

GEMINI - ALPHA
Mu Alpha Theta National Convention 2003

For all questions, answer E. "NOTA" means none of the above answers is correct.

1. A "really odd number" is one such that all the digits in the number are odd. For example, the number 975 is really odd. How many really odd numbers are there between 0 and 10000?

- A. 75 B. 780 C. 3900 D. 5000 E. NOTA

2. Quadrilateral ABCD has interior angles $A=60^\circ$, $B=75^\circ$, $C=135^\circ$, and $D=90^\circ$. What is the minimum number of side lengths you need to know in order to calculate the area of ABCD?

- A. 1 B. 2 C. 3 D. 4 E. NOTA

3. What is the base 10 fractional equivalent of $.56\overline{7}_8$?

- A. $375/512$ B. $377/512$ C. $183/252$ D. $185/252$ E. NOTA

4. If $(7x - 3y)\sqrt{(2x + 3y)} = 6$, then what is the value of $(x - y)\sqrt{(x + y)}$?

- A. $-8/13$ B. $-13/8$ C. $8/13$ D. $13/8$ E. NOTA

5. If the distinct permutations of the word **NATIONALS** are listed in alphabetical order, what is the 2003rd permutation on the list?

- A. AASNOLTIN B. AASNOLNIT C. AATOLSINN D. AATNOLSIN E. NOTA

6. The geometric mean of two numbers is 14; whereas, the harmonic mean of the two numbers is 12. What is the arithmetic mean of the two numbers?

- A. $49/6$ B. 8 C. $49/3$ D. 16 E. NOTA

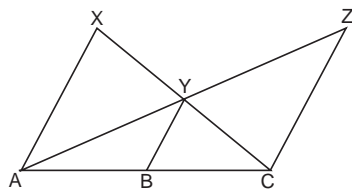
7. If the perimeter of an isosceles triangle is 36 and the altitude to the base is 9, then what is the length of the altitude to one of the legs?

- A. $48/17$ B. $96/17$ C. $54/5$ D. $27/5$ E. NOTA

8. If the probability of Alex answering a chemistry question correctly is 0.6, then what is the probability that he will pass a five question chemistry test (that is, get a 60% or higher on the test)?

- A. 1 B. $3/5$ C. $513/3125$ D. $2133/3125$ E. NOTA

9. In the figure below $AX \parallel BY \parallel CZ$, $AX = 5$, and $CZ = 4$. What is the length of BY ?



- A. $9/20$ B. $20/9$ C. $3/7$ D. $7/3$ E. NOTA

10. What is the largest integer n such that 7^n is a factor of $2003!$?

- A. 331 B. 343 C. 376 D. 2003 E. NOTA

11. Given that $\cos x + \cos 2x + \cos 3x = 0$, what is the sum of the roots for $0 < x < 2\pi$?

- A. 3π B. 4π C. 5π D. 6π E. NOTA

12. What is the length of the largest median in a triangle whose sides are 7, 8, and 9?

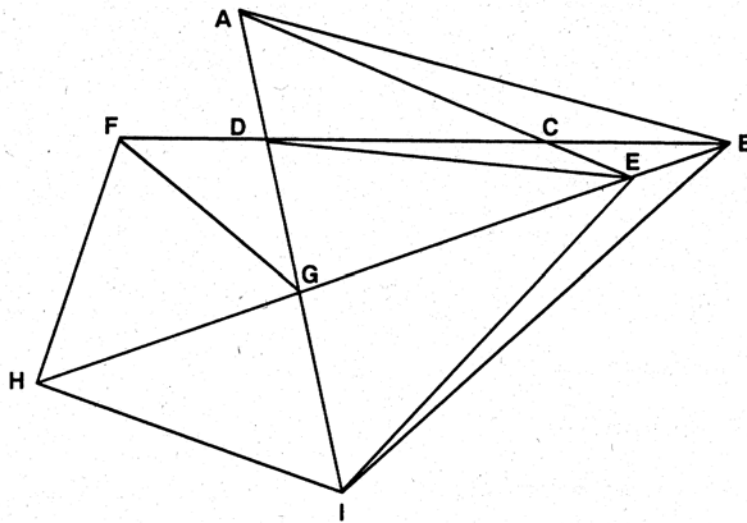
- A. $241/4$ B. $179/4$ C. $\sqrt{179}/2$ D. $\sqrt{241}/2$ E. NOTA

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13. The roots of the equation $x^3 + 4x^2 + 7x + 2 = 0$ are a , b , and c . Which equation below has roots that are $(1/a)+2$, $(1/b)+2$, and $(1/c)+2$?
- A. $2x^3 + 7x^2 + 4x + 1 = 0$ B. $2x^3 - 5x^2 + 5 = 0$
 C. $2x^3 + 14x^2 + 42x + 8 = 0$ D. $2x^3 + 5x^2 + 2x - 1 = 0$ E. NOTA
14. Newton's law of cooling states that the rate at which the temperature of an object changes is roughly proportional to the difference between the temperature of the object, T , and the temperature of the surrounding medium, T_s . If the initial temperature is T_0 , then the temperature of the object as a function of time, t , is given by the formula $T = T_s + (T_0 - T_s)e^{kt}$, where k is a constant. Use this information to answer the following problem: A pan of warm water (46°C) was put in a refrigerator. Ten minutes later, the water's temperature was 39°C ; 10 minutes after that, it was 33°C . How cold is the refrigerator?
- A. 0°C B. -3°C C. 3°C D. 11°C E. NOTA
15. Rank the following investments in order from largest to smallest yielding after two years.
- I. \$5000 principal at 8% interest rate compounded semi-annually
 II. \$5100 principal at 7% interest rate compounded quarterly
 III. \$5200 principal at 6% interest rate compounded daily
 IV. \$5300 principal at 5% interest rate compounded continuously
- A. IV, III, II, I B. I, II, III, IV C. III, II, IV, I D. I, IV, II, III E. NOTA
16. What are the asymptotes of the graph of $y = \frac{x^5 + 5x^4 - 9x^3 - 49x^2 - 8x + 60}{x^4 - 5x^3 - 7x^2 + 41x - 30}$?
- A. $x = -3, x = 1, x = 2$, and $x = 5$ B. $x = -3, x = 1, x = 2, x = 5$, and $y = x + 10$
 C. $x = -3, x = 2$, and $x = 5$ D. $x = -3, x = 2, x = 5$, and $y = x + 1$ E. NOTA
17. What are the roots of the equation: $\begin{vmatrix} x & 7 & 0 \\ 2 & x-6 & 7 \\ 0 & -1 & x \end{vmatrix} = 0$?
- A. $x = 0, 1$, and 7 B. $x = -1, 0$, and 7 C. $x = 0$ and $3 \pm \sqrt{2}$ D. $x = 0$ and $2 \pm \sqrt{3}$ E. NOTA
18. What is the sum of a , b , and c that satisfies the following system of equations:
- $(a/2) - (b/3) + (c/6) = 10$
 $(a/3) + (b/4) - (c/2) = -10$ A. -12 B. -6 C. 6 D. 12 E. NOTA
 $(a/6) + (b/6) - (c/6) = -4$
19. How many terms are there in the simplified expansion of $(w+x+y+z)^5$?
- A. 56 B. 70 C. 35 D. 28 E. NOTA
20. What is the sum of all natural numbers, n , such that $n!$ ends in exactly 2003 zeroes?
- A. 40110 B. 40135 C. 8020 D. 8025 E. NOTA
21. Points $(2,4)$, $(2,10)$, and $(10,4)$ are vertices of a triangle. What is the y -intercept of the line containing the angle bisector to the smallest side of the triangle?
- A. $(0,22)$ B. $(0,-14)$ C. $(0,22/3)$ D. $(0,7/3)$ E. NOTA

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22. The medians of a triangle are concurrent at a point (the centroid) that divides each median into what ratio?
 A. 4:1 B. 3:1 C. 2:1 D. 1:1 E. NOTA
23. What is the point on the plane $3x + 2y + 6z = 6$ that is closest to the point $(1, 1, 3)$?
 A. $(-2/49, 15/49, 45/49)$ B. $(1, 1, 1/6)$ C. $(5/6, 7/6, 5/2)$ D. $(4/25, 8/25, 51/25)$ E. NOTA
24. The planet Mars is brightest in the night sky when it is closest to Earth. Assuming that the Earth's and Mars's orbit around the sun (at the center) are perfectly circular, in the same plane, and have a period of 365 days and 687 days, respectively, how often (to the nearest day) is Mars at its brightest?
 A. 365 days B. 526 days C. 687 days D. 779 days E. NOTA
25. The equation $5x^2 - 2xy + y^2 = 1$ represents an ellipse. What is the area of this ellipse?
 A. $\pi\sqrt{6}/6$ B. $\pi\sqrt{6}$ C. $\pi/2$ D. 2π E. NOTA
26. The diagram below is filled with triangles small, medium, and large. How many distinct triangles are there?



- A. 10 B. 16 C. 22 D. 24 E. NOTA
27. What is the sine of the acute angle between the vectors $\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$ and $3\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$?
 A. $2\sqrt{15}/15$ B. $4\sqrt{15}/15$ C. $\sqrt{15}/15$ D. $\sqrt{165}/15$ E. NOTA
28. Find $\sum_{n=1}^{\infty} \frac{2n}{6^n}$.
 A. $2/5$ B. $12/25$ C. $1/2$ D. Does not converge E. NOTA
29. What is the coefficient of the term without x in the expansion of $\left[2x + \left(\frac{3}{x^2}\right)\right]^6$?
 A. 51840 B. 4320 C. 2880 D. 2160 E. NOTA
30. What is the focal width (the length of the line segment through the focus and perpendicular to the axis of the parabola) of the parabola governed by the equation $8x^2 - 32x - y + 36 = 0$?

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A. 0 B. $\frac{1}{8}$ C. $\frac{1}{16}$ D. $\frac{1}{32}$ E. NOTA