

Complex Numbers (Alpha)

FAMAT State Convention 2004

Instructions

Choose the single correct answer to each question from the choices A–D. If no single correct answer exists for a particular question, choose E for None Of The Above (NOTA). On this test, $i = \sqrt{-1}$, and if $z = a + bi$, where a and b are real, then $\Re(z) = a$ (the real part) and $\Im(z) = b$ (the imaginary part). Also, $|z|$ represents the distance from z to the origin, and the complex conjugate of z is represented by \bar{z} .

1. If $\Re(z) = 4$ and $|z| = 9$, then which of the choices could be the value of $\Im(z)$?
A. $-\sqrt{65}$ B. -1 C. 5 D. 8 E. NOTA
2. Let $\frac{m}{n} = \left| \frac{6 + 8i}{-20 + 21i} \right|$, where m and n are positive integers with a greatest common divisor of 1. What does $m + n$ equal?
A. 34 B. 39 C. 941 D. 1175 E. NOTA
3. Which answer choice is equal to i^{2004} ?
A. $-i$ B. -1 C. 1 D. i E. NOTA
4. Simplify: $\frac{(2 + 3i)(4 + 5i)}{i}$.
A. $-7 - 22i$ B. $-7 + 22i$ C. $-22 + 7i$ D. $22 + 7i$ E. NOTA
5. 1 is one of the fifth roots of 1. What is the sum of the other fifth roots of 1?
A. -1 B. 0 C. 1 D. There are no other fifth roots of 1. E. NOTA
6. Assume n is a positive even integer. How many distinct n th roots of -1 are there?
A. 0 B. 1 C. $\frac{n}{2}$ D. $n - 1$ E. NOTA
7. What is the measure, to the nearest degree, of the smaller angle between the position vectors of $1 + 3i$ and $-3 - i$ in the complex plane?
A. 108° B. 127° C. 135° D. 150° E. NOTA

8. Which of the following answer choices is not a sixth root of 1?
- A. $-\frac{1}{2} + \frac{i\sqrt{3}}{2}$ B. $-i$ C. -1 D. $\frac{1}{2} + \frac{i\sqrt{3}}{2}$ E. NOTA
9. $r \operatorname{cis} \theta = 2 \operatorname{cis} 56^\circ + 3 \operatorname{cis} 249^\circ$. Which choice could be the value of r (to the nearest hundredth)?
- A. 0.04 B. 1.14 C. 3.61 D. 5.00 E. NOTA
10. How many members of the following set of five numbers are complex? The set: $\{27, \frac{2}{3}, 4\pi, 7i, 6 - 3i\}$.
- A. 1 B. 2 C. 3 D. 4 E. NOTA
11. $\left(\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}\right)^{2004} =$
- A. 1 B. -1 C. $\frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$ D. $-\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}$ E. NOTA
12. What is the shape of the graph of $z^2 + \bar{z}^2 = 2$ in the complex plane?
- A. a parabola B. a circle C. an ellipse (but not a circle) D. a hyperbola E. NOTA
13. $r \operatorname{cis} \theta = 7 - 6i$. Which choice could be θ (to the nearest hundredth)? Assume r and θ are both real.
- A. -0.86 B. 0.71 C. 2.43 D. 9.22 E. NOTA
14. $(\cos 30^\circ + i \sin 30^\circ)(e^{11\pi i/6}) =$
- A. -1 B. $-\frac{1}{2}$ C. $\frac{1}{2} - \frac{\sqrt{3}}{2}$ D. 1 E. NOTA
15. Which answer choice could not be the fourth vertex of a parallelogram, in the complex plane, with the points $3 + i$, $1 - 2i$, and $-2 + 4i$ as the other three vertices?
- A. $-4 + i$ B. $7i$ C. $8 - 2i$ D. $6 - 5i$ E. NOTA
16. Simplify: $\frac{1}{i} + \frac{1}{i^2} + \frac{1}{i^3} + \frac{1}{i^4}$.
- A. $-i$ B. -1 C. 1 D. i E. NOTA
17. What is the shape of the graph of $|z - 1| + |z + i| = 2$ in the complex plane?
- A. two points B. a line C. a parabola D. an ellipse E. NOTA

18. If $z = a + bi$, where $a = -2$ and $b = 7$, then which of the answer choices is equal to $z\bar{z}$?
- A. $-45 - 28i$ B. -45 C. 45 D. $45 + 28i$ E. NOTA
19. Which answer choice is a value for $\ln(-i)$?
- A. $\frac{\pi i}{2}$ B. πi C. $\frac{3\pi i}{2}$ D. $2\pi i$ E. NOTA
20. If you connect the following six points in the complex plane in order (and then connect the last one to the first), what type of polygon is formed? The points: $4, 3 - i, -1 - 2i, -5 - 3i, -1 - i, 3 + i$.
- A. line segment (degenerate polygon) B. triangle C. quadrilateral D. pentagon E. NOTA
21. Simplify: $\sqrt{-5} \times \sqrt{-28} \times \sqrt{-35}$.
- A. $-70i$ B. -70 C. 70 D. $70i$ E. NOTA
22. What is the value of $m + n \div p$ given that $m = 1 + 2i$, $n = 2 - 3i$, and $p = -3 + 4i$?
- A. $-\frac{13}{25} - \frac{9i}{25}$ B. $\frac{7}{25} + \frac{51i}{25}$ C. $\frac{19}{25} + \frac{33i}{25}$ D. $\frac{31}{25} + \frac{33i}{25}$ E. NOTA
23. How many negative real roots does the polynomial $p(z) = 573z^4 - 892z^3 + 2z^2 + 573z - 89$ have?
- A. 0 B. 1 C. 2 D. 3 E. NOTA
24. If $f(z) = z^2 + 4iz - 4$ and $g(z) = \bar{z}$, then what is the value of $f(g(3 + 2i))$?
- A. $-21 + 20i$ B. $-7 - 24i$ C. $-7 + 24i$ D. 17 E. NOTA
25. Let \mathbb{C} , \mathbb{Q} , \mathbb{R} , and \mathbb{Z} denote the complex numbers, the rational numbers, the real numbers, and the integers, respectively. Which of the following statements are true? (The notation $\{a : b\}$ means the set of elements of the form a having the property b .)
- I. \mathbb{C} is equivalent to $\{x + yi : x, y \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$.
- II. \mathbb{C} is equivalent to $\mathbb{R} \cup \{yi : y \in \mathbb{R} \text{ and } i = \sqrt{-1}\}$.
- III. $\mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.
- A. I & II only B. II & III only C. I & III only D. I, II, & III E. NOTA
26. Which choice is the complex conjugate of $r \operatorname{cis} \theta$? Assume r and θ are both nonzero real numbers.
- A. $-r \operatorname{cis} \theta$ B. $-\frac{1}{r} \operatorname{cis} \theta$ C. $\frac{1}{r} \operatorname{cis}(-\theta)$ D. $r \operatorname{cis}(-\theta)$ E. NOTA

27. Solve for n : $(1 - i)^n = 4096$.
A. 8 B. 12 C. 16 D. 24 E. NOTA
28. If $1 - i$ is one of the fourth roots of the complex number z , then what is the value of z^3 ?
A. -64 B. -4 C. $-2 - 2i$ D. $-2 + 2i$ E. NOTA
29. Which answer choice describes the product of any pair of imaginary numbers?
A. real and not negative B. real and not positive C. imaginary D. not complex E. NOTA
30. What is the area of the triangle formed by the following three points in the complex plane? The points: $-2i$, $4 + 3i$, $-4 + 3i$.
A. 12 B. 20 C. 24 D. 40 E. NOTA

THIS IS THE END OF THE TEST.