Mu Alpha Theta National Convention 2003
ALPHA SEQUENCES AND SERIES

For all questions, answer "E. NOTA" means 'None Of The Above' answers is correct.

1. Find the next term for the sequence:
3, 6, 11, 18, 27, ____

A. 30 B. 36 C. 38 D. 40 E. NOTA

2. Find the sum: $\sum_{n=4}^{8}(2^n+5)$

A. 496 B. 501 C. 516 D. 521 E. NOTA

3. $3 + 6 + 12 + 24 + \ldots\ldots + 1536$ can be written as:

A. $\sum_{n=0}^{8}(3\cdot 2^n)$ B. $\sum_{n=1}^{10}(3\cdot 2^{n-1})$ C. $\sum_{n=1}^{512}(3n)$ D. $\sum_{n=1}^{10}(3n)$ E. NOTA

4. In a bake-off contest, the first place winner gets $200,000 and the twelfth-place winner gets $94,400. If the winning amounts form an arithmetic sequence, find the positive difference between each prize.

A. $8800 B. $8900 C. $9000 D. $9100 E. NOTA

5. Let the sum of the first n terms of an arithmetic series with first term -13 and the common difference 3.5 be -17.5. Let the sum of the first m terms of a geometric series with first term $\frac{1}{3}$ and the common ratio $\frac{2}{3}$ be $\frac{211}{243}$. Find $n + m$.

A. 10 B. 11 C. 12 D. 13 E. NOTA

6. Find the sum of the odd integers from 11 to 101, inclusive.

A. 2,520 B. 2,576 C. 5,040 D. 5,096 E. NOTA

7. Susan drops a ball from her balcony. The balcony is 80 feet above the ground. If the ball rebounds 80% of the height from which it fell on each bounce, find the vertical distance that the ball travels before coming to rest.

A. 400' B. 700' C. 720' D. 800' E. NOTA

8. The following sequence of numbers are called triangular numbers: 1, 3, 6, 10, 15, ...
Let f(n) be the formula that will generate any triangular number t_n in terms of n.
The following sequence of numbers are called square numbers: 1, 4, 9, 16, 25, ...
Let y be the sum of the first 20 square numbers. Find $y\cdot f(n)$.

A. $200n^2 + 200n$ B. $400n^2 + 400n$ C. $1235n^2 + 1235n$ D. $1435n^2 + 1435n$ E. NOTA

9. A bank pays 3.75% interest per year compounded monthly. A customer opens a new account and deposits $200 on the first day of each month into the account. How much to the nearest dollar is in the account after 11 months before a deposit for the 12th month is made?

A. $42 B. $2235 C. $2242 D. $2449 E. NOTA

10. Find the next term in the following sequence.

5, 7, 11, 19, 35, 67, _____

A. 101 B. 127 C. 131 D. 139 E. NOTA

11. Find the number of diagonals in a regular nonagon.

A. 24 B. 27 C. 36 D. 45 E. NOTA

12. At 3:00 p.m., a patient is given one doseage of 2,000 mg of medicine. At the end of each hour, the concentration of the medication is 70% of the amount present at the beginning of the hour. How many mg of the medication to the nearest tenth remains in the patient's body at 8:00 p.m., if the patient has not taken any more medicine and has no residual medicine from before the doseage given at 3:00 p.m.?

A. 115.3 B. 235.3 C. 336.1 D. 480.2 E. NOTA

13. The sum of the first four terms of an arithmetic sequence is -80. The sum of the first eight terms of the same arithmetic sequence is 80. Find the sum of the first six terms of this sequence.

A. -60 B. -30 C. 0 D. 15 E. NOTA

14. The formula $f(n) = \frac{n}{2}(3n-1)$ generates the _____ numbers.

A. Lucas B. triangular C. pentagonal D. Fibonacci E. NOTA

15. An 8 by 8 checkerboard has sixty-four 1 by 1 squares and many other squares of various sizes. How many 7 by 7, 5 by 5, 3 by 3, and 1 by 1 squares are there altogether?

A. 114 B. 116 C. 120 D. 204 E. NOTA

16. Which is not a correct expression for the series: $\frac{1}{2}+\frac{1}{3}+\frac{1}{2}+\frac{6}{5}+4+\frac{120}{7}+\ldots$

A. $\sum_{n=2}^{\infty}\frac{(n-2)!}{n}$ B. $\sum_{n=0}^{\infty}\frac{n!}{n+2}$ C. $\sum_{n=1}^{\infty}\frac{(n-1)!}{(n+1)}$ D. $\sum_{n=1}^{\infty}\frac{n!}{n(n+1)}$ E. NOTA

17. A retired teacher receives $28,000 from her retirement fund the first year that she retires. Each year that amount will increase 3%. What is the total amount that she will receive during the first 10 years of her retirement to the nearest dollar?

A. $284,455 B. $286,597 C. $320,989 D. $933,333 E. NOTA

18. Find the 50th term of the following sequence (Fibonacci).
1, 1, 2, 3, 5, 8, 13, 21, ...

A. 7,778,742,049 B. 10,000,000,001 C. 12,586,269,025 D. 20,365,011,074 E. NOTA

19. Use the first 5 terms of the power series for e^x to estimate e to the nearest ten-thousandth.

A. 1.7167 B. 2.6667 C. 2.7083 D. 2.7167 E. NOTA

20. The following sequence converges to _____.

$$10, 5, \frac{5}{2}, \frac{5}{4}, \frac{5}{8}, \ldots\ldots$$

A. 20 B. 10 C. $\frac{1}{2}$ D. 0 E. NOTA

21. Change the repeating decimal, .37545454... to a ratio, $\frac{a}{b}$, where a and b are natural numbers that are relatively prime. Find $b-a$.

A. 687 B. 1513 C. 4019 D. 6183 E. NOTA

22. The midpoints of the sides of a square that has a side of length 8 have been connected to form another square. Then the midpoints of the sides of the new square are connected to form another square, and so on. Find the sum of the perimeters of the first 10 squares, including the original square.

A. 64 B. $62+30\sqrt{2}$ C. $62+31\sqrt{2}$ D. $63+31\sqrt{2}$ E. NOTA

23. Find the sum of the two real geometric means inserted between 15 and 234.375.

A. 56.25 B. 109.6875 C. 126.5625 D. 131.25 E. NOTA

24. Find the limit as $n \to \infty$ for the sequence $a_n = \frac{3n^3+2n^2+5}{5n^2+7n^3+11}$.

A. 0 B. $\frac{3}{7}$ C. $\frac{5}{11}$ D. $\frac{3}{5}$ E. NOTA

25. Find the constant term for the binomial expansion of $(x^2 - \frac{5}{x})^6$.

A. −9375 B. −75 C. 75 D. 9375 E. NOTA

26. In an arithmetic sequence of complex numbers the first term is $3+4i$ and the second term is 7. Find the sum of the first 50 terms.

A. $5050-4700i$ B. $5000-7100i$ C. $5000+7100i$ D. $5050+7100i$ E. NOTA

27. Find the sum of all positive 3-digit numbers divisible by 7.

A. 49,320 B. 69,786 C. 70,336 D. 82,350 E. NOTA

28. The sum of the series $10+5+\frac{5}{2}+\frac{5}{4}+\ldots$ is _____.

A. divergent B. 19 C. 20 D. 100 E. NOTA

29. Find the sum: $\sum_{n=4}^{8} 2^n + 5$

A. 496 B. 501 C. 516 D. 521 E. NOTA

30. The first three terms of an arithmetic sequence are 76, 72, and 68. There are 2 values such that the sum of the first n terms is 448. Find the absolute value of the difference of these two values.

A. 23 B. 25 C. 39 D. 51 E. NOTA

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Solutions Page 1

1. pattern: add 3, 5, 7, 9, 11, term will be 38 **C**
2. $2^4+2^5+2^6+2^7+2^8+5\cdot5=521$ **D**
3. $\sum_{n=0}^{8}(3\cdot2^n)$ = 3+6+24+...+768 $\sum_{n=1}^{10}(3\cdot2^{n-1})$ = 3+6+12+24+...+1536
   $\sum_{n=1}^{512}(3n)$ = 3+6+9+...+1536 $\sum_{n=1}^{10}(3n)$ = 3+6+9+...+30 **B** is the correct choice
4. 94,400=200,000+(11)d d=|−9600| **E**
5. $-17.5=\frac{n}{2}(2\cdot-13+(n-1)(3.5))$ $n=7$ $\frac{211}{243}=\frac{\frac{1}{3}(1-(\frac{2}{3})^m)}{1-\frac{2}{3}}$ $m=5$ 7+5=12 **C**
6. 101=11+(n-1)(2) n=46 S=$\frac{46}{2}(11+101)$ S=2576 **B**
7. down + up $\frac{80}{1-.8}+\frac{80\cdot.8}{1-.8}=720$ **C**
8. use finite differences to find the formula for triangular numbers: $f(n)=\frac{n^2+n}{2}$

   use the sum of the squares formula: $\sum_{i=1}^{n}i^2=\frac{n(n+1)(2n+1)}{6}$; n = 20, sum is 2870

   correct choice is $1435n^2+1435n$ **D**
9. $S=\frac{200.625(1-1.003125^{11})}{1-1.003125}\approx\$2242$ **C**
10. pattern: 2^n+3, so next term will be 131 **C**
11. formula for number of diagonals in a polygon is $\frac{n(n-3)}{2}$, since n=9, answer is 27 **B**
12. $y=2000(.70)^5\approx336.1$ **C**
13. $-80=\frac{4}{2}(2a_1+3d)$ and $80=\frac{8}{2}(2a_1+7d)$, so $a_1=-42.5$ and $d=15$

    $S_6=\frac{6}{2}(2\cdot-42.5+5\cdot15)$; $S_6=-30$ **B**
14. $f(1)=1$ $f(2)=5$ $f(3)=12$ $f(4)=22$ These are the pentagonal numbers **C**
    FYI The Lucas numbers are 1,3,4,7,11,18,29,...
15. 64 1X1 36 3X3 16 5X5 4 7X7 total of 120 **C**
16. all of the given answers are correct **E**
17. $S_{10}=\frac{28000(1-1.03^{10})}{1-1.03}\approx320{,}989$ **C**
18. $F_n=\frac{1}{\sqrt{5}}(\frac{1+\sqrt{5}}{2})^n-\frac{1}{\sqrt{5}}(\frac{1-\sqrt{5}}{2})^n$ $F_{50}=12{,}586{,}269{,}025$ **C**
19. $e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+...$ using 5 terms $1+1+\frac{1}{2}+\frac{1}{6}+\frac{1}{24}\approx2.7083$ **C**

20. each term is being multiplied by $\frac{1}{2}$, so sequence will approach 0 **D**

21. $\frac{37}{100}+\frac{.0054}{1-.01}=\frac{37}{100}+\frac{3}{550}=\frac{413}{1100}$, so $1100-413=687$ **A**

22. $S_{10}=\frac{32(1-(\frac{1}{\sqrt{2}})^{10})}{1-\frac{1}{\sqrt{2}}}=62+31\sqrt{2}$ **C**

23. $234.375=15r^3$ $r=2.5$ $g_2=37.5$; $g_3=93.75$ sum is 131.25 **D**

24. $\lim_{n\to\infty} a_n=\frac{3}{7}$ **B**

25. constant term ${}_6C_4(x^2)^2(\frac{-5}{x})^4=9375$ **D**

26. $S_{50}=\frac{50}{2}(2(3+4i)+49(4-4i))$; $S_{50}=5050-4700i$ **A**

27. $994=105+(n-1)(7)$; $n=128$; $S_{128}=\frac{128}{2}(105+994)$; $S_{128}=70336$ **C**

28. $S_\infty=\frac{10}{1-\frac{1}{2}}$; $S_\infty=20$ **C**

29. $\sum_{n=4}^{8} 2^n+5=2^4+2^5+2^6+2^7+2^8+5=501$ **B**

30. $448=\frac{n}{2}(2\cdot 76+(n-1)(-4))$ n=32 or n=7 $32-7=25$ **B**