ALPHA APPLICATIONS
FAMAT State Convention 2004

For all questions, E. NOTA means none of the above answers is correct.

1. What is the period of $y = \frac{\sin(2x)}{3}$?

   A. $\frac{1}{\pi}$ B. $\frac{2\pi}{3}$ C. π D. 2π E. NOTA

2. What is the period of $y = \frac{\sin^{-1}(2x)}{3}$?

   A. $\frac{1}{\pi}$ B. $\frac{2\pi}{3}$ C. π D. 2π E. NOTA

3. In her Algebra II class, Mrs. Singleton writes a quadratic equation on the board for the class to solve in the form $x^2 + bx + c = 0$. Andy miscopies the value of b and arrives at the correct (for his miscopied version of the problem) solution $\{3, 4\}$. Westerdale miscopies the value of c and arrives at the correct (for his miscopied version of the problem) solution $\{4 \pm 2\sqrt{3}\}$. Christine correctly solves the quadratic, what is her solution?

   A. $\{-4, -4\}$ B. $\{4, 4\}$ C. $\{-2, -6\}$ D. $\{2, 6\}$ E. NOTA

4. A train leaves Orlando at 1:00 PM traveling at 150 miles per hour towards Huntsville, which is 600 miles away. A second train leaves Huntsville at 2:00 PM traveling at 100 miles per hour towards Orlando, on the same track as the first train. At what time do the trains collide?

   A. 3:00 PM B. 3:24 PM C. 3:48 PM D. 4:24 PM E. NOTA

5. Mr. Bantz begins walking to his classroom from the teacher parking lot, which is 100 feet away (in a perfectly straight line). Mr. Bantz walks at a rate of 5 feet per second. Right as Mr. Bantz begins his walk, a fly leaves his room and flies back and forth between Mr. Bantz and the classroom at 10 feet per second. Whenever the fly reaches either Mr. Bantz or the classroom, it immediately turns around and goes at 10 feet per second in the opposite direction. The fly continues in this fashion until Mr. Bantz squashes the fly at the door. How many feet does the fly travel in total?

   A. 100 B. 200 C. 400 D. Infinitely many E. NOTA

6. Factor the expression $a^2 + 2ab + b^2$.

   A. $(a+b)^2$ B. $(a-b)^2$ C. $(a+b)(a-b)$ D. Already factored E. NOTA

7. The Smiths have 5 children. Given that the first 3 are all boys, what is the probability that both of the other children are girls (assume that each child has a 50% chance of being a boy and a 50% chance of being a girl).

   A. $\frac{1}{2}$ B. $\frac{5}{16}$ C. $\frac{1}{4}$ D. $\frac{11}{16}$ E. NOTA

8. Factor the expression $a^2 - b^2$.

A. $(a+b)^2$ B. $(a-b)^2$ C. $(a+b)(a-b)$ D. Already factored E. NOTA

9. 15 identical snacks are to be placed in Danella's, Arun's, Abe's, and Paula's lunchboxes. If each child's lunchbox receives at least one snack (we don't want to let any of them starve!), how many such arrangements are possible?

A. 364 B. 816 C. 1365 D. 3876 E. NOTA

10. Jon has a 23-pound rubber band ball. When dropped from a height of h feet, the ball rebounds $\frac{3h}{5}$ feet when it bounces. If the ball is dropped from a height of 50 feet, how far does it travel in total before coming to rest?

A. 80 feet B. 125 feet C. 200 feet D. 250 feet E. NOTA

11. What is the sum of the slopes of the asymptotes of the graph of the equation $7x^2 - 5y^2 + 23x + y - 105 = 0$?

A. -1 B. 0 C. 1 D. It has no asymptotes E. NOTA

12. For $a, b, c, d, e > 0$, what is the greatest number of real roots that the following equation could have? $ax^6 + bx^4 - cx^3 - dx^2 + ex + 1 = 1$

A. 6 B. 4 C. 3 D. 2 E. NOTA

13. 2^{678} is congruent to which of the following?

A. $0 \bmod 7$ B. $1 \bmod 7$ C. $2 \bmod 7$ D. $4 \bmod 7$ E. NOTA

14. Mauro Braunstein is standing at the origin of the Cartesian coordinate plane. He needs to go to his house, which is located at $(-6,16)$, but first he wants to go to the beach, which is located at $x = 6$. What is the shortest distance he can travel to accomplish this?

A. $2\sqrt{73}$ B. $2\sqrt{82}$ C. 20 D. $12 + 2\sqrt{73}$ E. NOTA

15. Find the shortest distance between the point $(2,4,0)$, and the plane containing the points: $(5,0,1)$, $(-2,3,1)$, and $(3,9,-2)$. Round to the nearest integer.

A. 1 B. 2 C. 3 D. 4 E. NOTA

16. How many digits are in 2004^{2004}?

A. 2004 B. 2005 C. 6617 D. 6618 E. NOTA

17. What is the smallest positive angle of intersection of the vectors $\hat{i}+2\hat{j}$ and $2\hat{i}+4\hat{k}$? (Round to the nearest tenth of a radian)

A. 0.0 B. .2 C. 1.0 D. 1.4 E. NOTA

18. What is the eccentricity of the graph of $r^2=\dfrac{64}{4\sin^2\theta-16\cos^2\theta}$?

A. 0 B. $\dfrac{\sqrt{3}}{2}$ C. $\dfrac{\sqrt{5}}{2}$ D. 4 E. NOTA

19. What is (3141592664)(3141592644)-(3141592674)(3141592634)?

A. 300 B. 100 C. 50 D. 0 E. NOTA

20. If n is a natural number, then what is the smallest positive n such that $n!$ ends with at least 499 zeros?

A. 2004 B. 2005 C. 2009 D. 2495 E. NOTA

21. Given $M=\begin{bmatrix}7&13&12&6\\-2&0&1&3\\-1&4&8&4\\8&2&4&2\end{bmatrix}$ (I assure you that M is invertible) and that $M\times M^{-1}=\begin{bmatrix}a&b&c&d\\e&f&g&h\\i&j&k&l\\m&n&o&p\end{bmatrix}$, what is the value of: $(g+o+o+d)+(b+a+d)$?

A. 0 B. 1 C. 6 D. not enough info. E. NOTA

22. Simplify: $\sqrt{2070+\sqrt{2070+\sqrt{2070+\sqrt{2070+\sqrt{2070+\ldots}}}}}$

A. ∞ B. 45 C. 46 D. 2004 E. NOTA

23. If Danny randomly picks a choice (A, B, C, D, or E) for all 30 questions on this test, what is his expected score?

A. –30 B. 0 C. 30 D. cannot be determined E. NOTA

24. How many of the graphs of the following equations are two-dimensional circles?

$x^2 + y^2 = 1$

$x^2 + y^2 = 0$

$x^2 + y^2 = -1$

$x^2 - y^2 = 1$

$x^2 - y^2 = 0$

$x^2 + 2y^2 = 1$

$x^2 + y^2 = 2$

A. 2 B. 3 C. 4 D. 5 E. NOTA

25. What is the sum of the integral values of x which satisfy: $\left(x^2 + 5x + 5\right)^{\left(x^2 - 3x - 10\right)} = 1$

A. 3 B. –2 C. –5 D. –7 E. NOTA

26. Which of the following is equivalent to $\frac{1}{2}$?

A. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{vmatrix}$ B. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ \frac{1}{2} & 0 & 2 & -\frac{1}{2} \\ 2 & 4 & 6 & 8 \\ 0 & 1 & 2 & 1 \end{vmatrix}$ C. $\ln\left(e^{\pi}\right)$ D. $\sin(30°)$ E. NOTA

27. Evaluate the determinant of the following (hint: use row-reduction methods):

$$\begin{vmatrix} 8 & 4 & 9 & 18 & 3 \\ 4 & 3 & 6 & 12 & 2 \\ 6 & 4 & 12 & 18 & 3 \\ 2 & 1 & 3 & 6 & 1 \\ 8 & 4 & 9 & 18 & 4 \end{vmatrix}$$

A. 0 B. 1 C. 16 D. 36 E. NOTA

28. A boat, starting at the origin, travels at 20 miles per hour with a bearing of $060°$. After 30 minutes, the boat turns due south for another 30 minutes at a speed of 10 miles per hour. What is the boat's final bearing?

A. 060° B. 090° C. 120° D. 180° E. NOTA

29. Evaluate the infinite series: $\frac{1}{3} + \frac{2}{9} + \frac{3}{27} + \frac{4}{81} + \ldots + \frac{n}{3^n} + \ldots$

A. $\frac{1}{2}$ B. $\frac{2}{3}$ C. D. E. NOTA

30. Solve: $x^2 = 49$

A. {7} B. {–7} C. {ø} D. $\{\log_{\pi} \pi^e\}$ E. NOTA