## Alpha Bowl 2004 Answers & Solutions: Page 1 of 2

Question # 1:  $f(x) = 128 x^{15} - 2x^{13} - 2x^{10} + 3x^8 - x^6 - x^5 + x^3 - 5x^2 + 120$ A)  $\frac{\pm factors of 120}{\pm factors of 128} = 0$ : B) # of sign changes of f(x) = 6: C) # of sign changes in f(-x) is 7, therefore 7, 5, 3, or 1 possible negative, least # = 1: D) Based on B, 6, 4, 2, or 0, therefore 0 is one of the possible choices, so False = 2: E) sum  $= \frac{-b}{a} = \frac{0}{128} = 0$ : F) odd function so  $\frac{-d}{a} = \frac{-120}{128} = \frac{-15}{16}$ . Sum of A+B+C+D+E+F =  $\left< 8.0625 \text{ or } \frac{129}{16} \right>$ Question # 2: A)  $PE = \sqrt{119}$ :  $Area = \frac{1}{2}(5)(\sqrt{119}) \approx 27.2718$ : B)  $\frac{9\sin 79^{0}}{12} = \frac{\sin 0}{9}$ :  $O = \sin^{-1} \left( \frac{9\sin 79^{0}}{12} \right)$ ,  $N = 180^{0} - 79^{0} - O$ :  $Area = \frac{1}{2}(9)(12)(\sin N) \approx 39.7559$ C)  $s = \frac{18 + 11 + 15}{2}$ :  $area = \sqrt{s(s - 18)(s - 15)(s - 11)} \approx 82.3165$ D)  $abs \left( \frac{1}{2} \begin{vmatrix} 2 & 5 & 1 \\ -9 & 3 & 1 \\ 4 & 2004 & 1 \end{vmatrix} \right) = -10992.5$ :  $A + B + C + D \approx \langle 11145.547 \rangle$ Question # 3: case 1:  $\sin\left(\frac{\pi}{2}\right) + \log_{5} 125 + \ln e^{15} - \log_{2} 8 + \log_{x} x^{3}$ . 1 + 3 + 15 - 3 + 3 = 19 = 8

case 2: The smallest factor, other than 1 is 11 = K: case 3:  $\cos(105^\circ) = \frac{\sqrt{2} - \sqrt{6}}{4} = 4 + 1 = 5 = E$ case 4:  $(x-2)^2 + (y+3)^2 = 6^2 = 4(6) - 1 = 23 = W$ :  $\langle SKEW \rangle$ 

 $\frac{\sin 5^{\circ} \sin 10^{\circ} \sin 15^{\circ} \sin 20^{\circ} \sin 30^{\circ} \sin 45^{\circ} \sin 60^{\circ} \sin 70^{\circ} \sin 75^{\circ} \sin 80^{\circ} \sin 85^{\circ}}{\cos 5^{\circ} \cos 5^{\circ} \cos 10^{\circ} \cos 15^{\circ} \cos 20 \cos 30^{\circ} \cos 45^{\circ} \cos 60^{\circ} \cos 70^{\circ} \cos 75^{\circ} \cos 80^{\circ} \cos 85^{\circ}}$ Looking carefully everything cancels to 1 except:  $\frac{\sin 70^{\circ}}{\cos 20 \ radians} \approx \frac{.9397}{.4081} \approx 2.303$ 

Question # 5: consecutive angle EHG =  $60^{\circ}$ : Similar triangles:  $\frac{20}{27} = \frac{16}{HG}$ : HG = 21.6: Split of two equal  $\Delta^{s}$ , each with area:  $\frac{1}{2}(27)(21.6)\sin 60^{\circ} = times(2) = 583.2\frac{\sqrt{3}}{2} = \left\langle 291.6\sqrt{3} \text{ or } \frac{1458\sqrt{3}}{5} \right\rangle$ Question # 6:  $(_{2004}C_{2003})(2004)^{1}(-y)^{2003} = -4016016$ :  $2\left[\frac{20(20+1)(40+1)}{6} + 20(-4016016)\right]$  $2(-80,317,450) = \left\langle -160,634,900 \right\rangle$ 

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**Question #7:** Only  $\langle \mathbf{R} \& \mathbf{L} \rangle$  are true. F(even degree has nothing to do with the signs of the solutions), M(exactly once implies that it cannot cross more than once), A(less than once implies that it will never cross), I(a polynomial with an even degree does imply that every exponent is even, therefore it is not necessarily an even function), N(a polynomial with an odd degree does not imply that every exponent is odd, therefore it is not necessarily an odd function, which would be symmetric to the origin), S(odd functions would approach positive infinity in one direction and negative infinity in the other)

Question # 8: From the foci points of (-2, 5), (-2, 11) we know that c = 3, the eccentricity is between zero and one tells us that this is an ellipse and the area will be  $ab\pi$ , minor axis with length of 12 = 2b, b = 6.  $a^2 = b^2 + c^2$ :  $a^2 = 36 + 9 = 45$ ;  $a = 3\sqrt{5}$ ;  $(3\sqrt{5})(6)(\pi) = \frac{118\sqrt{5}\pi}{2}$ 

$$a^{-} = 36 + 9 = 45: a = 3\sqrt{5}: (3\sqrt{5})(6)(\pi), \quad \langle 18\sqrt{5\pi} \rangle$$
  
Question # 9:  $A = \lim_{x \to 4} \left( \frac{(x-4)(x+6)}{(x-4)} \right) = 10: B) \quad \lim_{x \to 3} (5f(x)) = (5)(8) = 40$   
C)  $\lim_{x \to \pi} (f(x)), \text{ if } f(x) = 12, \text{ constant} = 12: D) = \lim_{x \to -3} \left( \frac{(x+3)(x^{2}-3x+9)}{x+3} \right) = 27, \text{ A+B+C+D} = \langle 89 \rangle$   
Question # 10:  $\begin{vmatrix} 1 & -6 & 5 \\ 3 & -2 & 1 \\ x & y & z \end{vmatrix} = (4,14,16): (1,-6,5) \cdot (4,14,16) = 2: \sqrt{8^{2}+28^{2}+32^{2}} = \langle 12\sqrt{13} \rangle$   
Question # 11: A)  $-3 - 9(2003) = -18030: B) \quad \begin{vmatrix} 888-64 \\ 4 \end{vmatrix} + 1 = 207: C) \quad \frac{90}{1-\frac{1}{3}} = 135: \text{ sum} = \langle -17688 \rangle$ 

Question # 12: I) 
$$\begin{bmatrix} 7 & -11 \\ 3 & 6 \\ 2 & -12 \end{bmatrix} = 6: II) \begin{bmatrix} -38 & 26 \\ 24 & -88 \\ -14 & 38 \end{bmatrix} = -88: III) \begin{bmatrix} -280 & 360 \\ -240 & 80 \end{bmatrix} = 80: IV)$$
 Expand over minors = -  
4: V) -12024: VI) 36: 6 - 88 + 80 - 4 - 12024 + 36 =  $\langle -11994 \rangle$ 

Question # 13:  $(x-2-3i)(x-2+3i) = (x^2-4x+13) : (x-2i)(x+2i) = (x^2+4) : (x-2)(x+2) = (x^2-16) : (x^2-4x+13)(x^2+4)(x^2-16) = \langle x^6-4x^5+x^4+48x^3-220x^2+256x-832 \rangle$ 

Question # 14: Distance between the foci is 52, so c = 26.  $e = \frac{c}{a} = \frac{13}{x} = \frac{26}{2x}$ : From  $9^{x+1} = 8^{2x}$ :  $(x+1)\ln 9 = 2x\ln 8$ :  $x = \frac{-\ln 9}{\ln 9 - 2\ln 8}$ :  $c^2 = a^2 + b^2$ :  $26^2 = (2x)^2 + b^2$ :  $2b \approx \langle 51.914 \rangle$ 

Question # 15: M) 
$$\frac{2003!(5!)}{2004!} = \frac{10}{167} = 10$$
: A)  $\frac{11!}{(3!)(6!)(2!)} = 4620$ : T)  $\frac{6}{23} \cdot \frac{5}{22} \cdot \frac{9}{21} \cdot \frac{8}{20} \cdot \frac{7}{19} = \frac{18}{4807} = 10$   
H)  $\left(\frac{1}{2}\right)^8 = \frac{1}{256}$ :  $(10 + 4620 + 18 + 1) = \langle 4649 \rangle$ 

## Alpha Bowl 2004 Team Answers ONLY

1)	$\frac{129}{16}$ or 8.0625	5	
2)	11145.547		
3)	SKEW		
4)	2.303		
5)	$\frac{1458\sqrt{3}}{5}$		
6)	-160,634,900		
7)	R & L		
8)	$18\pi\sqrt{5}$		
9)	89		
10)	$12\sqrt{13}$		
11)	) -17,688		
12)	) -11994		
	6 . 5	Λ	 3

- 13)  $x^{6} 4x^{5} + x^{4} + 48x^{3} 220x^{2} + 256x 832$
- 14) 51.914
- 15) 4649