P What is the length of the segment of the line y + 7x = 26 which lies inside the circle $x^2 + y^2 - 2y = 24$?

Solution: $y=26-7x$, $x^2 + y^2 - 2y - 24 = 0$ by substitution $x^2 + (26-7x)^2 - 2(26-7x) - 24 = 0$; $50x^2 - 350x + 600 = 0$; $x^2 - 7x + 12 = 0$; $(x-3)(x-4)=0$; x = 3 then y = 5; x = 4 then y = -2
d((3, 5), (4, -2)) = $\sqrt{(4-3)^2 + (-2-5)^2} = \sqrt{50} = 5\sqrt{2}$

1. The equation $x^6 + 5x^5 + 5x^4 - 7x^3 - 9x^2 + 3x + 2 = 0$ has 2 rational roots and 4 irrational roots. Find the sum of the irrational roots.

Solution: The sum of the roots of the given equation is $\frac{-b}{a}$. Checking the possible rational roots ± 2, ± 1 you'll find $x^6 + 5x^5 + 5x^4 - 7x^3 - 9x^2 + 3x + 2 = 0$ $(x-1)(x+2)(x^4 + 4x^3 + 3x^2 - 2x - 1) = 0$
Then the sum of the 4 irrational roots is -4.

2. Solve for x: $\sum_{r=0}^{4} \binom{4}{r} 5^{4-r} x^r = 64$

Solution: In sigma notation $(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$ therefore, $\sum_{r=0}^{4} \binom{4}{r} 5^{4-r} x^r = 64$ is the same as $(x+5)^4 = 64$; x = $-5 \pm 2\sqrt{2}$

3. The equation $5^x + \frac{10}{5^x} = 7$ has x=1 as a solution. Find another solution.

Solution:
$5^x + \frac{10}{5^x} = 7 \quad \therefore \quad (5^x)^2 - 7(5^x) + 10 = 0 \quad \therefore \quad (5^x - 5)(5^x - 2) = 0$ x=1; $5^x = 2$ x=$\log_5 2 = \frac{\log 2}{\log 5}$

4. If Cos (9x) – Cos (7x) = 0, find the number of solutions for x, where $0 < x \le \frac{\pi}{2}$.

Solution: Cos (9x) – Cos (7x) = 0; Cos (8x + x) – Cos (8x – x) = 0;
Cos (8x) Cos x – Sin (8x) Sin x – [Cos (8x) Cos x + Sin (8x) Sin x] = 0
-2Sin (8x) Sin x = 0; Sin (8x) Sin x = 0; 8x = $\pi, 2\pi, 3\pi, 4\pi$; $x = \frac{\pi}{8}, \frac{\pi}{4}, \frac{3\pi}{8}, \frac{\pi}{2}$, so the number of solutions is 4.

5. What is the coefficient of x^2 in the expansion of $\left(4x^2 + \frac{1}{2x}\right)^7$?

Solution: $\left(4x^2 + \frac{1}{2x}\right)^7 = \ldots + \binom{7}{4}\left(4x^2\right)^3\left(\frac{1}{2x}\right)^4 + \ldots$ $\quad \frac{7!}{4!3!}\left(2^6\right)\left(\frac{1}{2^4}\right)x^2 = 140x^2$,

so coefficient is 140.

6. What expression must be used in order to rationalize the denominator in $\frac{1}{3-\sqrt[3]{2}}$?

Solution: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$ where a = 3 and b = $\sqrt[3]{2}$. The numerator and denominator must be multiplied by $9 + 3\sqrt[3]{2} + \sqrt[3]{4}$ in order to get $a^3 - b^3 = (3)^3 - (\sqrt[3]{2})^3 = 27-2=25$

7. If $N^{\log_2 3} = 8$, what is $N^{(\log_2 3)^2}$?

Solution: $N^{\log_2 3} = 8 \quad \left[N^{\log_2 3}\right]^{\log_2 3} = 8^{\log_2 3} \quad N^{(\log_2 3)^2} = 2^{3\log_2 3} \quad N^{(\log_2 3)^2} = 2^{\log_2 3^3} = 27$

8. Find the slope(s) of the line(s) formed when graphing y = |2x - 4| + |4 - x| ?

Solution: y = |2x - 4| + |4 - x| = $\begin{cases} -3x+8, & \text{if } x \le 2 \\ x, & \text{if } 2 \le x \le 4 \\ 3x-8, & \text{if } x \ge 4 \end{cases}$ y=mx+b where m = -3, 1, 3

9. Find x if $\sqrt{5+2\sqrt{6}} - \sqrt{5-2\sqrt{6}} = x\sqrt{x}$.

Solution: Squaring both sides of $\sqrt{5+2\sqrt{6}} - \sqrt{5-2\sqrt{6}} = x\sqrt{x}$ you get
$5+2\sqrt{6} - 2\sqrt{25-24} + 5 - 2\sqrt{6} = x^3 \quad 8 = x^3 \quad x = 2$

10. Solve $x^4 - x^3 - 19x^2 + 49x - 30 < 0$. State your final answer in interval notation.

Solution: The possible rational roots are $\pm(1,2,3,5,6,10,15,30)$. Using synthetic division,
$x^4 - x^3 - 19x^2 + 49x - 30 < 0 \quad \Rightarrow \quad (x+5)(x-1)(x-2)(x-3) < 0$ +++|---------|----|++|---|
(-5, 1) $\cup$ (2,3) -5 0 1 2 3