1. $$\begin{array}{l} 4x-3y+2z=6 \\ \underline{-6x+y-2z=-24} \\ -2x-2y=-18 \\ x+y=9 \end{array}$$
ANSWER: B

2. $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \dfrac{5\cdot 2+8\cdot -1+3\cdot 4}{\sqrt{5^2+8^2+3^2}\sqrt{2^2+(-1)^2+4^2}} =$
$\dfrac{14}{7\sqrt{42}} = \dfrac{\sqrt{42}}{21}$
ANSWER: C

3. $|B| = 27+20+16-15-32-18=-2$

ANSWER: A

4. In statement I, the determinant of the new matrix is equal to r det(A), not r^2 det(A). Statements II, III, and IV are correct.
ANSWER: C

5. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 3 & -4 & -1 \\ 0 & 6 & 2 \end{vmatrix} = (-8i+18k)-(-6i+6j) = -2i-6j+18k$
ANSWER: A

6. A and B are not because not all pivots are 1.
D is not because the entry above a pivot is not 0.
C and E are.
ANSWER: B

7. a point on the first plane is (5,0,0)
distance from (5,0,0) to plane $x+2y+2z=10$ is
$\left|\dfrac{1\cdot 5+2\cdot 0+2\cdot 0-10}{\sqrt{1^2+2^2+2^2}}\right| = \dfrac{5}{3}$
ANSWER: C

8. $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|^2}\cdot\vec{b} = \dfrac{-5}{14}(2,-3,1) = \dfrac{1}{14}(-10,15,-5)$
ANSWER: B

9. dimension of column space = rank
$$\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 0 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 6 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & 3 & 3 & 2 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$
rank=2
ANSWER: B

10. Taking the derivatives of the basis vectors, $1, t, t^2$, and t^3,
$Ap_1 = 0, Ap_2 = p_1, Ap_3 = 2p_2, Ap_4 = 3p_3$
Now with $p_1 = (1,0,0,0)$, $p_2 = (0,1,0,0)$, etc.,
A becomes $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, where Ap_1 is A's first column, Ap_2 is A's second column, etc.
ANSWER: B

11. dimensions of A^{T} are 6×5
dimensions of $A^{\mathrm{T}}B$ are 6×7
dimensions of $A^{\mathrm{T}}BC$ are 6×1
ANSWER: B

12. (6,22,15)-(4,-5,8)-(0,2,-13)=(2,27,7)-(0,2,-13)=(2,25,20)
ANSWER: B

13. $AB = \begin{bmatrix} 30 & 24 & 18 \\ 84 & 69 & 54 \\ 138 & 114 & 90 \end{bmatrix}$
trace(AB)=30+69+90=189
ANSWER: B

14. Sum of eigenvalues = trace of matrix = 4+8+2 = 14
ANSWER: B

15. Product of eigenvalues = determinant of matrix = 64+15+72-(48+72+20) = 11
ANSWER: A

16. $((AB)^{\mathrm{T}}C)^{\mathrm{T}} = C^{\mathrm{T}}((AB)^{\mathrm{T}})^{\mathrm{T}} = C^{\mathrm{T}}AB$
ANSWER: C

17. The 1 in the upper right will not affect any of the main diagonal elements, so $\mathrm{tr}(A^5) = 1^5+2^5+3^5 = 276$
ANSWER: A

18. All 5 of the conditions are equivalent and imply A is nonsingular.
ANSWER: D

19.
$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 6 & 1 & 2 & 5 & 1 \\ 0 & 4 & 0 & 2 & -2 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 2 & 5 & 1 \\ 4 & 0 & 2 & -2 \end{vmatrix} = -\begin{vmatrix} 1 & 0 & 0 \\ 1 & 5 & 1 \\ 4 & 2 & -2 \end{vmatrix} = -\begin{vmatrix} 5 & 1 \\ 2 & -2 \end{vmatrix}$$
$= -(-10-2) = 12$
ANSWER: D

20. The coefficients of the t terms determine the direction of the line, which are (-1,6,3) in A and B.
In A, $\ell(1)=(-1+7)i+(6-4)j+(3+2)k=(6,2,5)$, so it contains the point in question.
In B, the line contains the point (6,2,-5), not (6,2,5)
ANSWER: A

21. We seek $\|f\|_\infty=\max(|f(t)|)$. Since $f(t)$ is differentiable on [0,3], we can find the critical points. $f'(t)=2t-4$ and the max occurs where $f'(t)=0$ $(@\, t=2)$ or at an endpoint (0 or 3). Checking those points, we get $f(2)=-4, f(0)=0, f(3)=-3$. Thus, $\|f\|_\infty=|f(2)|=|-4|=4$.
ANSWER: D

22. $\det((A^{\mathrm{T}}B)^{-1})=\det(A^{\mathrm{T}})^{-1}\det(B)^{-1}=\det(A)^{-1}\det(B)^{-1}$
ANSWER: A

23. $5163274\to1563274\to1263574\to1236574\to1234576\to1234567$
The process took 5 steps, so answer C is odd.
The rest of the permutations take an even number of steps to return to 1234567, so they are even.
ANSWER: C

24. A linear transformation requires all 3 properties to be true by definition.
ANSWER: D

25. $\text{area}=\left|10\cdot\begin{vmatrix}2 & 1\\ 1 & -2\end{vmatrix}\right|=|10(-4-1)|=|-50|=50$
ANSWER: D

26. I is the definition of a positive definite matrix.
II would be true if it read $\lambda_i>0$, and III would be true if it read positive determinants. II and III are conditions for a matrix to be semidefinite. Therefore only I is correct.
ANSWER: A

27. The sum of the elements in A^{H} will just be the conjugate of the sum of the elements in *A*.
$\overline{4-3i}=\overline{4+3i}$
ANSWER: B

28. Let the rotation matrix be *Q*.
$$Q\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}\cos\theta\\ \sin\theta\end{bmatrix}$$
$$Q\begin{bmatrix}0\\1\end{bmatrix}=\begin{bmatrix}-\sin\theta\\ \cos\theta\end{bmatrix}$$
Combining these, $Q=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$
ANSWER: E

29. The length of the vector must be 1, and only answer D satisfies this condition.
$$\sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{3}{7}\right)^2+\left(\frac{6}{7}\right)^2}=1$$
ANSWER: D

30. Choose a transformation matrix A, and let $B=M^{-1}AM$ for any invertible matrix *M*.
$A\vec{x}=\lambda\vec{x}\Rightarrow MBM^{-1}\vec{x}=\lambda\vec{x}\Rightarrow B(M^{-1}\vec{x})=\lambda(M^{-1}\vec{x})$
Therefore the eigenvalue of B is still λ, but the eigenvector has been multiplied by M^{-1}.
This means I is true and II is not.
ANSWER: A