Mu Alpha Theta National Convention 2003 Theta Equations & Inequalities Solutions

① $\log_{10} a + \log_{10}(a+21) = 2$

$\log_{10} a(a+21) = 2$

$a^2 + 21a = 100$

$a^2 + 21a - 100 = 0$

$(a+25)(a-4) = 0$

$\cancel{a = -25}$ $a = 4$ $\boxed{B}$

② Substituting into $y = ax^2 + bx + c$

$(0,-5) \Rightarrow -5 = c$

$(-1,-7) \Rightarrow -7 = a - b - 5$

$(-4,-1) \Rightarrow -1 = 16a - 4b - 5$

By linear combination,

$a = 1, b = 3, c = -5 \therefore abc = -15$ $\boxed{B}$

③ $b^2 - 4ac = 0 \Rightarrow K^2 - 4 \cdot 2 \cdot 3K = 0$

$K(K-24) = 0$ $K = 24$ $\boxed{C}$

④ Completing the square gives:

$(x-2)^2 + 9(y+3)^2 = 36$

$\frac{(x-2)^2}{36} + \frac{(y+3)^2}{4} = 1$

$K = -3,\ 2a = 12 \Rightarrow -36$ $\boxed{C}$

⑤ conjugate roots are $4-2i$, $4+2i$

Their sum $= -\frac{b}{a} = 8$

product $= \frac{c}{a} = 20$ $\Big\}\ x^2 - 8x + 20$

$(x-6)(x^2 - 8x + 20) =$

$x^3 - 14x^2 + 68x + 120$ $\boxed{A}$

⑥ $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ -11 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 8 \\ -20 \end{bmatrix} \therefore \begin{matrix} x = 2 \\ y = -5 \end{matrix}$

$xy = -10$ $\boxed{C}$

⑦ $5x^2 + 2x = 3$

$5x^2 + 2x - 3 = 0$

$(5x-3)(x+1)$

$\{\frac{3}{5}, -1\}$

$\frac{3}{5}$ $\boxed{B}$

⑧ Add 1st + 3rd equations: $2x = 14 \therefore x = 7$

Substituting gives $y = \frac{14}{3}$, $z = -1$

$\frac{7}{\frac{14}{3}} - 5(-1) = 6\frac{1}{2}$ $\boxed{B}$

⑨ $2800 = 2000 \cdot e^{4r}$

$\frac{7}{5} = e^{4r}$

$r = \frac{\ln 1.4}{4}$

$A = 2000 \cdot e^{16r}$

$A \approx 7683$ $\boxed{C}$

⑩ $y = \dfrac{\begin{vmatrix} 2 & 5 & -1 \\ 1 & 7 & 1 \\ 3 & 1 & -3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & -1 \\ 1 & 1 & 1 \\ 3 & -2 & -3 \end{vmatrix}} = \frac{6}{-3} = -2$ $\boxed{A}$

⑪ $(x+2)(x+3)(x-1) = x^3 + 4x^2 + x - 6$ $\boxed{C}$

⑫ $2 \cdot 3^{2x} + 3^x \cdot 5^x = 5^{2x}$ let $a = 3^x$, $b = 5^x$

$2a^2 + ab - b^2 = 0$

$(2a-b)(a+b) = 0$

$2a = b$ or $a = -b$

$2 \cdot 3^x = 5^x$ or $3^x = -(5$

$\left(\frac{3}{5}\right)^x = \frac{1}{2} = m$ not possi

$\therefore -\frac{2}{m} = -4$ $\boxed{C}$

⑬ roots are $2+3i$ and $2-3i$

sum $= -\frac{b}{a} = 4$ product $= \frac{c}{a} = 13$; $a = 1, b = -4, c=$

$b^2 - 4ac = -36$ $\boxed{C}$

⑭ $\log 8^{x-2} = \log 5^x$

$(x-2)\log 8 = x \cdot \log 5$

$x\log 8 - x\log 5 = 2\log 8$

$x(\log 8 - \log 5) = \log 64$

$x = \frac{\log 64}{\log 8/5} \approx 8.849$ $\boxed{B}$

⑮ radius must be 5, center must be (1,4)

so $(x-1)^2 + (y-4)^2 = 16$

$x^2 - 2x + y^2 - 8y - 1 = 0$ $\boxed{D}$

(16) $3^{2x+3} \cdot 5^{2x+3} = 3^{3x} \cdot 5^{y} \cdot 25$

$\dfrac{3^{2x} \cdot 27 \cdot 5^{2x} \cdot 5^{3}}{3^{3x} \cdot 25} = 5^{y}$

$\dfrac{3^{2x+3}}{3^{3x}}$ must $=1$ $\therefore x = 3$

so $\dfrac{5^{6} \cdot 5^{3}}{25} = 5^{y}$ $\therefore$ $y=7$ $\boxed{E}$

(17) Multiply through by $(x+4)(x-7)$

$\Rightarrow A(x-7) + B(x+4) = 11x-22$

$Ax - 7A + Bx + 4B = 11x - 22$

$A+B=1$

$-7A+4B=-22$

$\therefore B=5,\ A=6$

$3A-2B = 8$ $\boxed{D}$

(18) $x-9=4x+3$ or $x-9=-4x-3$

$-12=3x$

$x=-4$ extraneous

$5x=6$

$x=\frac{6}{5}$ $\boxed{A}$

(19) $(a-b)(a^2+ab+b^2)=24$

$2 \cdot 12$

$(a-b)^2 = a^2-2ab+b^2=4$

$a^2+ab+b^2=12$

$-3ab = -8 \Rightarrow ab=\frac{8}{3}$

$(a+b)^2=a^2+2ab+b^2 = 12+\frac{8}{3} = \frac{44}{3}$

$a+b=\sqrt{\frac{44}{3}} = \dfrac{\pm 2\sqrt{33}}{3}$ $\boxed{B}$

(20) $7 - \dfrac{1}{\frac{1}{7}+\frac{1}{x}} = \dfrac{1}{7} \Rightarrow 7 - \dfrac{7x}{x+7} = \dfrac{1}{7}$

$\dfrac{7x+49-7x}{x+7} = \dfrac{1}{7} \Rightarrow x+7=343$ $\boxed{D}$

$x=336$

(21) $\left.\begin{aligned} \frac{1}{y}+\frac{2}{x}&=\frac{11}{12} \\ \frac{2}{y}-\frac{3}{x}&=\frac{2}{3}\end{aligned}\right\}$ $\left.\begin{aligned} x&=6 \\ y&=\frac{12}{7}\end{aligned}\right\}$ $x-y=\dfrac{30}{7}$ $\boxed{B}$

(22) $3\{2x-[1-6x]\}=10x-18$

$3(8x-1)=10x-18$ $\therefore x=\dfrac{-15}{14}$ $\boxed{A}$

(23) $f(x+1) = 2(x+1)+1$

$= 2x+3$

$2x+3 = \dfrac{12}{2x+1-2}$

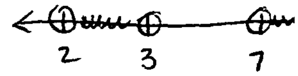
$4x^2+4x-15=0$

sum $= -\frac{b}{a} = -1$ $\boxed{A}$

(24)
$$\begin{array}{r|rrrr} 2 & 1 & -12 & 41 & -42 \\ & & 2 & -20 & 42 \\ \hline & 1 & -10 & 21 & 0 \end{array}$$

$(x-2)(x-7)(x-3)=0$

critical points are 2, 3 and 7



intervals w/solutions have values of $x^2+1 = 5 \quad 10 \quad 50$

so 51 $\boxed{D}$ is possible value

(25) $x^3-1=(x-1)(x^2+x+1)$

$x = \dfrac{-1 \pm \sqrt{1-4}}{2} = -\dfrac{1}{2} \pm$

$\therefore b=\frac{\sqrt{3}}{2}$ and $b^2=\frac{3}{4}$ $\boxed{A}$

(26) $(x-\frac{4}{9})(x-\frac{4}{9}+x-\frac{1}{3}) = (x-\frac{4}{9})(2x-\frac{7}{9})=0$

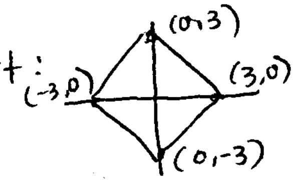
$x=\frac{4}{9}$ or $x=\frac{7}{18}$ $\left|\frac{4}{9}-\frac{7}{18}\right| = \frac{1}{18}$ $\boxed{E}$

(27) $x - x\sqrt{3} = 4$

$x(1-\sqrt{3})=4$

$x = \dfrac{4}{1-\sqrt{3}} = \dfrac{4(1+\sqrt{3})}{-2} = -2-2\sqrt{3}$

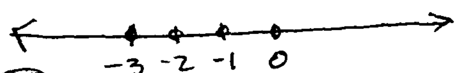
(28) sketch it:



Area $=18$ $\boxed{C}$

(29) $x+2 \le 2x+7$ and $x+2 \ge -2x-7$

$x \ge -5$ and $x \ge -3$



4 non-negative integers $\boxed{B}$

(30) intersection points:

$\left.\begin{matrix}(0,2)\\(2,0)\\(4,3)\end{matrix}\right\}$ substituted into $f(x,y)$ $\Rightarrow$ minimum value $=2$

$\boxed{D}$