Question #1
Theta State Bowl – Mu Alpha Theta National Convention 2003

Find the product of all real values of x that satisfy the given equation:

$$4(\log_{16} x)^2 - \log_{16} x^7 + \log_{16}\left(\frac{1}{256}\right) = 0$$

Question #2
Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = the sum of all values excluded from the range of g[f(x)] if $f(x) = \frac{2}{x-2}$ and $g(x) = \frac{x+1}{x-3}$

Let B = the numerical value of $x^3 + y^3$ if $x + y = 4$ and $x^2 + y^2 = 2$

Let C = the value of the constant term in the expansion of $\left(3x^3 - \frac{1}{x}\right)^8$

Let D = f(1), given that f is a linear function, $f(3) = -1$ and $f(-1) = 3$

Find: AC + BD

Question #3
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Let A = the perimeter of a regular hexagon inscribed in a circle of radius 6

Let B = the number of sides of a regular polygon if each angle of the polygon measures 168°

Let C = the area of a rectangle with perimeter 320 and a pair of adjacent sides in the ratio 3:17

Let D = $\frac{x}{y}$, given that the lengths of the legs of a right triangle are x and $4x + y$ and the hypotenuse length is $5x - y$

Find: A + B + CD

Question #4
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Given: $x * y = xy + 1$ and $x \# y = x + y$

Let A = the numerical value of $5 * [(6 \# 8) \# (4 * 5)]$

Find the number of positive integral factors of A.

Question #5

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Find the sum of all values of x and y that satisfy the following equations:

$$|4 + 5x| = 3 + 2x$$

$$125^{y+2} = \left(\frac{1}{5}\right)^{3-y}$$

Question #6

Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = the number of diagonals that can be drawn in a convex decagon

Let B = the number of distinguishable permutations of the letters in the word ATLANTA

Let C = the number of ways four Georgia Tech fans and four University of Georgia fans can stand alternately in a line

Let D = the number of ways a committee of three student delegates can be chosen from a group of twenty student delegates

Find: $D - C + \frac{B}{A}$

Question #7

Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = the sum of all integral values of x that satisfy the given inequality: $\frac{|5 - x|}{3} < 2$

Let B = the sum of the numerical coefficients in the expansion of $(2x - 3y)^{2003}$

Let C = the ratio of the first term to the common difference for an arithmetic series in which the sum of the first 10 terms is 4 times the sum of the first 5 terms

Let D = the remainder when $4x^{2003} - 3x^{200} + 4x^3 - 2003$ is divided by $x + 1$

Find: $AB + CD$

Question #8

Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = the number of cubic inches in the volume of a sphere inscribed in a cube if the surface area of the cube is 96 square inches

Let B = the number of square inches in the area of a certain circle if a rectangle inscribed that circle has length 16 inches and a perimeter of length 56 inches

Let C = the total number of subsets that can be formed from the set $\{a, b, c, d, e, f\}$

Find: $\frac{BC}{A}$

Question #9
Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = $\frac{d}{m}$, where d is the distance between the points $P(-16,\ 3)$ and $Q(-4,\ -2)$ and m is the slope of $\overline{PQ}$

Let B = the number of pounds of water that must be evaporated from 80 pounds of a salt and water solution that is 12% salt in order to obtain a solution that is 20% salt

Let C = the value of k so that $x - 2$ is a factor of $2x^4 - (kx)^2 + 6kx - 41$

Let D = $(\log_7 25)\left(\log_2\left(\frac{1}{7}\right)\right)(3\ \log_3 8)\left(\log_5 \sqrt{27}\right)$

Find: $\frac{5A}{C} + B + D$

Question #10
Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = the length of the radius of the circle with equation $3x^2 + 3y^2 + 6x - 12y - 5 = 0$

Let B = the sum of the x– and y–coordinates of the focus of the parabola with equation $x = \frac{1}{12}y^2 - y + 4$

Let C = the distance between the foci of the graph with equation $9x^2 + 25y^2 + 72x + 50y - 56 = 0$

Let D = the product of the slopes of the asymptotes for the graph of $9x^2 - 4y^2 - 36x + 24y - 36 = 0$

Find: $\frac{A^2CD}{B}$

Question #11
Theta State Bowl – Mu Alpha Theta National Convention 2003

Let A = $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots$

Let B = $1 + 4 + 2 + 8 + 3 + 12 + 4 + 16 + \cdots + 20 + 80$

Let C = the value of the discriminant of the quadratic equation $x^2 + 4x - 2 = 1$

Let D = the number of entries in the product MN if M is a 3×4 matrix and N is a 4×7 matrix

Find: $\frac{B}{A} + C + D$

Question #12
Theta State Bowl–Mu Alpha Theta National Convention 2003

Let A = the numerical coefficient of the 7th term in the expansion of $(\log_3 9^x + \log 10^y)^{10}$

Let B = $\frac{p}{q}$, if $p = \log_8 225$ and $q = \log_2 15$

Let C = the numerical value of $\frac{2^{x+5} - 2 \cdot 2^x}{2 \cdot 2^{x-4}}$

Let D = the simplified form of

$$\frac{1}{3-\sqrt{8}} - \frac{1}{\sqrt{8}-\sqrt{7}} + \frac{1}{\sqrt{7}-\sqrt{6}} - \frac{1}{\sqrt{6}-\sqrt{5}} + \frac{1}{\sqrt{5}-2}$$

Find: $AB + \frac{C}{D}$

Question #13
Theta State Bowl – Mu Alpha Theta National Convention 2003

Find the product of all real values of x that satisfy

$$\begin{vmatrix} x & 1 & -2 \\ -2 & x+1 & 1 \\ -1 & 3 & x-6 \end{vmatrix} = 0$$

Question #14
Theta State Bowl – Mu Alpha Theta National Convention 2003

Natalie rode her bicycle for 20 miles at a constant rate. If she had ridden 2 miles per hour faster, she would have reduced her time by 20 minutes. How many hours did her original trip take?

Queston #15
Theta State Bowl – Mu Alpha Theta National Convention 2003

In a certain infinite geometric series, each term value is four times as large as the sum of all succeeding terms. If the first term $(t_1) \neq 0$, determine the ratio of the second term to the first term $\left(\frac{t_2}{t_1}\right)$.

① Let $y = \log_{16} x$

$4y^2 - 7y - 2 = 0$

$(4y+1)(y-2) = 0$

$y = -1/4$ or $y = 2$

$\log_{16} x = -\frac{1}{4}$ or $\log_{16} x = 2$

$x = 1/2$ or $x = 256$

product = **128**

② A: Since $f(x)$ cannot $= 0$, $-\frac{1}{3}$ is excluded from the range of $g[f(x)]$.
Since $g(x)$ cannot $= 1$, 1 is excluded from the range of $g[f(x)]$.

$1 + (-\frac{1}{3}) = \frac{2}{3}$

B: $x + y = 4 \Rightarrow x^2 + 2xy + y^2 = 16$

$2xy = 16 - (x^2+y^2) = 16 - 2 = 14$

$xy = 7$

$x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$x^3 + y^3 = 4(2-7) = -20$

C: $\frac{8!}{6!2!}(3x^3)^2\left(-\frac{1}{x}\right)^6$

$28(9x^6)\left(\frac{1}{x^6}\right)$

252

D: $(3,-1)$ $(-1,3)$

$m = -1$

$(1, y)$

$\frac{y-3}{1-(-1)} = -1$

$y - 3 = -2$

$y = 1$

$AC + BD = \left(\frac{2}{3}\right)(252) + (-20)(1) = $ **148**

③ A:

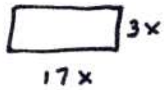
perimeter = 36

B: $\frac{(n-2)180}{n} = 168 \Rightarrow 180n - 360 = 168n$

$12n = 360$

$n = 30$

C:



$40x = 320$

$x = 8$

$(24)(136) = 3264$

D: $x^2 + (4x+y)^2 = (5x-y)^2$

$x^2 + 16x^2 + 8xy + y^2 = 25x^2 - 10xy + y^2$

$17x^2 + 8xy = 25x^2 - 10xy$

$18xy = 8x^2$

$18y = 8x \quad (x \neq 0)$

$\frac{18}{8} = \frac{x}{y} \Rightarrow \frac{x}{y} = \frac{9}{4}$

$A + B + CD = 36 + 30 + 3264\left(\frac{9}{4}\right)$

7410

④ $6\#8 = 6+8 = 14$ | $4*5 = (4)(5)+1 = 21$

$14\#21 = 14+21 = 35$ | $5*35 = (5)(35)+1 = 176$

$176 = 2^4 \cdot 11^1 \Rightarrow 5(2) = 10$ factors **10**

⑤ $|4+5x| = 3+2x \Rightarrow 4+5x = 3+2x \Rightarrow x = -1/3$

or $4+5x = -3-2x \Rightarrow x = -1$

$(5^3)^{y+2} = (5^{-1})^{3-y} \Rightarrow 3y + 6 = -3 + y \Rightarrow y = \frac{-9}{2}$

sum = **$-35/6$**

⑥ A: ${}_{10}C_2 - 10 = 45 - 10 = 35$

B: $\frac{7!}{3!2!} = 420$

C: $8 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 = 1152$

D: ${}_{20}C_3 = \frac{20 \cdot 19 \cdot 18}{3 \cdot 2 \cdot 1} = 1140$

$D - C + \frac{B}{A} =$

$1140 - 1152 + \frac{420}{35}$

0

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(7) A: $\frac{|5-x|}{3} < 2$

$-6 < 5-x < 6$

$-11 < -x < 1$

$11 > x > -1$

sum $= 0+1+\dots+10 = 55$

B: $(2-3)^{2003} = -1$

C: $\frac{10}{2}[2t_1+9d] = 4\left(\frac{5}{2}[2t_1+4d]\right)$

$10t_1+45d = 20t_1+40d$

$5d = 10t_1$

$\frac{5}{10} = \frac{t_1}{d} \Rightarrow \frac{1}{2}$

D: $4(-1)^{2003} - 3(-1)^{200} + 4(-1)^3 - 2003$

$-4-3-4-2003 = -2014$

$AB + CD = (55)(-1) + \left(\frac{1}{2}\right)(-2014)$

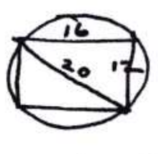
-1062

(8) A: diameter of sphere = edge of cube

$6e^2 = 96 \Rightarrow e = 4$

$r = 2 \Rightarrow V = \frac{4}{3}\pi(2)^3 = \frac{32\pi}{3}$

B:



$r = 10$

$A = 100\pi$

C: $2^6 = 64$ | $\frac{BC}{A} = \frac{100\pi(64)}{\frac{32\pi}{3}} = 600$

(9) A: $d = \sqrt{(-16+4)^2+(3+2)^2} = 13$ | $\frac{d}{m} = \frac{-156}{5}$

$m = -\frac{5}{12}$

B: $80(.12) - x(0) = (80-x)(.2)$

$x = 32$

C: $2(2)^4 - (2k)^2 + 12k - 41 = 0$

$-4k^2 + 12k - 9 = 0$

$4k^2 - 12k + 9 = 0$

$(2k-3)^2 = 0$ | $k = \frac{3}{2}$

D: $\frac{2\log 5}{\log 7} \cdot \frac{-\log 7}{\log 2} \cdot \frac{9\log 2}{\log 3} \cdot \frac{\frac{3}{2}\log 3}{\log 5} = -27$

$\frac{5A}{C} + B + D = \frac{-156}{\frac{3}{2}} + 32 + (-27) = -99$

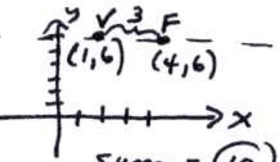
(10) A: $3(x^2+2x+1)+3(y^2-4y+4) = 5+3+12$

$3(x+1)^2 + 3(y-2)^2 = 20 \Rightarrow r = \sqrt{\frac{20}{3}}$

B: $x = \frac{1}{12}y^2 - y + 4 \Rightarrow 4p = 12 \Rightarrow p = 3$

$y_{vertex} = \frac{1}{\frac{1}{6}} = 6$

$x_{vertex} = \frac{1}{12}(6)^2 - 6 + 4 = 1$



sum $= 10$

C: $9(x^2+8x+16) + 25(y^2+2y+1) = 56 + 144 + 25$

$\frac{(x+4)^2}{25} + \frac{(y+1)^2}{9} = 1$

$c^2 = 16 \Rightarrow 2c = 8$

D: $9(x^2-4x+4) - 4(y^2-6y+9) = 36 + 36 - 36$

$\frac{(x-2)^2}{4} - \frac{(y-3)^2}{9} = 1$

slopes $= \frac{3}{2}, -\frac{3}{2} \Rightarrow$ product $= -\frac{9}{4}$

$\frac{A^2CD}{B} = \frac{\left(\frac{20}{3}\right)(8)\left(-\frac{9}{4}\right)}{10} = -12$

(11) A: $R = -\frac{1}{3} \Rightarrow S = \frac{1}{1-(-\frac{1}{3})} = \frac{3}{4}$

B: $1+2+3+\dots+20 = \frac{20(21)}{2} = 210$

$4+8+12+\dots+80 = \frac{20}{2}(4+80) = 840$

$210 + 840 = 1050$

C: $x^2 + 4x - 3 = 0$

$4^2 - 4(1)(-3) = 28$

D: product is a 3×7 matrix $\Rightarrow 21$

$\frac{B}{A} + C + D = \frac{1050}{\frac{3}{4}} + 28 + 21 = 1449$

(12) A: $(\log_3 3^{2x} + \log 10^y)^{10} = (2x+y)^{10} \Rightarrow \frac{10!}{6!4!}(2x)^4(y)^6$

$210(16) = 3360$

B: $\frac{2\log 15}{3\log 2} \cdot \frac{\log 2}{\log 15} = \frac{2}{3}$

C: $\frac{2^x(2^5-2^1)}{2^x(2^1)(2^{-4})} = \frac{30}{\frac{1}{8}} = 240$

$AB + \frac{C}{D}$

$(3360)\left(\frac{2}{3}\right) + \frac{240}{5}$

2288

D: $(3+\sqrt{8}) - (\sqrt{8}+\sqrt{7}) + (\sqrt{7}+\sqrt{6}) - (\sqrt{6}+\sqrt{5}) + (\sqrt{5}+2) = 5$

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(13)

$$\begin{vmatrix} x & 1 & -2 \\ -2 & x+1 & 1 \\ -1 & 3 & x-6 \end{vmatrix} = 0$$

$$x[(x+1)(x-6)-3]-1[-2(x-6)-(-1)]-2[-6-(-1)(x+1)]=0$$

$$x[x^2-5x-6-3]-1[-2x+12+1]-2[-6+x+1]=0$$

$$x^3-5x^2-9x+2x-13+10-2x=0$$

$$x^3-5x^2-9x-3=0$$

-1	1	-5	-9	-3
		-1	6	3
	1	-6	-3	0

$$(x+1)(x^2-6x-3)=0$$

⇓ discriminant > 0

All values of x are real, so product = $\boxed{3}$

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(14)

	R	T	D
orig rate	R	20/R	20
new rate	R+2	20/(R+2)	20

$$3R(R+2)\left[\frac{20}{R+2} = \frac{20}{R} - \frac{1}{3}\right]3R(R+2)$$

$$60R = 60(R+2) - R(R+2)$$

$$60R = 60R + 120 - R^2 - 2R$$

$$R^2 + 2R - 120 = 0$$

$$(R+12)(R-10) = 0$$

$R=-12$ (reject) or $R=10 \Rightarrow \frac{20}{R} = \boxed{2}$

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(15)

$$t_1 = 4\left(\frac{t_2}{1-R}\right)$$

$$R = \frac{t_2}{t_1} \Rightarrow t_2 = t_1 R$$

$$t_1 = \frac{4t_1 R}{1-R}$$

$$t_1 - t_1 R = 4t_1 R$$

$$t_1 = 5t_1 R$$

$$\frac{1}{5} = R \quad (t_1 \neq 0)$$

$$\therefore \frac{t_2}{t_1} = \boxed{\frac{1}{5}}$$