{3}{x} + \frac{2}{y} + \frac{6}{z} = 1$$

$$\frac{5}{x} + \frac{5}{y} + \frac{1}{z} = 4$$

Solution (3, 2, -6)

15. Use Pythagorean Property

$$\sqrt{[2(\sin 15\ -\ \cos 15)]^2 + [\cos 60]^2} = 1.5$$