

1. C - There are $2 \cdot 2 = 4$ cases: 1) both even- product is even, 2) both odd- product is odd, 3) 1^{st} even, 2^{nd} odd- product is even, and 4) 1^{st} odd, 2^{nd} even- product is even. $\frac{3}{4}$ of these cases yield even products.

2. A - Definition: two events are “mutually exclusive” when their intersection is the null set.

3. D - The order of the 3 people must have been MFM or FMF. $\frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} = \frac{6}{27} = \frac{2}{9}$ and $9 - 2 = 7$.

4. D - There are $\frac{6!}{3! \cdot 2!}$ permutations of “BANANA,” and 5 of them have consecutive N's. Therefore the probability is $\frac{5}{60} = \frac{1}{12}$.

5. C - Because the 2^{nd} throw was worse than the 1^{st} , the 3 throws could have ranked $\{1,2,3\}$, $\{1,3,2\}$, or $\{3,1,2\}$ (from best to worst). In $\frac{2}{3}$ of these possibilities, the 3^{rd} throw is worse than the 1^{st} .

6. B - There are $10^2 - 9^2 = 19$ integers on $[1, 100]$ that contain a 7, $10^3 - 9^3 = 271$ integers on $[1, 1000]$ that contains a 7, ..., and $10^n - 9^n$ integers out of 10^n integers on $[1, 10^n]$ that contain a 7. The probability is therefore $\frac{10^n - 9^n}{10^n}$.

7. B - 13 possible ranks for the group of 3, and ${}_4C_3 = 4$ ways to pick the suits for those 3. 12 possible remaining ranks for the group of 2, and ${}_4C_2 = 6$ ways to pick the suits for those 2. So there are $13 \cdot 4 \cdot 12 \cdot 6 = 3744$ ways to draw a full house out of ${}_{52}C_5$ possible 5-card hands.

8. A - $P(F \cap E) = P(E) + P(F) - P(E \cup F) = \frac{2}{13}$.
 $P(F \cap \bar{E}) = P(F) - P(F \cap E) = \frac{3}{13}$.
 $P(F|E) = \frac{P(F \cap E)}{P(E)} = \frac{3}{10}$.

9. E - The product of any two integers each greater than 1 is always composite, so the probability is 1.

10. B - The probability of the event that at least one match will be made is the complement of the event that no matches will be made.

$1 - P(\text{Not Dena} \cap \text{Not Courtney} \cap \text{Not Leigh Ann}) = 1 - \left[\left(\frac{11}{12}\right)^5\right]^3 \approx .729$.

11. B - There are $5 \cdot 5 = 25$ total two-number sets and 6 of those whose elements sum to 4 or less: $\{1, 1\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 1\}$, $\{2, 2\}$, and $\{3, 1\}$. So there are $25 - 6 = 19$ sets whose element sum to greater than 4, and $\frac{19}{25} = .76$.

12. C - 84% of the distribution lies below the point 1 standard deviation above the mean of a normal distribution.

13. C - Initially, there is a $\frac{1}{3}$ chance that the car lies behind any of the 3 doors. Also, it is equally likely that the host shows you what lies behind either door #2 or #3. If the host shows door #2, then the probability of the car lying behind door #1 is $\frac{\frac{1}{3}}{\frac{1}{2}} = \frac{1}{3}$, while with door #3 it is $\frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$. Therefore, the best strategy is to switch to door #3.

14. B - $\frac{364}{365} \cdot \frac{363}{365} \cdot \dots \cdot \frac{1}{365} = \frac{364!}{365^{364}}$.

15. C - $P(\text{rains on two specific days in a week}) = \frac{1}{7C_2}$.
 $P(\text{rains on two specific consecutive weeks}) = \left(\frac{1}{7C_2}\right)^2$.
 Imagining that there are 5 two-week periods (with 1 week overlapping a week of another two-week period) within a 6-week period, the probability is $1 - P(\text{doesn't rain on two specific consecutive weeks})^5 = 1 - \left[1 - \left(\frac{1}{7C_2}\right)^2\right]^5 \approx .011$.

16. D - $\frac{h+h(1-h)}{\frac{h}{10} + \frac{h}{10}(1-\frac{h}{10})} = \frac{200-100h}{20-h}$.

17. A - $P(\text{“Captain Steve” doesn't place}) = \frac{6}{6+1} = \frac{6}{7}$.
 $P(\text{“Fantastic Light” doesn't place}) = \frac{11}{11+1} = \frac{11}{12}$.
 $P(\text{neither places}) = \frac{6}{7} \cdot \frac{11}{12} = \frac{11}{14}$.

18. D - $|f(x)| < 1$ on $[-\frac{9\pi}{4}, -\frac{7\pi}{4}]$, $[-\frac{5\pi}{4}, -\frac{3\pi}{4}]$, $[-\frac{\pi}{4}, \frac{\pi}{4}]$, $[\frac{3\pi}{4}, \frac{5\pi}{4}]$. These intervals add up to a length of 4π out of a total domain length of 8π , and $\frac{4\pi}{8\pi} = \frac{1}{2}$.

19. C - There are 4 prime numbers less than 10: 2, 3, 5, and 7. There are 10 whole numbers on $[0, 9]$ and so $\frac{4}{10} = \frac{2}{5}$.

20. C - If ${}_{30}C_c \cdot \left(\frac{1}{5}\right)^c \cdot \left(\frac{4}{5}\right)^{30-c} \approx .154$, where c is the number of questions answered correctly, then $c = 7$.
 $n = 4c - (30 - c) = 28 - 23 = 5$.

21. C - There are 151 total integers on $[-30, 150]$ and 6 unattainable scores: 109, 113, 114, 117, 118, and 119. So the probability is $\frac{6}{151}$.

22. E - There exist $2^4 = 16$ possible combinations and 8 of them include onions. So the probability is $\frac{8}{16} = \frac{1}{2}$.

23. C - A chord of length 2 (length of the shorter dimension of the rectangle) has endpoints at points of intersection between the circle and rectangle: $\{-1, \sqrt{3}\}$ and $\{1, \sqrt{3}\}$. This chord, combined with a line segment drawn from the center of the circle to each of these intersections (radii), forms an equilateral triangle. Therefore, this arc is defined by an angle of $\frac{\pi}{3}$. Because there are two other intersection points at the other end of the rectangle, the total angle measure that defines the parts of the circle inside the rectangle is $2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}$ and $\frac{\frac{2\pi}{3}}{2\pi} = \frac{1}{3}$.

24. D - $\sum_{k=14}^{22} {}_{22}C_k \cdot (.75)^k \cdot (1 - .75)^{22-k} \approx .925$.

25. C - $\frac{1}{2N} \cdot N^3 = \frac{N^2}{2} = 1$.
 $N^2 = 2$ and so $N = \sqrt{2}$.

26. C - $1 - .4 = .6$ probability of heads. $P(\text{at most 2 heads}) = 1 - P(3 \text{ heads})$.
 $1 - (.6)^3 = .784 = \frac{98}{125}$.

27. A - $P(\text{pick red} \cap \text{return red}) + P(\text{pick orange} \cap \text{return orange}) + P(\text{pick blue} \cap \text{return blue}) = \frac{3}{10} \cdot \frac{6}{11} + \frac{3}{10} \cdot \frac{3}{11} + \frac{4}{10} \cdot \frac{4}{11} = .390$.

28. A - # work & private only = $15 - 12 = 3$.
 # work & state only = $38 - 12 = 26$.
 # private & state only = $100 - 12 = 88$.
 # private only = $115 - 3 - 88 - 12 = 12$.
 # state only = $160 - 88 - 26 - 12 = 34$.
 # work only = $50 - 26 - 3 - 12 = 9$.
 # remaining = $200 - 12 - 34 - 9 - 88 - 26 - 3 - 12 = 16$.
 $\frac{16}{200} = .08$.

29. A - 1^{st} digit can start with any of $\{0, 1, 2, 3\}$ for increasing numbers. These 4 possibilities can also be reversed for decreasing numbers, which makes $4 \cdot 2 = 8$ numbers out of a total 10^7 . So the probability is $\frac{8}{10^7}$.

30. D - $P(\text{winning point on } 1^{st} \text{ serve}) = .75 \cdot .8 = .6$ of her total points. $P(\text{winning point on } 2^{nd} \text{ serve}) = (1 - .75) \cdot .9 \cdot .35 = .07875$ of her total points. Therefore $P(1^{st} \text{ serve} | \text{won point}) = \frac{.6}{.6 + .07875} = \frac{160}{181}$.