

POLYGONS/CIRCLES – THETA  
National Mu Alpha Theta Convention 2003

Diagrams are not drawn to scale.

For all questions, answer E is NOTA, none of the above answers is correct.

1. Five distinct points lie on a circle. The number of possible convex polygons having five or less of these points is

- A. 12
- B. 15
- C. 16
- D. 18
- E. NOTA

2. Points A, B, and C are points on circle O.

Given minor arcs:  $m\widehat{AB} = 5x + 20$ ,  $m\widehat{BC} = 6x + 30$ ,  $m\widehat{AC} = 3x - 40$ , find  $m\widehat{ABC}$ .

- A. 25
- B. 35
- C. 215
- D. 325
- E. NOTA

3. A circle with center C has a diameter of 20.

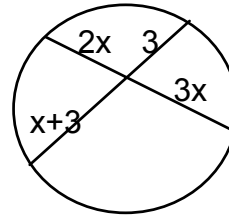
Chord  $\overline{RS}$  with a length of 16 intersects and is perpendicular to diameter  $\overline{AB}$  at point M. Find the length of  $\overline{BM}$  if  $AM < BM$ .

- A. 4
- B. 6
- C. 10
- D. 16
- E. NOTA

4. Given regular polygon ABCDEFGH with side length 1, find the area of ACEG.

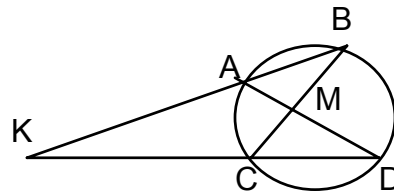
- A. 1
- B.  $\sqrt{2}$
- C.  $2 + \sqrt{2}$
- D. 4
- E. NOTA

5. Find the numerical value of the product of the length of the chords in this circle.



- A. 56.25
- B. 182.25
- C. 250
- D. 600
- E. NOTA

6. Given the figure with points A, B, C, and D on the circle,  $\overline{AD}$  and  $\overline{BC}$  intersect at M. Find the ratio of  $m\angle BKD$  to  $m\widehat{BD}$  if  $m\angle BMD$  is twice as large as  $m\angle BKD$ .



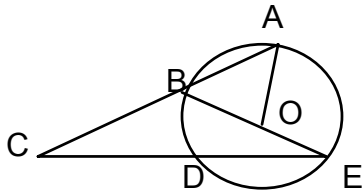
- A. 1:4
- B. 1:3
- C. 1:2
- D. 2:3
- E. NOTA

7. Starting with a square piece of paper, I trim the corners so as to leave the largest possible circular piece of paper. Starting again, I trim the circular piece of paper so as to leave the largest possible square piece of paper. How much of the original square piece of paper was cut off?

- A.  $\frac{1}{8}$
- B.  $\frac{1}{4}$
- C.  $\frac{1}{2}$
- D.  $\frac{\sqrt{2}}{4}$
- E. NOTA

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8. Circle O has  $m\angle AOB = 80$ ,  $m\angle BEC = 30$ . Find  $m\angle ACE$ .



- A. 10  
B. 15  
C. 25  
D. 30  
E. NOTA

9. Find the area of a circle with equation  $4x^2 + 4y^2 + 24x - 16y - 24 = 0$ .

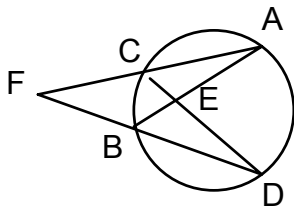
- A.  $4\pi$   
B.  $12\pi$   
C.  $19\pi$   
D.  $76\pi$   
E. NOTA

10. Circles O and O' are internally tangent and O' is the smaller circle, passes through the center of O. If the area of circle O is  $16\pi$ , find the area of circle O'.

- A.  $2\pi$   
B.  $4\pi$   
C.  $8\pi$   
D.  $16\pi$   
E. NOTA

11. Given the circle with point F outside the circle with secant segments  $\overline{FA}$  and  $\overline{FD}$ , chords  $\overline{AC}$  and  $\overline{BD}$ .  $m\angle AFD = 25$ ,

$m\angle AED = 95$ ,  $m\widehat{AB} = 2x$ ,  $m\widehat{CD} = 3x$ . Find  $m\widehat{BCD}$ .

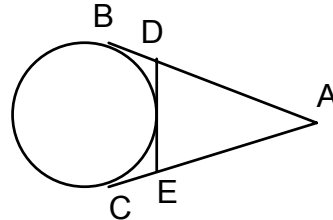


- A. 168  
B. 170  
C. 172  
D. 174  
E. NOTA

12. If each interior angle of a regular polygon is  $177^\circ$ , how many diagonals does it have?

- A. 6990  
B. 7020  
C. 7050  
D. 7080  
E. NOTA

13.  $\overline{AB}$ ,  $\overline{AC}$ , and  $\overline{DE}$  are all tangent segments. What is the perimeter of  $\triangle ADE$  if  $AB=20$ ?



- A. 40  
B. 50  
C. 60  
D. cannot be determined  
E. NOTA

14. A chord the length of this circle's radius cuts off a segment of a circle with area of  $\pi$  square units. The length of the chord is the same as the radius of the circle. What is the length of the radius of the circle rounded to the nearest hundredth? (A segment of a circle is a region bounded by an arc of the circle and the chord of the arc.)

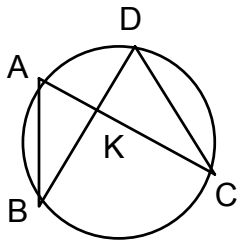
- A. 2.54  
B. 5.89  
C. 6.14  
D. 8.47  
E. NOTA

15. A gardener has a circular garden with a regular hexagonal lawn inscribed in it. The hexagonal lawn contains  $\sqrt{108}$  square meters of turf. What is the diameter of the garden?

- A. 2  
B.  $\sqrt{3}$   
C.  $\sqrt{6}$   
D. 4  
E. NOTA

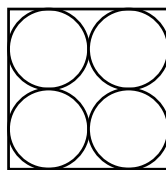
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16. In circle O, chords  $\overline{AC}$  and  $\overline{BD}$  intersect at K. Find the length of segment DC given  $KA = 2x + 1$ ,  $KB = 3x - 1$ ,  $AB = 2x + 2$ ,  $KC = 5x + 2$ , and  $KD = 5x - 2$ .



- A. 4  
B. 5  
C. 10  
D. 20  
E. NOTA
17. What is the area of a circle in which a chord of length 10 is 5 units from the center of the circle ?
- A.  $25\pi$   
B.  $50\pi$   
C.  $75\pi$   
D.  $100\pi$   
E. NOTA
18. A convex polygon has 119 diagonals. How many sides does the polygon have?
- A. 14  
B. 17  
C. 19  
D. 23  
E. NOTA
19. ABCD is a square with  $AB=12$ . Point P is an interior point whose distances to A, B, and  $\overline{CD}$  are equal. Find the distance from P to A.
- A.  $3\sqrt{3}$   
B. 6  
C. 7.5  
D.  $6\sqrt{3}$   
E. NOTA

20. Four congruent circles are packed into a square of side s so that each of the circles is tangent to two sides of the square and to two other circles, as shown. What is the ratio of the total area of the four circles to the area of the square?



- A.  $\frac{\pi}{8}$   
B.  $\frac{\pi}{4}$   
C.  $\frac{\pi}{3}$   
D.  $\frac{\pi}{2}$   
E. NOTA
21. P and Q are points of tangency of tangents drawn from a point A in the exterior of circle O. If  $m\angle A = 72$  and the radius of the circle is 10, find the area of the sector bounded by  $\widehat{PQ}$  and radii PO and PQ.
- A.  $5\pi$   
B.  $10\pi$   
C.  $20\pi$   
D.  $25\pi$   
E. NOTA
22. A regular pentagon ABCDE is inscribed in a circle with center O. If the radius of the circle is 10, find the length of  $\widehat{ABC}$ .
- A.  $4\pi$   
B.  $8\pi$   
C.  $10\pi$   
D.  $20\pi$   
E. NOTA

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23. Find the length of a side of a regular hexagon that has an area equal to the area of an equilateral triangle with perimeter of 36.

- A. 26
- B.  $3\sqrt{3}$
- C.  $6\sqrt{2}$
- D.  $2\sqrt{6}$
- E. NOTA

24. Three circles with radii 1, 2, and 3 are tangent externally to each other in pairs. The area of the triangle having the center of the circles as vertices is

- A. 6
- B. 7.5
- C. 10
- D. 12
- E. NOTA

25. The ratio of the measure of an interior angle of a regular pentagon to that of a regular decagon is

- A. 1:2
- B. 3:8
- C. 3:4
- D. 2:1
- E. NOTA

26. In regular hexagon ABCDEF, diagonals  $\overline{BF}$  and  $\overline{AE}$  are drawn intersecting at G. Find  $m\angle AGB$ .

- A. 30
- B. 45
- C. 50
- D. 60
- E. NOTA

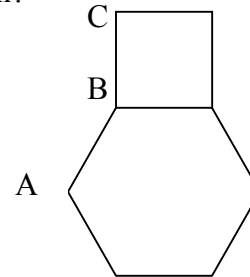
27. The diagonals of a rhombus are 12 and 16. Find the perimeter of the rhombus.

- A. 10
- B. 20
- C. 30
- D. 40
- E. NOTA

28. The measures of the angles in a quadrilateral are in the ratio 2:3:4:6. Find the measure of the largest exterior angle.

- A. 24
- B. 132
- C. 144
- D. 156
- E. NOTA

29. Point B is a mutual vertex of a regular hexagon, a square, and a third regular polygon. If 2 of the sides of this third polygon are  $\overline{AB}$  and  $\overline{BC}$ , what is this polygon?



- A. octagon
- B. decagon
- C. dodecagon
- D. not possible
- E. NOTA

30. A rhombus contains a 120 degree angle. Find the ratio of the length of the longer diagonal to the length of the shorter diagonal.

- A.  $\sqrt{3} : 3$
- B. 1:2
- C.  $\sqrt{3} : 1$
- D. 2:1
- E. NOTA