Limits and Derivatives - Calculus - **SOLUTIONS**
Mu Alpha Theta National Convention 2003

1. The deriv is $3\cdot f^2(x)\cdot f'(x) = 3\cdot 1^2\cdot -5 = -15$ **C**

2. The deriv is **A**
$\frac{(x^2+1)\cdot 2g'(x) - 2g(x)(2x)}{(x^2+1)^2} = \frac{(1^2+1)\cdot 2(-1) - 2(6)(2)}{(1^2+1)^2} = -7$

3. $\sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}\cdot|r|}{2}$ **B**

4. \$4.75 - \$1.60 = \$3.15 **A**

5. $\lim_{x\to 0^-}(sx) = 0$, so the limit of the entire exp. = 0 **C**

6. Differentiate p(t) with respect to t.
$v(t) = gt - v_0$ **C**

7. F(x) looks like sin(x), so its derivative will look like cos(x) **A**

8. Solve simultaneously:
a = b + 2 and 4a = 3b => a=-6,b=-8 **D**

9. B(t) = 1,270,00 – 26,000(t – 1)
B(3) = 1,218,000 **A**

10. tan(Θ) = 5/10.5 => Θ ≈ .4444
Differentiate tan(Θ) = 5/x wrt t:
$\sec^2(\Theta)\frac{d\Theta}{dt} = \frac{-5}{x^2}\cdot\frac{dx}{dt}$. Solving for dΘ/dt =>
20.33 rad/hour **B**

11. Maximize P(x) = R(x) – C(x) => x = 89 **C**

12. $\frac{dy}{dt}\cdot\frac{dt}{dx} = 4t^3\cdot\frac{1}{3t^2} = \frac{4t}{3} @ t = 3 \Rightarrow = 4$ **D**

13. $n - \sqrt{n^2 - n} = \frac{n}{n + \sqrt{n^2 - n}}$, then multiply top and bottom by 1/n =>

$$\lim_{n\to\infty}\left[\frac{1}{1+\sqrt{1-1/n}}\right] = \frac{1}{2}$$ **B**

14. The Δ with max. area is a right triangle. Area $= \frac{r^2}{2}$ **B**

15. Since the slope of a line is constant, the rate of change of the slope is zero **E**

16. This limit approaches *e* **C**

17. $Y'(x) = \sin(x^2)$. $Y'(-1) = \sin(1) > 0$ **C**

18. The value of a where Y(x) is NOT existent is (-b, b) **A**

19. Y'(x) = F'(G(x))G'(x). Since G is a parabola, G'(π) = 0. Y'(π) = 0 **B**

20. Let x be the distance to ground.
$\frac{x}{2} = r\cdot\sin(\frac{\theta}{2})$ => $\frac{1}{2}\cdot\frac{dx}{dt} = r\cdot\cos(\frac{\theta}{2})\cdot\frac{1}{2}\cdot\frac{d\theta}{dt}$
Solving for $\frac{dx}{dt}$ = .058 m/s **B**

21. $\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{24}$ => cos(1.1) ≈ .456
$R_4(x) = \frac{(1.1)^6}{6!} \approx .00246 \Rightarrow$
.456 – .002246 = .454 and .456 + .002246 = .458
So, 0.454 < cos(1.1) < 0.458 **E**

22. The max possible value is 14. **D**

23. The slope of this line is $\frac{-\sqrt{3}}{9}$, so the line equation is $(y - \sqrt{12}) = \frac{-\sqrt{3}}{9}(x-6)$.
a – b = $\frac{8\sqrt{3}}{3} - 24$ **E**

24. $\frac{d^2y}{dx^2} = \frac{40x(x^2-3)}{(x^2+1)^3} - \frac{2}{x^3}$ **D**

25. $\frac{dy}{dx} = 4xy \Rightarrow \int\frac{dy}{y} = \int 4x\,dx \Rightarrow \ln y = 2x^2 + c$.
Solving for y gives $y = Ce^{2x^2}$ and C could be 2. **C**

26. r(t) = $\frac{5}{2}t^2$. $\theta(t) = 2t + \frac{\pi}{2}$ =>
x(t) = $\frac{5}{2}t^2\cdot\cos(\theta) = \frac{5}{2}t^2\cdot\cos(2t + \frac{\pi}{2})$
x''(6.5) ≈ 57.5 m/s^2 **E**

27. $\frac{ds}{dt} = \frac{-s}{50} \Rightarrow s(t) = 10e^{\frac{-1}{50}t}$. And $s(10) \approx 8.19$ **D**

28. Let A = Area of the triangle, L = Distance from centroid to x-axis.
V(s) = $2\pi AL = 2\pi\left(\frac{\sqrt{3}}{4}s^2\right)\left(\frac{s\cdot\sin 60}{3}\right) = \frac{\pi\sqrt{3}}{12}s^3$. =>
$V'(s) = \frac{3\pi\sqrt{3}}{6}s^2\frac{ds}{dt}$ **E**

29. $\lim_{t\to 0^-}\left[\frac{\sin(2t)}{t}\right] = 2$ => $\lim_{t\to 0^+}\vec{P}(t) = \langle 2,1,1\rangle$ **B**

30. The graph will have a horizontal tangent when $\frac{dy}{d\theta} = 0$. $y = r\cdot\sin(\theta) = 2(1-\cos(\theta))\sin(\theta)$
=> $\frac{dy}{d\theta} = -4\cos^2(\theta) + 2\cos(\theta) + 2 = 0$ =>
$\theta = \left\{\frac{2\pi}{3}, \frac{4\pi}{3}\right\}$ **C**