

Mu Alpha Theta National Convention 2003  
Advanced Calculus – BC Level

\*\*\*\*\* For all problems on this test, answer choice "E" - NOTA means none of the above. \*\*\*\*\*

1.  $\lim_{x \rightarrow 0} \frac{3\sin x}{e^x}$  is
  - A. divergent
  - B. 0
  - C. 1
  - D. 3
  - E. NOTA
  
2.  $\int \frac{dx}{\sqrt{(3x+5)^2}} =$ 
  - A.  $\ln |3x+5| + C$
  - B.  $2\sqrt{(3x+5)^2} + C$
  - C.  $\frac{1}{3} \ln |3x+5| + C$
  - D.  $\sec^{-1}(3x+5) + C$
  - E. NOTA
  
3. A particle moving along a path  $r(t) = t\mathbf{i} + \left(\frac{1}{t}\right)\mathbf{j}$  at  $t = 1$  has acceleration
  - A.  $-\sqrt{2}$
  - B.  $\sqrt{2}$
  - C.  $\mathbf{i} - \mathbf{j}$
  - D.  $2\mathbf{j}$
  - E. NOTA
  
4. The arc length of the graph of  $y = 5 - x^{3/2}$  for  $1 \leq x \leq 4$  is
  - A.  $\frac{8}{27} \left[ (10)^{3/2} - \frac{(13)^{3/2}}{8} \right]$
  - B.  $\frac{8(10)^{3/2}}{27}$
  - C.  $\frac{8(3)^{3/2}}{27}$
  - D.  $\frac{8}{27} \left[ (11)^{3/2} - (13)^{3/2} \right]$
  - E. NOTA
  
5. The series  $r^2 + r^4 + r^6 + \dots$ 
  - A. always converges
  - B. always diverges
  - C. converges if  $r^2 \leq 1$
  - D. diverges if  $r \geq 1$
  - E. NOTA
  
6.  $\int x \sec^2 x \, dx$ 
  - A.  $x^2 \sec^3 \frac{x}{6} + C$
  - B.  $\tan \frac{x}{2}$
  - C.  $x \tan x + \ln |\cos x| + C$
  - D.  $\sec^2 x + 2x \sec^2 x \tan x + C$
  - E. NOTA
  
7. The first four terms of the Taylor expansion of  $y = e^{-x^2}$  about  $x = 0$  are
  - A.  $1 + x^2 + \frac{x^4}{2!} + \frac{x^6}{3!}$
  - B.  $1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!}$
  - C.  $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!}$
  - D.  $1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!}$
  - E. NOTA

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8.  $\int_{-1}^0 \frac{dx}{(x-1)(x-2)(x-3)}$
- A.  $\frac{1}{2}\ln 3 - \ln 2$       B.  $\frac{3}{2}\ln 3 - \frac{5}{2}\ln 2$       C.  $-\ln \frac{\sqrt{3}}{2}$       D.  $\frac{3}{2}\ln 2 - \ln 3$       E. NOTA
9. The area of the region bounded by the polar curve  $r = 4 + \sin \theta$  is
- A.  $\frac{3\pi}{2}$       B.  $\frac{11\pi}{2}$       C.  $8\pi$       D.  $\frac{33\pi}{2}$       E. NOTA
10.  $\lim_{x \rightarrow 0} \frac{1}{x} - \frac{1}{xe^{2x}}$
- A. divergent      B. 0      C. 1      D. 2      E. NOTA
11. Which of the following is true regarding  $\sum_{n=1}^{\infty} \frac{(-1)^n}{3n-2}$ ?
- A. converges absolutely      B. converges conditionally      C. diverges  
D. convergence/divergence cannot be determined      E. NOTA
12.  $\int t^3 \sin t^2 dt =$
- A.  $\frac{t^3 \cos t^2}{3} - \frac{t^2 \sin t^2}{2} + \frac{t \cos t^2}{1} + C$       B.  $\frac{t^3 \cos t^2}{3} - \frac{t^2 \sin t^2}{2} + \frac{t \cos t^2}{1} - \sin t^2 + C$   
C.  $\frac{t^2 \cos t^2}{2} - \frac{t \sin t^2}{2} + C$       D.  $\frac{t^2 \cos t^2}{-2} + \frac{\sin t^2}{2} + C$       E. NOTA
13.  $\int_{-2}^0 \frac{dx}{x^2 + 4x + 8} =$
- A.  $\frac{\pi}{4}$       B.  $\frac{\pi}{2}$       C.  $\frac{\pi}{6}$       D.  $\frac{\pi}{8}$       E. NOTA
14. The area of the region bounded by  $r^2 = 4 \sin 2\theta$  is
- A.  $\frac{\pi}{6}$       B.  $\frac{\pi}{3}$       C.  $\frac{\pi}{4}$       D.  $\frac{\pi}{2}$       E. NOTA

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22.  $\int \frac{dx}{(x+2)(x^2+1)}$

- A.  $\frac{1}{5} \ln|x+2| - \frac{1}{10} \ln(x^2+1) + C$   
B.  $\frac{1}{5} \ln|x+2| - \frac{1}{10} \ln(x^2+1) + \frac{2}{5} \tan^{-1} x + C$   
C.  $\ln|x+2| + \ln(x^2+1) + C$   
D.  $\frac{1}{5} \ln|x+2| - \frac{x}{5} \ln(x^2+1) + \frac{2}{5} \ln(x^2+1) + C$   
E. NOTA

23.  $\int \ln x \, dx =$

- A.  $x \ln x + C$       B.  $x \ln x - x + C$       C.  $\ln x + C$       D.  $e^x + C$       E. NOTA

24.  $\int_0^{\pi} \cos 3x \sin 3x \, dx =$

- A.  $\frac{1}{6}$       B.  $-\frac{1}{6}$       C. 1      D. 0      E. NOTA

25.  $\int_0^{\pi/2} \cos 3x \sin 3x \, dx =$

- A.  $\frac{1}{6}$       B.  $-\frac{1}{6}$       C. 1      D. 0      E. NOTA

26. The interval of convergence of  $\sum_{n=1}^{\infty} \frac{n(n+1)x^n}{5^n}$  is

- A.  $(-5, 5)$       B.  $[-5, 5)$       C.  $(-5, 5]$       D.  $[-5, 5]$       E. NOTA

27.  $\int \frac{x^2+2}{x^2+2x} \, dx =$

- A.  $\ln|x| + C$       B.  $\ln|x| - 3\ln|x+2| + C$       C.  $\frac{x^3}{3} \ln|x| + 2\ln|x+2| + C$   
D.  $\ln\left|\frac{x}{(x+2)^3}\right| + x + C$       E. NOTA

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15.  $\int_{-1}^1 \frac{dx}{x^2 - 1}$  is

- A. diverging to  $\infty$       B. diverging to  $-\infty$       C. 0      D. 1      E. NOTA

16. Which of the following statements is true about the series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ ?

- A. The series converges by the ratio test  
B. The series diverges by the ratio test  
C. The ratio test is inconclusive for this series  
D. The series diverges by the alternating series test  
E. NOTA

17.  $\int_{\infty}^3 \frac{x}{1-x^2} dx$  is

- A. divergent      B. 0      C. 1      D.  $-\frac{1}{2}$       E. NOTA

18. A particle moving along a path  $r(t) = (t+1)\mathbf{i} - t^3\mathbf{j}$  has an initial velocity of

- A.  $\mathbf{i}$       B.  $\mathbf{i} - 3\mathbf{j}$       C. 1      D.  $\sqrt{10}$       E. NOTA

19.  $\lim_{x \rightarrow 0^+} x^2 \ln x$  is

- A. divergent      B. 0      C.  $\frac{1}{2}$       D. 2      E. NOTA

20.  $\sum_{n=1}^{\infty} \frac{n}{5n-1}$

- A. converges to 0      B. converges to 1      C. converges to  $\frac{1}{5}$   
D. diverges      E. NOTA

21. If a particle is moving according to  $r(t) = \frac{t^2}{2}\mathbf{i} + (\sqrt{2}t)\mathbf{j} + (\ln t)\mathbf{k}$ , then the speed of the particle at  $t = 1$  is

- A.  $i + \sqrt{2}j + k$       B.  $i - k$       C. 0      D. 1      E. NOTA

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28.  $\int \cos(\ln x) dx$

A.  $x \cos(\ln x) + x \sin(\ln x) - \cos(\ln x) + \sin(\ln x) + C$

B.  $\frac{x}{2} [\cos(\ln x) + \sin(\ln x)] + C$

C.  $2x \sin(\ln x) + C$

D.  $\sin(\ln x) - \left( \frac{\cos x}{x} \right) \left( \frac{x^2}{x^2 - 1} \right) + C$

E. NOTA

29. The length of the curve represented by  $x = e^t \sin t$  and  $y = e^t \cos t$  from  $t = 0$  to  $t = \pi$  is

A.  $\sqrt{2}e^\pi - 2$

B.  $e^\pi - \sqrt{2}$

C.  $\sqrt{2}(e^\pi - 1)$

D.  $\frac{e^\pi}{\sqrt{2}}$

E. NOTA

30. Obtain the first three terms for the Taylor expansion of  $y = \sin^{-1}x$  about  $x = 0$ .

A.  $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$

B.  $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$

C.  $x - \frac{x^3}{6} + \frac{3x^5}{40} - \dots$

D.  $x + \frac{x^3}{6} + \frac{3x^5}{40} + \dots$

E. NOTA