ALPHA ANALYTIC GEOMETRY 2004
SOLUTIONS

1) **A** Applying the distance formula from a general point (x, y) to the two points mentioned:
$\sqrt{(x-1)^2+(y-2)^2}=\sqrt{(x-3)^2+(y-7)^2}$
Solving, we get $4x+10y-53=0$

2) **B**

Conic section	Eccentricity
Circle	$e=0$
Ellipse	$0<e<1$
Parabola	$e=1$
Hyperbola	$e>1$

3) **B** The three cube roots of 1 can be found by:

$z_1=1\cdot cis(360/3)=\frac{-1}{2}+\frac{\sqrt{3}}{2}i$

$z_2=1\cdot cis(2\cdot 360/3)=\frac{-1}{2}-\frac{\sqrt{3}}{2}i$, and $z_3=1$, which is a triangle with base $\sqrt{3}$ and height 3/2.

4) **A** Using Green's theorem, the area can be found by:
$A=(x_1y_2+x_2y_3+\ldots+x_{n-1}y_n+x_ny_1$
$-y_1x_2-y_2x_3-\ldots-y_{n-1}x_n-y_nx_1)$

5) **D** The centroid is (x,y), where x is the average of the x-coordinates, and y is the average of the y-coordinates. So, $(x,y)=(-1/7,13/7)$. The distance from this point to the line $7x-7y-70=0$ can be found by $D=\frac{\left|7\cdot\frac{-1}{7}-7\cdot\frac{13}{7}-70\right|}{\sqrt{7^2+7^2}}=6\sqrt{2}$. Applying the formula given, $V=2\pi\frac{131}{2}\cdot 6\sqrt{2}=786\pi\sqrt{2}$

6) **B** Using the formula $\text{proj}_\text{v}\text{u}=\left(\frac{\mathbf{u}\bullet\mathbf{v}}{\|\text{v}\|^2}\right)\mathbf{v}$, we get

$\text{proj}_\text{v}\text{u}=\left(\frac{50}{100}\right)\langle 10,0\rangle=\langle 5,0\rangle$

7) **A** Setting $y=0$ in the equation to solve for the roots we get $27x^3=1$ or $(3x)^3=1$, so $3x$ is a cube root of 1 and has the form $3x=\frac{-1}{2}\pm\frac{\sqrt{3}}{2}i$

, and therefore $|x|=\frac{1}{3}\left|\frac{-1}{2}\pm\frac{\sqrt{3}}{2}i\right|=\frac{1}{3}$

8) **A** This problem is describing a parabola with directrix at $y=1/2$ and focus at $(3,4)$. The vertex is at $\left(3,\frac{9}{4}\right)$ and $p=\frac{\left(4-\frac{1}{2}\right)}{2}=\frac{7}{4}$. The parabola can now be written from the standard equation
$(x-h)^2=4p(y-k)\Rightarrow(x-3)^2=7\left(y-\frac{9}{4}\right)$

9) **C** The standard equation for a circle with center (h,k) and radius, r, is : $(x-h)^2+(y-k)^2=r^2$.

10) **A** This volume can be found by summing the volumes of the double-napped cone below and above the z-axis. Above the z-axis, the cone's base is at $x^2+y^2=9^2=81$ (a cone with base area = 81π), and below the z-axis, the cone's base is at $x^2+y^2=(-6)^2=36$ (with base area = 36π). So the sum of the volumes of the cones
$(V=\frac{bh}{3}=\frac{81\pi\cdot 9}{3}+\frac{36\pi\cdot 6}{3}=315\pi$

11) **B** The three points of intersection can be found at $\theta = 0°, 90°, 270°$. The point $(-3, 180°)$ is coincident with the point $(3, 0°)$ and should not be counted twice.

12) **C** The figure described is an ellipse with major axis equal to 5 (when the dog stretches the rope to be exactly 4 units away from each stake, making an isosceles triangle with them). and minor axis equal to 4 (when the dog is on the same line as the stakes, one unit to the outside of them) The area of the ellipse (yard) that he covers is therefore $A = 20\pi$

13) **C** The equation can be factored as follows:
$y = \dfrac{(\sin x + \cos x)^2}{(\sin x + \cos x)(\sin x - \cos x)}$. , so we have discontinuities whenever $\sin x + \cos x = 0$ or $\sin x - \cos x = 0$, or whenever $x = n \pm \dfrac{\pi}{4}$.

14) **D** Applying the distance formula to a general point, (x, y), we get $\dfrac{|4x - y|}{\sqrt{4^2 + (-1)^2}} = 2 \cdot \dfrac{|-x + 4y|}{\sqrt{4^2 + (-1)^2}}$, and solving, we get $|4x - y| = 2|4y - x|$. This yields 4 equations, $6x = 9y, -2x = 7y, 2x = -7y, -6x = -9y$, which reduces to $y = \dfrac{2}{3}x$ and $y = \dfrac{-2}{7}x$.

15) **E** Two of these vectors lie on the same line and therefore cannot form a rectangular prism.

16) **B** The shortest distance from the center (-2,3) to the line $x - y = 7$ is found by
$D = \dfrac{|1 \cdot (-2) + (-1) \cdot 3 - 7|}{\sqrt{1^2 + (-1)^2}} = 6\sqrt{2} = r$. So our circle can be expressed as: $(x+2)^2 + (y-3)^2 = \left(6\sqrt{2}\right)^2$, or $x^2 + y^2 + 4x - 6y - 59 = 0$.

17) **D** From the standard definition of a parabola.

18) **E** The equation for this line, going through the origin, is $\theta = \dfrac{2\pi}{3}$ or $\theta = \dfrac{-\pi}{3}$.

19) **B** Any two diagonals of a cube have length $s\sqrt{3}$, where *s* is the length of the side. The triangles that are formed by the intersection of the diagonals all have sides $\dfrac{s\sqrt{3}}{2}, \dfrac{s\sqrt{3}}{2}, s$ or $\dfrac{s\sqrt{3}}{2}, \dfrac{s\sqrt{3}}{2}, s\sqrt{2}$. The first produces an inner angle of $2\sin^{-1}\left(\dfrac{1}{\sqrt{3}}\right) \approx 71$, and the second yields $2\sin^{-1}\left(\sqrt{\dfrac{2}{3}}\right) \approx 109$.

20) **C** The two equations to be solved simultaneously are $(R^2 - r^2)\pi = 10\pi$ and $(R + r) = 5$. Substituting, we get $(R - r) \cdot 5\pi = 10\pi$, and the two equations yield $R = 3.5, r = 1.5$.

21) **A** The cosine can be found using the formula: $\cos\theta = \frac{\mathbf{u} \bullet \mathbf{v}}{\lVert\mathbf{u}\rVert \cdot \lVert\mathbf{v}\rVert} = \frac{-2\cdot 1 + 3\cdot 7}{\sqrt{(-2)^2+3^2}\cdot\sqrt{1^2+7^2}} = \frac{19\sqrt{26}}{130}$	26) **D** Given two points $\langle r_1, \theta_1\rangle$ and $\langle r_2, \theta_2\rangle$, the distance between them can be found by: $d = \sqrt{r_1^2 + r_x^2 - 2r_2 r_1 \cos(\theta_2 - \theta_1)}$. In our case: $d = \sqrt{2^2 + 5^2 - 2\cdot 5\cdot 2\cdot\cos(215° - 35°)} = \sqrt{49} = 7$
22) **B** The two graphs intersect at the points $(1,0)$ and $\left(\frac{1}{2}, \frac{-1}{2}\right)$, the line through which has a slope of 1.	27) **A** In general: $r = \frac{2ep}{1 \pm e\sin\theta}$ or $r = \frac{2ep}{1 \pm e\cos\theta}$., where e = eccentricity. Since, for a parabola, $e = 1$, b), c), and d) fail. But a) can be rewritten $r\sin\theta = 6\cot\theta$ and converting: $y = 6\left(\frac{x}{y}\right)$, or $y^2 = 6x$
23) **C** The equation can be rewritten as $(x + y - 3)(2x - 2y + 5) = 0$, which is graphed as a pair of lines.	28) **C** If a = major axis and b = minor axis, then $\frac{a}{b} = 3$ and $ab\pi = 48\pi$. Substituting, we get $a = 12$ and $b = 4$. The length of the latus rectum is $\frac{2b^2}{a} = \frac{2\cdot 4^2}{12} = \frac{8}{3}$
24) **C** The graph $f(x)$ closely resembles the function $y = \ln(x)$. Therefore $f^{-1}(x) = e^x$ and $f^{-1}(-x) = e^{-x}$ which closely resembles the graph in c)	29) **B** The equation of the line through the two points is $y = 5x - 13$. Choosing $x = \frac{t}{5}$, we get $y = t - 13$.
25) **A** The radius of a circle given the side lengths of the inscribed triangle is found by: $r = \frac{ABC}{4\cdot Area}$. The area of the isosceles triangle is found by $\frac{bh}{2}$, where $h = \sqrt{4^2 - 3^2} = \sqrt{7}$, and therefore $r = \frac{6\cdot 4\cdot 4}{4\cdot(3\cdot\sqrt{7})} = \frac{8}{\sqrt{7}}$.	30) **B** Plugging $a = 7$ and $b = 4$ into the equation given, we get $C \approx (11.2055)\pi$. A circle with the same circumference would have radius $r = \frac{C}{2\pi} \approx 5.6028$ and therefore have area $A = \pi r^2 \approx \pi(5.6028)^2 \approx 98.6$